PC481 Lab Manual
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[1]Much of this information is taken from OptoSci documentation
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Chapter 1

Power Measurements

1.1 Purpose

The purpose of the this exercise is to familiarize yourself with fibre optic measurements and measurement equipment.

1.2 Introduction

This exercise is intended to introduce basic concepts of measurement related to optical fibre networks.

1.3 Theory

$$P(dBm) = 10\log_{10}\left(\frac{P_s}{1mW}\right) \tag{1.1}$$

$$P(dB) = 10\log_{10}\left(\frac{P_A}{P_B}\right) \tag{1.2}$$

1.4 Procedure

1.4.1 Experimentation

Apparatus

- various meters
- various single mode cables
- various fibre light sources

Method

PROTECT EYES!!!!

- always keep sources capped unless in use
- never point at eyes (yours or anyone else's!)

PROTECT Equipment

- most pieces few to 10's of thousand dollars. (even used!)
- take your time
- don't move equipment unless absolutely necessary

Power is measured in three ways:

1. absolute, in **Watts**
2. relative, in **dB** (See Equation 1.2.)
3. absolute, in **dBm** (See Equation 1.1.)

This exercise will cover the following concepts:

1. Conversion between power units:
   - dBm to W
   - W to dBm

   Note that difference in dBm = difference in dB

2. Comparing sources:

   Which is most dangerous?

3. Comparing meters:

   How consistent are they?

In-lab Tasks

IT1: Measure the power of a single source in dBm using a single meter, and convert it to mW. Do this with it connected properly and improperly so you can see the difference. Use the results to fill in Table 1.1. *Demonstrate general results to the lab instructor.*

IT2: Measure the power through a single cable with a single meter using 3 different sources to determine the power of each source. Note any indications about what class of laser each source represents. (If a source produces two wavelengths, measure both.) Use the results to fill in the dBm columns of Table 1.1. (You'll convert to mW later.) *Demonstrate general results to the lab instructor.*

IT3: Measure the power through a single cable with a single source with 3 different meters to see how well the meters agree. Repeat the measurement with the first meter after the others to see how consistent it is. Use the results to fill in the dBm columns of Table 1.3. (You'll convert to mW later.) Will the different powers of the different sources affect this? Explain. *Demonstrate general results to the lab instructor.*

1.4.2 Analysis

Post-lab Discussion Questions

Q1: What is a class I laser? Do your power measurements agree for the ones which identify themselves as such?

Q2: What is the dB value of

1. 10 % loss?
2. 50 % loss?
3. 90 % loss?

Q3: What percentage of the input power is lost if the cable is improperly connected?

Q4: What is the advantage of measuring power in dB over mW?

Post-lab Tasks

T1: Fill in the conversions from dBm to mW in Tables 1.2 and 1.3.

1.5 Recap

By the end of this exercise, you should know how to :

- Connect optical fibre components properly.
- Measure optical power in
  - dBm
  - Watts

  and convert between both units.

1.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	0	________	0
Post-lab Questions	4	________	30
Pre-lab Tasks	0	________	0
In-lab Tasks	3	________	60
Post-lab Tasks	1	________	10

1.7 Template

Source:			
Meter:			
Properly connected		Improperly connected	
dBm	mW	dBm	mW

Table 1.1: Power conversion

Meter:				
Source	1550 nm		1310 nm	
	dBm	mW	dBm	mW

Table 1.2: Source variation

Source:				
Meter	1550 nm		1310 nm	
	dBm	mW	dBm	mW

Table 1.3: Meter variation

Chapter 2

Insertion Loss

2.1 Purpose

The purpose of the this experimentation is to practice taking measurements of insertion loss.

2.2 Theory

Insertion loss is the loss of transmitted light power when optical devices are inserted into the light path. An example of this would be the use of a patch cord, a fibre connector, imperfections in the fibre itself such as a bad splice, or an unclean fibre end. The total loss of light energy in the system is called the **insertion loss**.

Light traveling in the core of the fibre remains within the core due to the refractive index ratio of the core and cladding. This is due to the total internal reflection (TIR) relation. If the angle of the propagating light wave reflecting off of the cladding back into the core becomes less than the TIR angle, often called the critical angle, some of the light will escape into the cladding, and thus reduce the optical power of the signal.

2.3 Procedure

2.3.1 Preparation

Pre-lab Questions

PQ1: In order for the calculations below to work, should power be measured in mW or dBm? Explain.

2.3.2 Experimentation

Apparatus

- 2 patch cords
- patch cord adapter (connector)
- power meter
- 1550 nm laser source

Method

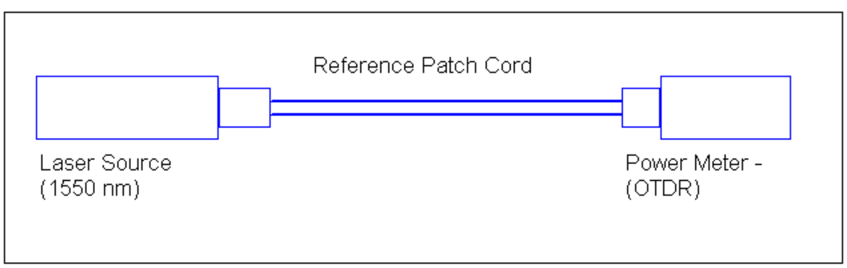

Figure 2.1: Simple Patch cord

Note: For all of the discussion following, power is assumed to be in dBm; otherwise power loss will be represented by multiplication by a factor less than one. Subraction only makes sense in logarithmic terms.

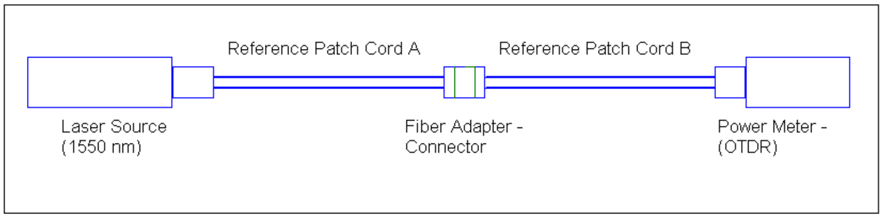

Figure 2.2: Two Patch Cords and Adapter

1. Measure the output power (P_A) of the 1550 nm laser diode or laser source with a power meter through one patch cord to be used as a reference. Actually, P_A is the output power of the source, *minus* the loss due to the cable. We could write this as:

$$P_A = P_0 - P_{c1}$$

   where P_0 is the source power and P_{c1} is the loss in the cable. The reference patch cord should then be marked, for this will be the used as the reference to measure the loss of other devices and cords. (See Figure 2.1). Repeat with a 1310 nm source.

2. Repeat the above, though this time use another patch cord of a defined length and measure the output power again (P_B). Similar to what was said above, P_B is the output power of the source, *minus* the loss due to the cable. We could write this as:

$$P_B = P_0 - P_{c2}$$

   where P_0 is the source power, (as before), and P_{c2} is the loss in the cable.

3. Connect the two patch cords together via the provided adapter and take the power output (P_{total}) reading of the system. (See Figure 2.2).

4. The overall loss should be noted as such; $P_{total} = P_A + P_B + P_{(adapter)}$. In this case, P_{total} is the output power of the source, *minus* the loss due to both cables *and the adapter*. We could write this as:

$$P_{total} = P_0 - P_{c1} - P_{c2} - P_{adapter}$$

where $P_{adapter}$ is the loss in the adapter. We can calculate the insertion loss in the second cable (almost) by subtracting $P_A - P_{total} = P_B + P_{(adapter)}$. Note that we can't get the adapter loss by itself. In this case

$$P_A - P_{total} = P_0 - P_{c1} - (P_0 - P_{c1} - P_{c2} - P_{adapter}) = P_{c2} + P_{adapter}$$

Note that this will give losses as positive values. Calculate the insertion loss due to cable 2 and the adapter.

5. Now that the power loss of the separate components of the system is known we are able to determine the loss due to other components used in the system if the original references are used. For instance, if we take $P_A + P_B - P_{total}$ then we should be able to *almost* get the output power of the source itself since:

$$P_A + P_B - P_{total} = P_0 + P_{adapter}$$

Calculate the output power of the source and the adapter.

6. Keeping the first patch cord and the adapter, replace the second patch cord with cord 3, measure the output and calculate the insertion loss for cable 3 and the adapter.

7. Keeping the first patch cord and the adapter, replace the second patch cord with cord 4, measure the output and calculate the insertion loss for cable 3 and the adapter.

In-lab Tasks

IT1: Fill in the dBm columns of Table 2.1. (You'll fill in the mW columns later.) *Demonstrate general results to the lab instructor.*

In-lab Questions

IQ1: Can you ever determine the insertion loss of the adapter itself? Explain.

2.3.3 Analysis

Post-lab Discussion Questions

Q1: If a device has an insertion loss of 3 dB, what percentage of the input power is being absorbed by the device?

Q2: Are the insertion losses for different cables in the same ballpark? If not, is there something which might explain the discrepancy?

Q3: What percentage of the incoming light is lost in each cable? Does that seem reasonable?

Post-lab Tasks

T1: Fill in the mW columns of Table 2.1.

2.4 Recap

By the end of this exercise, you should know how to :

- Measure the insertion loss of any component in a fibre optic system.

2.5 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	1	________	10
In-lab Questions	1	________	20
Post-lab Questions	3	________	10
Pre-lab Tasks	0	________	0
In-lab Tasks	1	________	40
Post-lab Tasks	1	________	20

2.6 Template

Source:				
Meter:				
cord	1550 nm		1310 nm	
	dBm	mW	dBm	mW
1				
2				
series				
1 and 3				
1 and 4				

Table 2.1: Cable variation

Chapter 3

Bending Loss

3.1 Purpose

The purpose of the this exercise is to study bending loss in optical fibres.

3.2 Theory

Over bending the optical fibre such that the bend radius decreases leads to signal attenuation, referred to as **bend radius attenuation**.

3.3 Procedure

3.3.1 Preparation

Pre-lab Questions

PQ1: From the webpage, read the description of **Backreflection** (BR) on page 12 of the *JDS RM3750B Backreflection/Power Meter* (which I'll refer to from now on as the RM3), and read the description of **Return loss** on pages 11 and 12 of the *JDS PS3 Polarization Dependent Loss Meter* (which I'll refer to from now on as the PS3), and explain how the two quantities are related.

PQ2: Is the term *Return loss* in PQ 1 the same as the term defined in section 3.3.2 of the text?

PQ3: Read section 3.4.1.2 of the textbook and determine how to identify **SC**, **ST**, and **FC** connectors. (These are the ones we'll be using in the lab.) Which is the most common one in our lab?

3.3.2 Experimentation

Apparatus

- various meters
- various single mode cables
- various fibre light sources

Method

Never bend the fibre around anything with a diameter less than 20 mm, for this will permanently damage the fibre.

1. For three different diameters of spindles, do the following:
   (a) Measure the power through a cable.

   (b) Wind one turn of cable around the spindle, and measure power again.

   (c) Add a turn and repeat the measurement, up to 5 turns.

2. Measure the spindle diameters.

3. Find how big the diameter has to be to produce no noticeable loss with one turn. Record this diameter.

4. Repeat the measurement for a different cable.

5. Create a new table, where each of the values has the *baseline* subtracted; i.e. the power measured with no bend in the cable. In other words, this table will just show the changes due to bending.

In-lab Tasks

IT1: Explain general results to the lab instructor:

- how loss varies with diameter
- how loss varies with the number of turns
- how the above losses vary between cables

3.3.3 Analysis

Use the table with the baseline subtracted to do each of the following:

- Plot loss versus diameter.
- Plot loss versus number of turns.
- Plot the above for both cables.

  You can put any or all of them on a single graph if it's not too cluttered.

Post-lab Discussion Questions

Q1: Is the relationship between loss and diameter linear? Would you expect it to be linear? Explain.

Q2: Is the relationship between loss and number of turns linear? If so, how much loss is there per turn for each diameter? Would you expect it to be linear? Explain.

Q3: Is the loss the same for both cables? Would you expect it to be? Explain.

Post-lab Tasks

T1: Photocopy your data, and hand in the graphs and question answers.

3.4 Recap

By the end of this exercise, you should know how bending affects optical fibres.

3.5 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	3	________	10
In-lab Questions	0	________	0
Post-lab Questions	3	________	30
Pre-lab Tasks	0	________	0
In-lab Tasks	1	________	40
Post-lab Tasks	1	________	20

3.6 Template

Source:				
Meter:				
diameter (mm)	turns			
	0	1	2	3
Cable one:				
Cable two:				

Table 3.1: Loss (dBm) measurements

Chapter 4

Measuring Refractive Index in Fibre Optics

4.1 Purpose

The purpose of this experiment is to study the propagation of light in optical fibres and to measure the refractive index of the fibre

4.2 Introduction

Without refraction, light waves would pass in straight lines through transparent substances, such as optical fibres, without any change of direction and quickly leave the fibres. Refraction, the change of direction of light, confines traveling light within the optical fibre. This bending depends on the velocity of the wave through different media. Knowing the velocity, the refractive index can be calculated.

4.3 Theory

Propagation of light through the core of an optical fibre depends on the materials of the core, the cladding, and the difference between their refractive indices. The speed of light traveling through an optical fibre and the refractive index are related in the following way:

$$n = \frac{c}{v}$$

where n is the refractive index of the fibre, v is speed of light in the fibre, and c is the speed of light in a vacuum.

4.4 Procedure

4.4.1 Preparation

Pre-lab Questions

PQ1: A refracted wave occurs when a wave passes from one medium into another medium. What determines the angle of refraction?

4.4.2 Experimentation

Apparatus

- set of four optical fibres of 15 cm, 10 m, 20 m and 40 m
- oscilloscope
- transmitter and receiver block

Method

Equipment setup

1. Turn on the oscilloscope.
2. Connect the probe of channel 1 to the test point marked “Reference” on the transmitter and receiver block (TRB).
3. Connect the probe of channel 2 to the “Delay” test point on the TRB.
4. Turn the power on.
5. Select the 15 cm sample fibre and insert one end of it in the LED D3 unit and the other in the D8 detector.

Calibration

1. Using the “Calibration delay knob” in the TRB, adjust the position of the peak of the second signal until it coincides with the peak of the first signal .

2. Use the calibration pulse as a reference pulse for subsequent measurements.

Measurement

1. For each of the different length optical fibres, measure the delay time with its uncertainty. Record the results for all three fibres and with their uncertainties.

In-lab Questions

IQ1: By adjusting the calibration so that the 15 cm fibre produced two peaks that coincide, what effect will that have on the delay times of the other fibres? How will this affect the value for the speed of light produced?

IQ2: Is there any evidence of dispersion in the fibres? Explain.

4.4.3 Analysis

1. Make a graph of delay time versus fibre length, and from the graph calculate the speed of light with its uncertainty.

2. Use the value for the speed of light to determine the index of refraction for the fibre with its uncertainty.

Post-lab Tasks

T1: Compare and comment on your result by comparison with the manufacturer’s value for the index of refraction of the cable (SH4001 Super ESKA Polyethylene Jacketed Optical Fibre Cord). Does it agree within experimental uncertainty?

4.5 Recap

By the end of this exercise, you should know how to :

- Measure speed of light and determine n of optical fibres

4.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	1	________	20
In-lab Questions	2	________	0
Post-lab Questions	0	________	0
Pre-lab Tasks	0	________	0
In-lab Tasks	0	________	60
Post-lab Tasks	1	________	20

4.7 Template

Fibre length	Uncertainty	Delay	Uncertainty
(m)	(m)	(nS)	(nS)

Table 4.1: Fibre delay as a function of length

Chapter 5

Numerical Aperture in Optical Fibres

5.1 Purpose

The purpose of this experiment is to study propagation of light in optical fibres, and to measure numerical aperture, (NA), of the fibre.

5.2 Introduction

Fibre optic cables are used in transmitting data in communication systems to make physical links between fixed points. Since it carries signals as light, an optical fibre cannot pick up electromagnetic interference. The center of fibre is the core, which has a higher refractive index compared to the outer coating, (the cladding), and this difference makes light propagate through the central core because of total internal reflection and is the means by which an optical signal is confined to the core of a fibre. In order for a light to be guided through the fibre it must enter the core with an angle that is less than the so called **acceptance angle** for the fibre. A ray entering the fibre with an angle greater than the acceptance angle will be lost in the cladding. The sine of the acceptance angle is also called the **numerical aperture**. Thus the numerical aperture (NA) is a measurement of the ability of an optical fibre to capture light.

5.3 Theory

Propagation of light through the core of an optical fibre depends on the materials of the core, the cladding and the difference between their refractive indices.

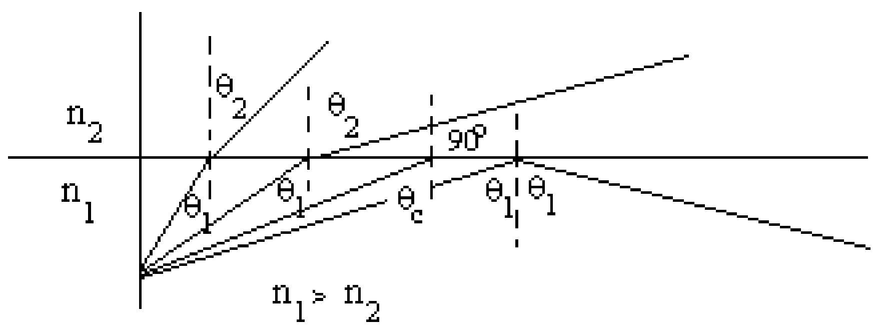

Figure 5.1: Total Internal Reflection

Snell's law explains the propagation of light along an optical fibre. This law explains relationship between angles of incidence and transmission at the interface between two dielectric media:

$$n_1 \sin \alpha = n_2 \sin \beta \tag{5.1}$$

If the angle of incident is increased, there will be a point when the angle of refraction will be equal to 90°, which is referred to as the **critical angle**. Therefore, Snell's law gives the relationship between the critical angle and the refractive indices of the core and cladding:

$$\sin \beta = \frac{n_2}{n_1} \tag{5.2}$$

If the angle of incidence is increased slightly beyond the critical angle, the refractive angle will also be increased beyond 90° level and 99.8% of incident light reflects toward the n_1 medium. So, light can propagate through a dielectric medium of refractive index n_1 surrounded by a cladding dialectic material with index n_2 where $n_1 > n_2$ in zigzag mode and for incident angles $\alpha > \beta$. The speed of light traveling through an optical fibre with refractive

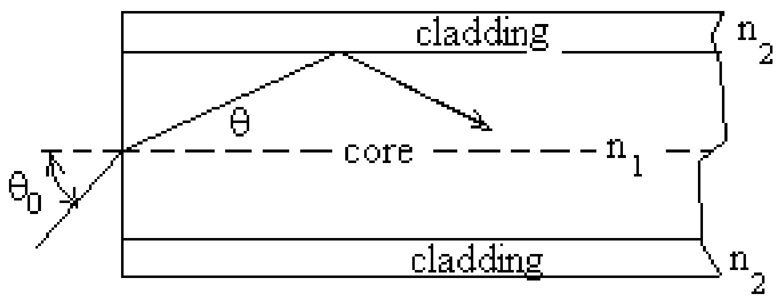

Figure 5.2: Light Propagation in Fibre

index $n = 1.5$ is calculated from $n = c/v$, where v is speed of light in the fibre and c is the speed of light in a vacuum.

The acceptance angle can be calculated from refractive indices of the core and cladding using the formula

$$\beta = \arcsin \sqrt{{n_1}^2 - {n_2}^2} \tag{5.3}$$

(where the input is from air or a vacuum).

The numerical aperture of the fibre is equal to the sine of the fibre acceptance angle and it is given by:

$$NA = \sqrt{{n_1}^2 - {n_2}^2} \tag{5.4}$$

5.4 Procedure

5.4.1 Preparation

Pre-lab Questions

PQ1: The theory talks about the *acceptance angle* for light coming *into* the fibre, but in the experiment you will measure light *leaving* the fibre. What relationship is there between those angles, and why?

5.4.2 Experimentation

Apparatus

- optical fibre
- optical transmitter

Method

The experiment should be carried out in semi darkness to make the spot easy to see. NA will be calculated by investigating the light leaving the fibre.

1. Switch on the transmitter.
2. Project the light output from the fibre on to the 5mm circle target.
3. Determine the circle diameter D of the light and the distance L from the fibre to the screen, along with their uncertainties.
4. Repeat this for each of the fibres and record the results.

In-lab Questions

IQ1: Did there seem to be any significant variation between the fibres? Does this fit your expectations?

IQ2: What determined the uncertainties in D and L?

5.4.3 Analysis

1. Calculate the spot radius, R, and its uncertainty, from each of the diameters.
2. Calculate the numeric aperture and the acceptance angle of each fibre, with their uncertainties.
3. Average the values of the numerical aperture and compare the result to the values in the data sheet for the cable (SH4001 Super ESKA Polyethylene Jacketed Optical Fiber Cord).

4. Given the value you obtained for the numerical aperture, and the value of the core index of refraction from the data sheet, calculate the index of the cladding with its uncertainty.

5. Calculate the fibre acceptance angle and its uncertainty.

Post-lab Discussion Questions

Q1: Does the value you determined for numerical aperture agree with the specifications within its experimental uncertainty?

5.5 Recap

By the end of this exercise, you should know how to :

- Measure NA of optical fibres

5.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	1	________	20
In-lab Questions	2	________	0
Post-lab Questions	1	________	0
Pre-lab Tasks	0	________	0
In-lab Tasks	0	________	40
Post-lab Tasks	0	________	40

5.7 Template

Fibre length	L	ΔL	D	ΔD
(m)	(cm)	(cm)	(cm)	(cm)

Table 5.1: Spot diameter of fibre output

Chapter 6

Fibre Bandwidth

6.1 Purpose

The purpose of the this exercise is to study the bandwidth limits of a fibre optical system.

6.2 Theory

The bandwidth of a fibre optic system depends on the

- transmitters
- cables
- receivers

and all other components in the system.

6.3 Procedure

6.3.1 Preparation

Pre-lab Questions

PQ1: Why is it important to see the input and output waveforms on an oscilloscope, rather than just using power meters for the input and output signals?

6.3.2 Experimentation

Apparatus

- wave generator
- modulated source
- demodulating receiver
- oscilloscope

Method

1. Determine the limits of amplitude and offset which can be transmitted undistorted for a low frequency (e.g. 1 kHz) signal. Use a sine wave.
   (a) Measure both input and output signal, and determine gain.
2. Over a wide range of frequencies, measure input and output amplitude to determine gain. Be sure to go to high enough frequencies that you can see the gain greatly reduced.

In-lab Tasks

IT1: Fill in Table 6.1 with sufficient data. Add rows if needed.

In-lab Questions

IQ1: Why is it important to use a sine wave when determining bandwidth?

IQ2: From previous measurements of the cable, how much of the observed attenuation is due to the cable?

6.3.3 Analysis

- Create a Bode plot of the results, and discuss.
- Comment on whether the limits of the input amplitude or offset changed with frequency.

Post-lab Discussion Questions

Q1: Since digital signals are *not* sinusoidal, why is it that transmission of digital signals is preferable? (Explain with reference to IQ1.)

6.4 Recap

By the end of this exercise, you should know how to test bandwidth.

6.5 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	1	________	10
In-lab Questions	2	________	0
Post-lab Questions	1	________	30
Pre-lab Tasks	0	________	0
In-lab Tasks	1	________	40
Post-lab Tasks	0	________	20

6.6 Template

frequency	input amplitude	input offset	output amplitude	distorted?
(Hz)	(V)	(V)	(V)	(y/n)

Table 6.1: Bandwidth data

Appendix A

Review of Uncertainty Calculations

A.1 Review of uncertainty rules

These are from the PC131 lab manual.

A.1.1 Repeated measurements

Arithmetic Mean (Average)

Note: In the following sections, each measurement x_i can be assumed to have an uncertainty pm, (i.e. the precision measure of the instrument used), due to measurement uncertainty.

The **arithmetic mean** (or average) represents the best value obtainable from a series of observations from "normally" distributed data.

$$\begin{aligned} Arithmetic\ mean &= \overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \\ &= \frac{x_1 + x_2 + \cdots + x_n}{n} \end{aligned}$$

Standard Deviation

The **standard deviation** of a number of measurements is a measurement of the uncertainty in an experiment due to reproducibility. The standard deviation is given by

$$\begin{aligned} \textit{Standard Deviation} = \sigma &= \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \\ &= \frac{1}{\sqrt{n-1}}\sqrt{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}} \end{aligned}$$

With random variations in the measurements, about 2/3 of the measurements should fall within the region given by $\overline{x} \pm \sigma$, and about 95% of the measurements should fall within the region given by $\overline{x} \pm 2\sigma$. (If this is not the case, then either uncertainties were not random or not enough measurements were taken to make this statistically valid.)

This occurs because the value calculated for $\overline{x}$, called the **sample mean**, may not be very close to the "actual" **population mean**, μ, which one would get by taking an infinite number of measurements.

Rule of thumb: For normally distributed data, an order of magnitude approximation for the standard deviation is 1/4 the range of the data. (In other words, take the difference between the maximum and minimum values and divide by 4 to get an approximate value for the standard deviation.)

Standard Deviation of the Mean

(In some texts this quantity is called the "standard *error* of the mean".) It is an *interval* around the *calculated* mean, $\overline{x}$, in which the *population* mean, μ, can be reasonably assumed to be found. This region is given by the *standard deviation of the mean*,

$$Standard\ deviation\ of\ the\ mean = \alpha = \frac{\sigma}{\sqrt{n}}$$

and one can give the value of the measured quantity as $\overline{x} \pm \alpha$. (In other words, μ should fall within the range of $\overline{x} \pm \alpha$.)

Uncertainty in the average

The uncertainty in the average is the *greater* of the uncertainty of the individual measurements, (i.e. *pm*, the precision measure of the instrument used), and α; i.e.

$$\Delta\overline{x} = \max(pm, \alpha)$$

If possible, when doing an experiment, enough measurements of a quantity should be taken so that the uncertainty in the measurement due to instrumental precision *is greater than or equal to* α*. This is so that the random variations in data values at some point become less significant than the instrument precision. (In practice this may require a number of data values to be taken which is simply not reasonable, but sometimes this condition will not be too difficult to achieve.)*

In any case, the uncertainty used in subsequent calculations should be the *greater* of the uncertainty of the individual measurements and α.

A.1.2 Rules for combining uncertainties

Basic arithmetic rules

The uncertainty in results can *usually* be calculated as in the following examples (if the percentage uncertainties in the data are small):

$$\begin{aligned}
(a) \quad & \Delta(A+B) = (\Delta A + \Delta B) \\
(b) \quad & \Delta(A-B) = (\Delta A + \Delta B) \\
(c) \quad & \Delta(A \times B) \approx |AB| \left(\left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta B}{B} \right| \right) \\
(d) \quad & \Delta(\frac{A}{B}) \approx \left| \frac{A}{B} \right| \left(\left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta B}{B} \right| \right)
\end{aligned}$$

Note that the first two rules above *always* hold true.

Uncertainties in functions, by algebra

$$\Delta f(x) \approx |f'(x)| \, \Delta x \tag{A.1}$$

Uncertainties in functions, by inspection

$$\Delta f(x) \approx f_{max} - f \tag{A.2}$$

or

$$\Delta f(x) \approx f - f_{min} \tag{A.3}$$

Sensitivity of Total Uncertainty to Individual Uncertainties

If $f = f(x, y)$, then to find the proportion of Δf due to each of the individual uncertainties, Δx and Δy, proceed as follows:

- To find Δf_x, let $\Delta y = 0$ and calculate Δf.
- To find Δf_y, let $\Delta x = 0$ and calculate Δf.

Uncertainties and Final Results

Always express final results with *absolute* uncertainties.

Mathematically, if two quantities a and b, with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if

$$|a - b| \leq \Delta a + \Delta b \tag{A.4}$$

A value with no uncertainty given can be assumed to have an uncertainty of zero.

If two numbers do not agree within experimental error, then the *percentage difference* between the experimental and theoretical values must be calculated as follows:

$$Percent\ Difference = \left| \frac{theoretical - experimental}{theoretical} \right| \times 100\% \tag{A.5}$$

Remember: Only calculate the percent difference if your results do not agree within experimental error.

Significant Figures in Final Results

Always quote final answers with one significant digit of uncertainty, and round the answers so that the least significant digit quoted is the uncertain one.

A.2 Discussion of Uncertainties

- Spend most time discussing the factors which contribute most to uncertainties in your results.
- Always give a measured value or a numerical bound on an uncertainty.
- State whether any particular factor leads to a systematic uncertainty or a random one. If it's systematic, indicate whether it would tend to increase or decrease your result.

A.2.1 Types of Errors

- Measurable uncertainties
- Bounded uncertainties
- Blatant filler

Don't use *"human error"*; it's far too vague.

A.2.2 Reducing Errors

1. Avoid mistakes.
2. Repeat for consistency, if possible.
3. Change technique
4. Observe other factors as well; including ones which you may have assumed were not changing or shouldn't matter.
5. Repeat and do statistical analysis.
6. Change equipment; the last resort.

A.2.3 Ridiculous Errors

Anything which amounts to a mistake is not a valid source of error. A serious scientist will attempt to ensure no mistakes were made before considering reporting on results.

Appendix B

Common Uncertainty Results

Following are some common results about uncertainties which you may find useful. If there are others which you feel should be here, inform the lab supervisor so that they may included in future versions of the lab manual.

$$\Delta(x^n) \approx n|x|^{n-1}\Delta x$$

$$\Delta(\sin x_R) \approx |\cos x_R| (\Delta x)_R$$

$$\Delta(\tan x_R) \approx (\sec x_R)^2 (\Delta x)_R$$

where x_R denotes x in *radians*.

$$\Delta \ln x \approx \frac{1}{x}\Delta x = \frac{\Delta x}{x}$$

$$\Delta x^y \approx \left|x^{y-1}y\right| \Delta x + |x^y \quad \ln x| \Delta y$$

$$\Delta \sqrt[y]{x} \approx \left|x^{\left(\frac{1}{y}-1\right)} \quad \frac{1}{y}\right| \Delta x + \left|x^{\frac{1}{y}} \quad \ln x\right| \frac{\Delta y}{y^2}$$

$$\Delta f(x,y,z) \approx \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y + \left|\frac{\partial f}{\partial z}\right| \Delta z$$

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