Electronics
Kirchhoff's Law Example

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Analyzing the behaviour of DC circuits involving *only* DC (i.e. unchanging in time) voltage sources and **ohmic** devices can be done using **Kirchhoff's Laws**.

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For many devices the relationship may be far more complex, and so the equations relating voltages and currents in the circuit become non-linear and are much harder to solve.

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$$\sum I = 0$$

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$$\sum V = \sum IR$$

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- those due to **sources**, such as batteries,
- those due to **sinks**, which are resistors.

  By Ohm’s law, the voltage across a resistor is IR.

The way that these laws are applied to analyze a circuit involves choosing **nodes** in the circuit for the first law and using **loops** in the circuit for the second and producing equations from each node and loop.

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where A is the coefficient matrix and X is the vector of the currents. (Because of the two types of equations used, B is a vector of voltages and zeros.)

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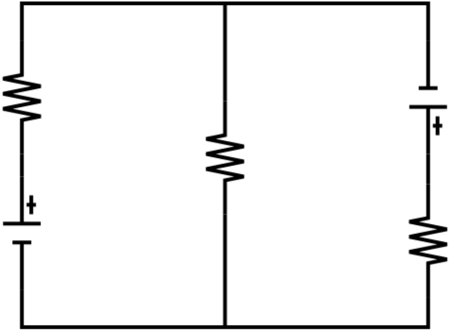
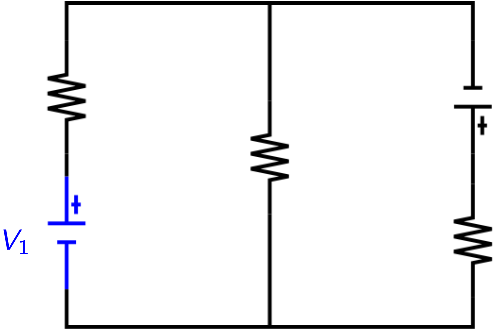


Label all V's and R's.



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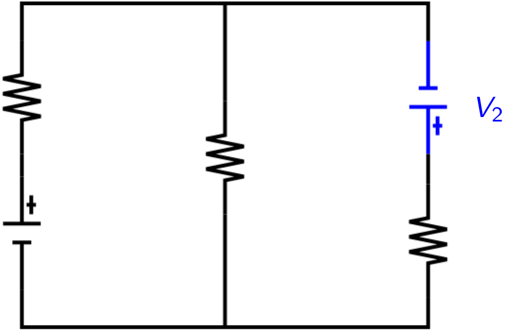

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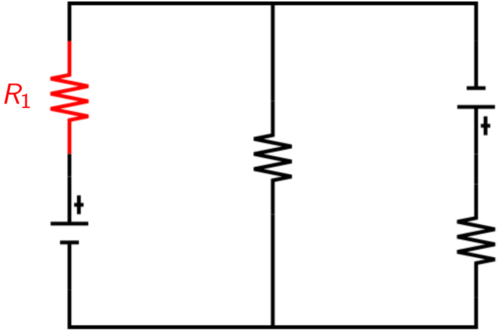

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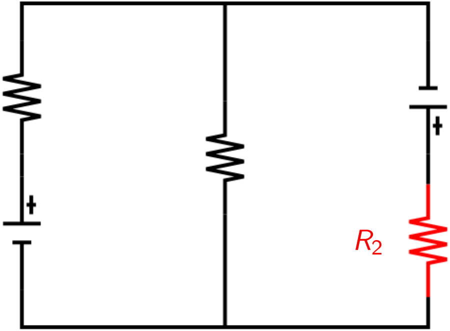

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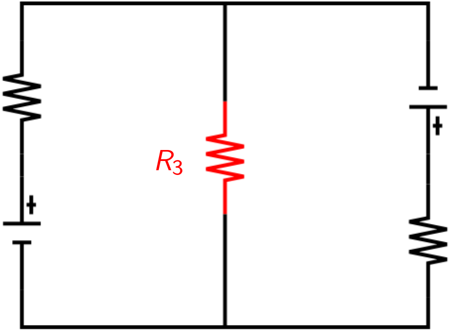

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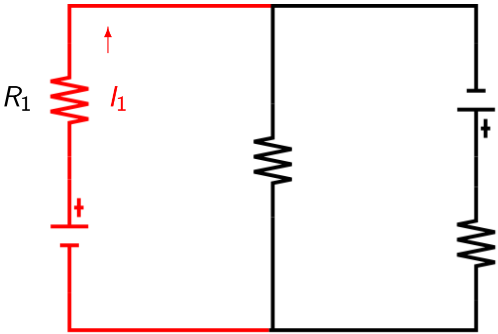
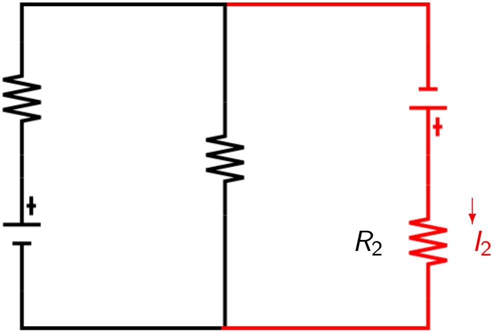


Label all I's to go with R's and pick directions for I's.



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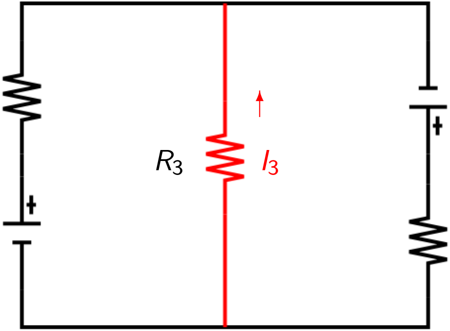

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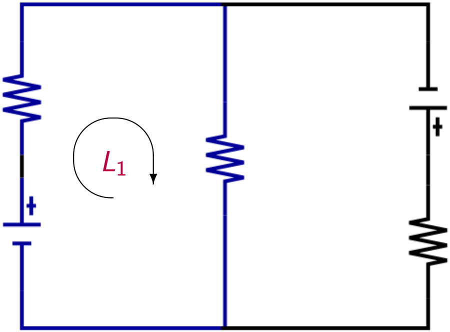
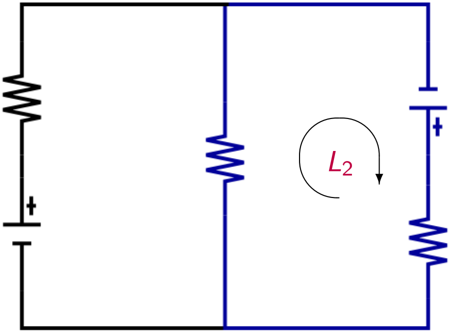


Label loops.



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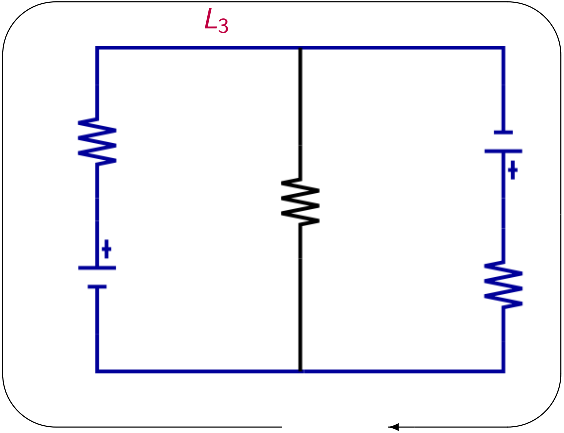

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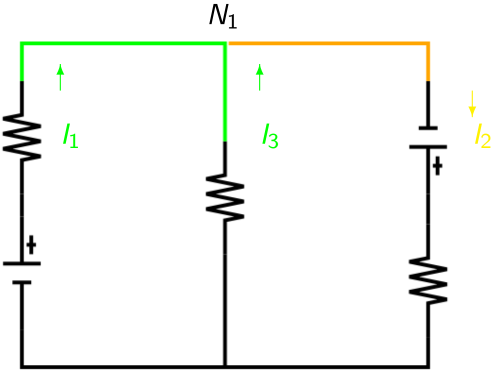
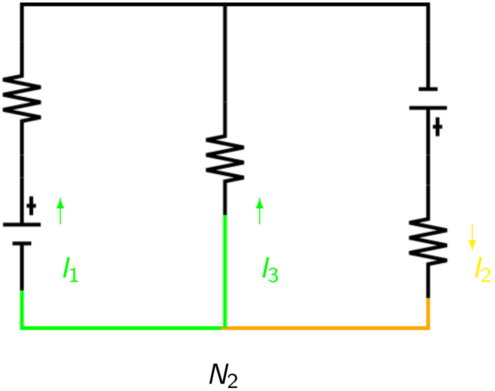


Label nodes.



Label nodes.

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4. Repeat for all components until you are back at the node.

Making loop equations

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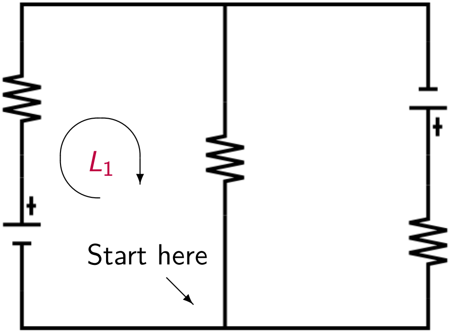
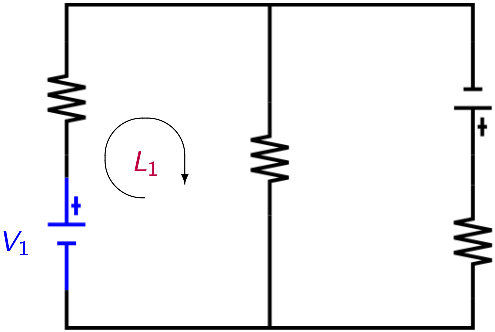
1. Start at node, go around to first component.
2. If component is a battery, count the voltage as *positive* if you come to the '-' terminal first.
3. If component is a resistor, count the IR as *positive* if you come to the resistor going *against* the current.
4. Repeat for all components until you are back at the node.
5. Set the sum of all of the contributions from 2 and 3 to zero.

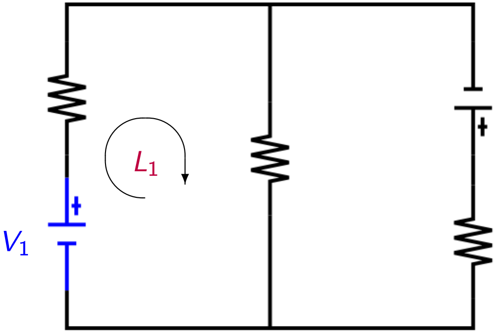
Here's how the equation for Loop 1 is created.



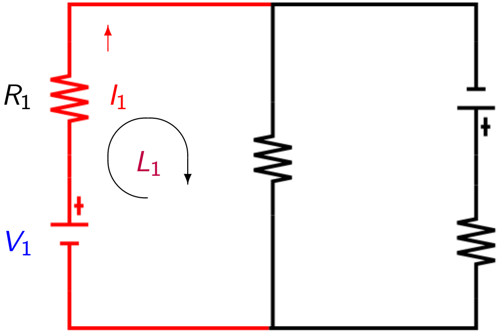
Start at the lower right corner of this loop.



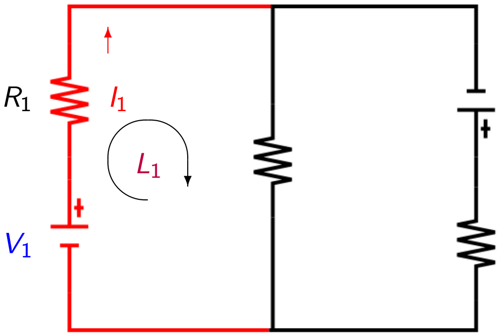
V_1 is the first thing we encounter.



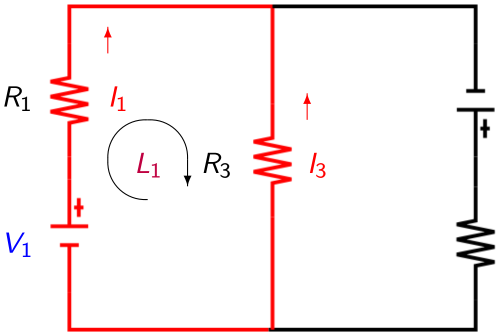
V_1 is *positive* since we hit the negative terminal first.



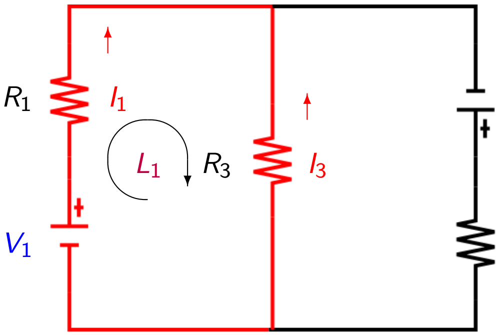
R_1 is the next thing we encounter.



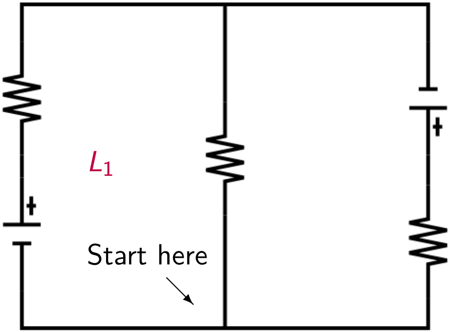
$I_1 R_1$ is *negative* since the loop direction *matches* the current direction.

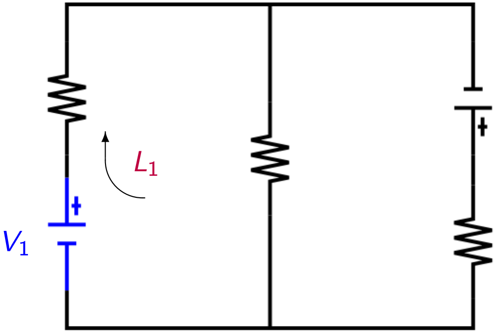


R_3 is next.

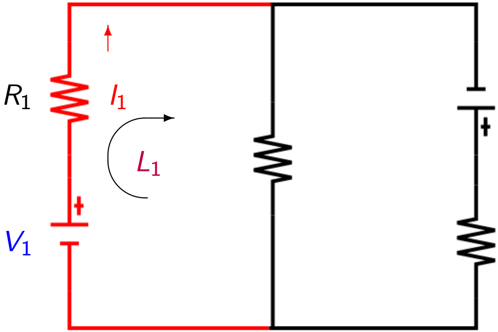


I_3R_3 is *positive* since the loop direction *opposes* the current direction.

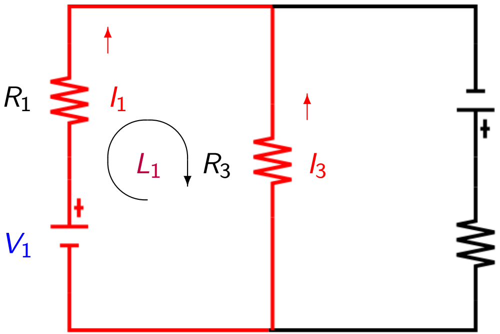

L_1
Start here



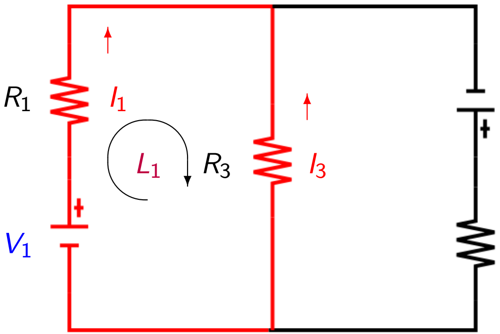
$+V_1$



$$+V_1 - I_1 R_1$$



$$+V_1 - I_1 R_1 + I_3 R_3$$



$$+V_1 - I_1 R_1 + I_3 R_3 = 0$$

Thus for the example above:

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Loop 1 (starting at Node 1)

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Loop 2 (starting at Node 2)

$$V_2 - I_2 R_2 - I_3 R_3 = 0$$

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Loop 2 (starting at Node 2)

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Loop 3 (starting at Node 1)

$$V_1 - I_1 R_1 + V_2 - I_2 R_2 = 0$$

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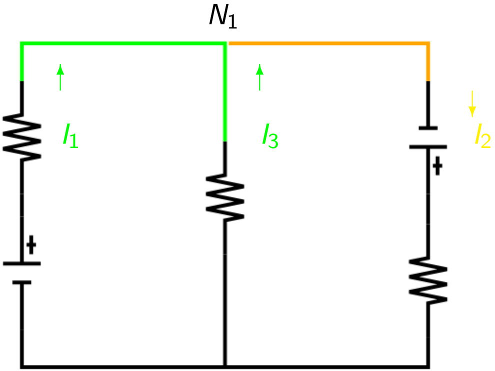

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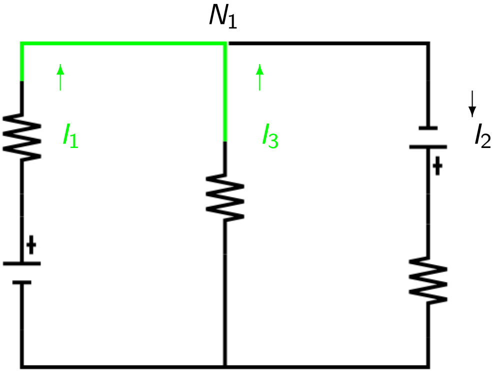
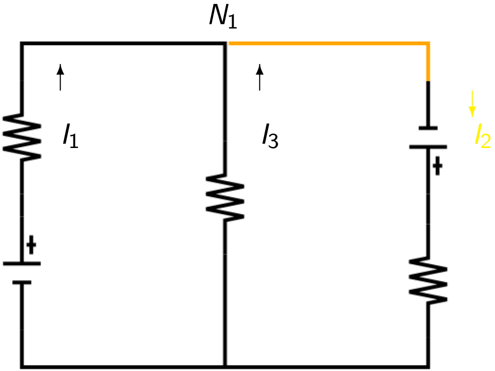
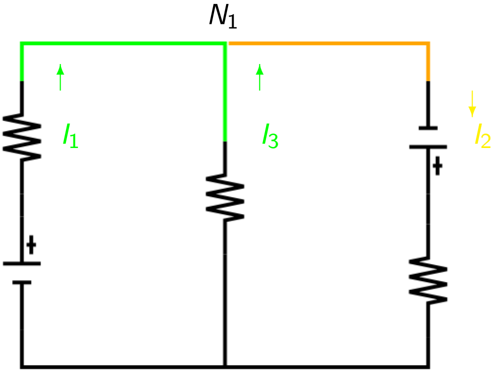


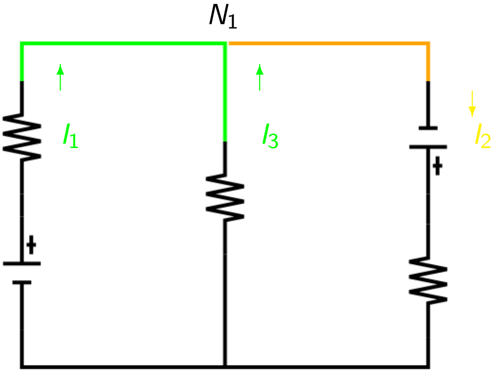
I_1 and I_3 go *into* Node 1.



I_2 comes *out of* Node 1.



$$I_1 + I_3 = I_2$$



$$I_1 + I_3 - I_2 = 0$$

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Node 2

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Set up a system of equations.

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$$\begin{array}{lccccc}
Loop\ 1: & -I_1R_1 & & +I_3R_3 & = & -V_1 \\
Loop\ 2: & & -I_2R_2 & -I_3R_3 & = & -V_2 \\
Loop\ 3: & -I_1R_1 & -I_2R_2 & & = & -V_1-V_2 \\
Node\ 1: & -I_1 & +I_2 & -I_3 & = & 0 \\
Node\ 2: & +I_1 & -I_2 & +I_3 & = & 0
\end{array}$$

Create matrices.

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We can rewrite the above system in matrix form as

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$$\begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -V_1 \\ -V_2 \\ -V_1 - V_2 \\ 0 \\ 0 \end{pmatrix}$$

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$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

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$$X = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

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$$X = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

$$B = \begin{pmatrix} -V_1 \\ -V_2 \\ -V_1 - V_2 \\ 0 \\ 0 \end{pmatrix}$$

Then

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$$AX = B$$

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$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

Here's the matrix

$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

Notice row 1

$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

Notice row 1 +row 2

$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

Notice row 1 +row 2 equal row 3, so we can get rid of row 3

$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

Also, row 4

$$A = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -R_1 & -R_2 & 0 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{pmatrix}$$

Also, row 4 = -row 5, so get rid of row 5

So we are left with

$$\mathcal{A} = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -1 & +1 & -1 \end{pmatrix}$$

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$$\mathcal{A} = \begin{pmatrix} -R_1 & 0 & +R_3 \\ 0 & -R_2 & -R_3 \\ -1 & +1 & -1 \end{pmatrix}$$

and

$$\mathcal{B} = \begin{pmatrix} -V_1 \\ -V_2 \\ 0 \end{pmatrix}$$

after adjustment

Now

$$X = \mathcal{A}^{-1}\mathcal{B}$$

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$$X = \frac{1}{R_2R_1 + R_2R_3 + R_1R_3}\begin{pmatrix} R_3V_1 + R_3V_2 + R_2V_1 \\ R_3V_1 + R_1V_2 + R_3V_2 \\ R_1V_2 - R_2V_1 \end{pmatrix}$$

Now

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Notice that, depending on the voltages involved, some of the currents could be negative.

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Notice that, depending on the voltages involved, some of the currents could be negative. In this case, it simply means that the actual current goes in the opposite direction from what you had chosen.