Electronics
Negative Feedback in Operational Amplifiers

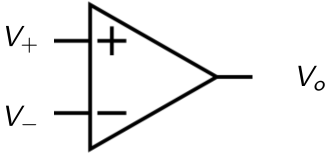
Terry Sturtevant

Wilfrid Laurier University

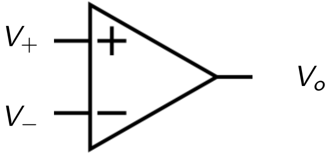
July 29, 2013

The equation for the output of an op amp is always true *as long as the output is not saturated*

The equation for the output of an op amp is always true *as long as the output is not saturated*



The equation for the output of an op amp is always true *as long as the output is not saturated*



$$V_o = A\,(V_+ - V_-)$$

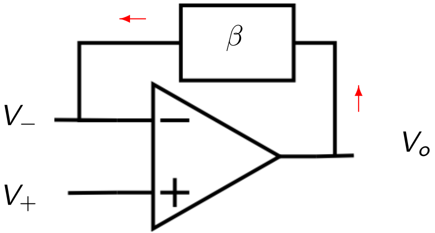
For negative feedback, make the voltage at V_- be some portion of V_o.

For negative feedback, make the voltage at V_- be some portion of V_o.

This ratio of V_- to V_o is called the **feedback factor**, β.

For negative feedback, make the voltage at V_- be some portion of V_o.

This ratio of V_- to V_o is called the **feedback factor**, β.



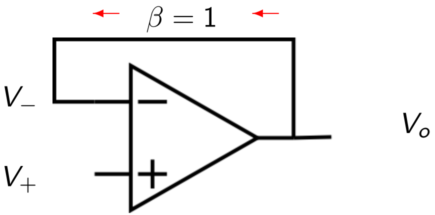
For instance, with just a wire from V_- to V_o, the feedabck factor is 1.

For instance, with just a wire from V_- to V_o, the feedabck factor is 1.

This is the case for a *voltage follower.*

For instance, with just a wire from V_- to V_o, the feedabck factor is 1.

This is the case for a *voltage follower.*



For feedback

For feedback

$$V_- = \beta V_o$$

For feedback

$$V_- = \beta V_o$$

where

For feedback

$$V_- = \beta V_o$$

where

$$\beta \in [0, 1]$$

For feedback

$$V_- = \beta V_o$$

where

$$\beta \in [0, 1]$$

So the op amp equation

For feedback

$$V_- = \beta V_o$$

where

$$\beta \in [0, 1]$$

So the op amp equation

$$V_o = A\,(V_+ - V_-)$$

becomes

For feedback

$$V_- = \beta V_o$$

where

$$\beta \in [0, 1]$$

So the op amp equation

$$V_o = A(V_+ - V_-)$$

becomes

$$V_o = A(V_+ - \beta V_o)$$

For feedback

$$V_- = \beta V_o$$

where

$$\beta \in [0, 1]$$

So the op amp equation

$$V_o = A\left(V_+ - V_-\right)$$

becomes

$$V_o = A\left(V_+ - \beta V_o\right)$$

$$V_o = AV^+ - \beta AV_o$$

$$V_o + \beta A V_o = A V_+$$

$$V_o + \beta A V_o = A V_+$$

$$V_o \left(1 + \beta A\right) = A V_+$$

$$V_o + \beta A V_o = AV_+$$

$$V_o\,(1 + \beta A) = AV_+$$

$$V_o\,(1 + \beta A) = AV_+$$

$$V_o + \beta A V_o = A V_+$$

$$V_o \left(1 + \beta A\right) = A V_+$$

$$V_o \left(1 + \beta A\right) = A V_+$$

$$V_o = V_+ \left(\frac{A}{1 + \beta A}\right)$$

$$V_o + \beta A V_o = A V_+$$

$$V_o \left(1 + \beta A\right) = A V_+$$

$$V_o \left(1 + \beta A\right) = A V_+$$

$$V_o = V_+ \left(\frac{A}{1 + \beta A}\right)$$

$$V_o = V_+ \left(\frac{1}{\frac{1}{A} + \beta}\right)$$

If the gain, A, is much greater than 1, then

If the gain, A, is much greater than 1, then

$$A \gg \beta$$

If the gain, A, is much greater than 1, then

$$A \gg \beta$$

and so

$$\frac{1}{A} \ll \beta$$

If the gain, A, is much greater than 1, then

$$A \gg \beta$$

and so

$$\frac{1}{A} \ll \beta$$

$$V_o = V_+ \left(\frac{1}{\cancel{1/A} + \beta} \right)$$

If the gain, A, is much greater than 1, then

$$A \gg \beta$$

and so

$$\frac{1}{A} \ll \beta$$

$$V_o = V_+ \left(\frac{1}{\cancel{\frac{1}{A}} + \beta} \right)$$

thus

$$V_o \approx V_+ \left(\frac{1}{\beta} \right)$$

If the gain, A, is much greater than 1, then

$$A \gg \beta$$

and so

$$\frac{1}{A} \ll \beta$$

$$V_o = V_+ \left(\frac{1}{\cancel{1/A} + \beta} \right)$$

thus

$$V_o \approx V_+ \left(\frac{1}{\beta} \right)$$

so V_o only depends on β.

$$V_o \approx V_+ \left(\frac{1}{\beta}\right)$$

$$V_o \approx V_+ \left(\frac{1}{\beta}\right)$$

So for a voltage follower, where $\beta = 1$,

$$V_o \approx V_+ \left(\frac{1}{\beta}\right)$$

So for a voltage follower, where $\beta = 1$,

$$V_o \approx V_+$$

$$V_o \approx V_+ \left(\frac{1}{\beta}\right)$$

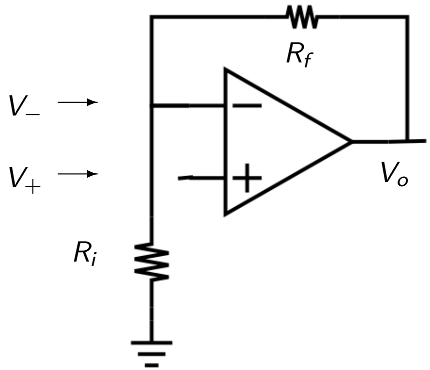
So for a voltage follower, where $\beta = 1$,

$$V_o \approx V_+$$

as expected.

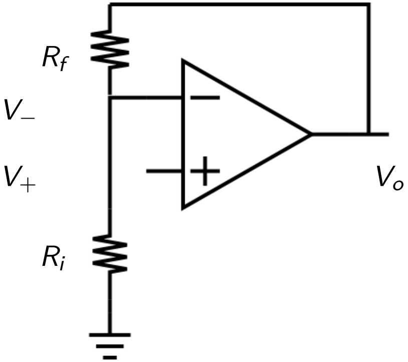
Non-inverting amplifier

Non-inverting amplifier



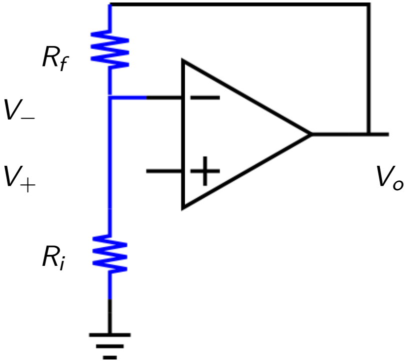
This can be redrawn

This can be redrawn



This can be redrawn

This can be redrawn



R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

By definition, this is the feedback factor, β.

R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

By definition, this is the feedback factor, β.

Thus

R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

By definition, this is the feedback factor, β.

Thus

$\beta = \frac{R_i}{R_f + R_i}$

R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

By definition, this is the feedback factor, β.

Thus

$\beta = \frac{R_i}{R_f + R_i}$

so

R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

By definition, this is the feedback factor, β.

Thus

$\beta = \frac{R_i}{R_f + R_i}$

so

$gain = \frac{1}{\beta} = \frac{R_f + R_i}{R_i}$

R_f and R_i form a voltage divider, so the voltage at V_- is a fraction of V_o

By definition, this is the feedback factor, β.

Thus

$\beta = \frac{R_i}{R_f + R_i}$

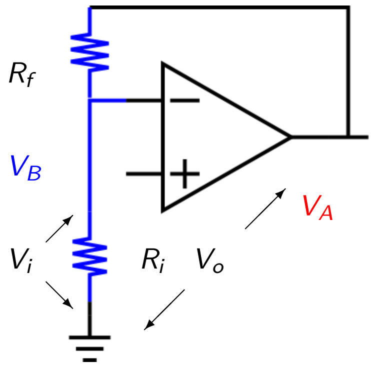
so

$gain = \frac{1}{\beta} = \frac{R_f + R_i}{R_i}$

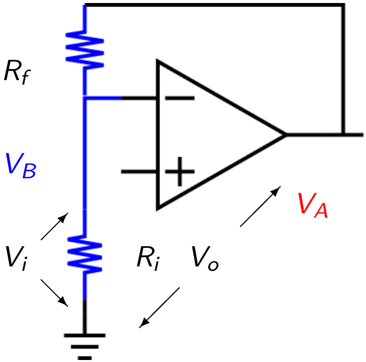
as expected

This can be redrawn

This can be redrawn



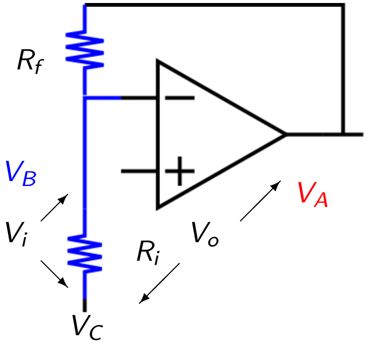
This can be redrawn



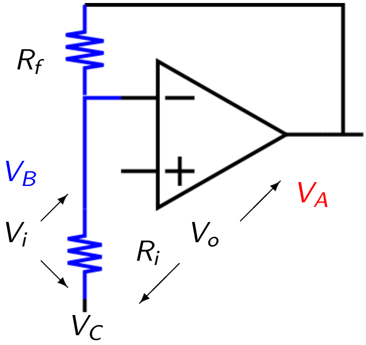
$$\frac{V_o}{V_i} = \frac{V_A}{V_B} = 1 + \frac{R_f}{R_i}$$

If R_i is not grounded, then it looks like this.

If R_i is not grounded, then it looks like this.

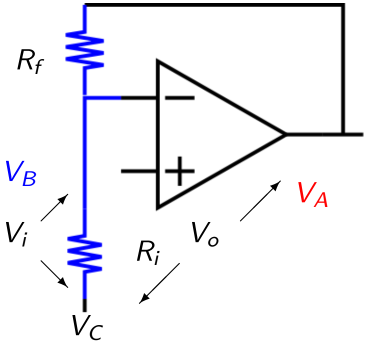


If R_i is not grounded, then it looks like this.



V_0 is V_A - V_C

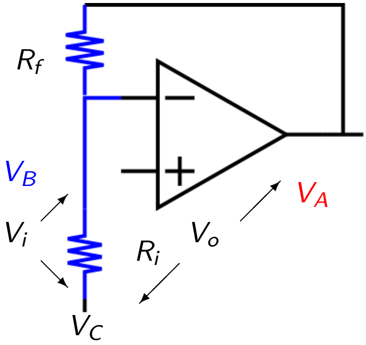
If R_i is not grounded, then it looks like this.



V_0 is V_A - V_C

V_i is V_B - V_C

If R_i is not grounded, then it looks like this.



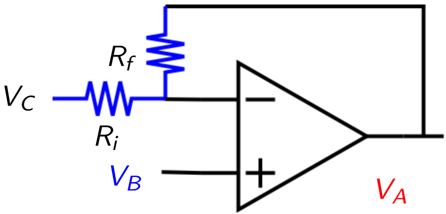
V_0 is V_A - V_C

V_i is V_B - V_C

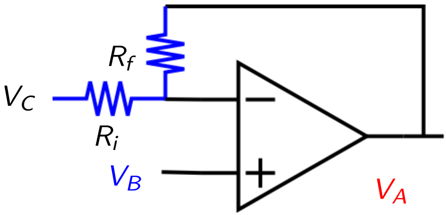
so $\frac{V_o}{V_i} = \frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$

We can redraw the circuit slightly differently.

We can redraw the circuit slightly differently.

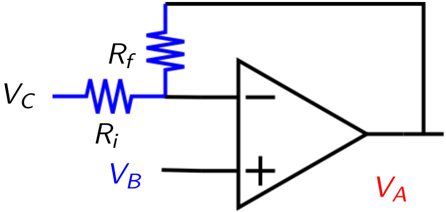


We can redraw the circuit slightly differently.



Since $\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$

We can redraw the circuit slightly differently.



Since $\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$

This can be rewritten $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

The derivation goes like this:

The derivation goes like this:

$$\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$$

The derivation goes like this:

$$\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$$

$$V_A - V_C = \left(1 + \frac{R_f}{R_i}\right)(V_B - V_C)$$

The derivation goes like this:

$$\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$$

$$V_A - V_C = \left(1 + \frac{R_f}{R_i}\right)(V_B - V_C)$$

$$V_A - \cancel{V_C} = \left(1 + \frac{R_f}{R_i}\right) V_B - \cancel{V_C} - \frac{R_f}{R_i} V_C$$

The derivation goes like this:

$$\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$$

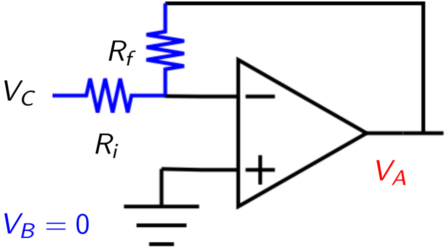
$$V_A - V_C = \left(1 + \frac{R_f}{R_i}\right)(V_B - V_C)$$

$$V_A - \cancel{V_C} = \left(1 + \frac{R_f}{R_i}\right) V_B - \cancel{V_C} - \frac{R_f}{R_i} V_C$$

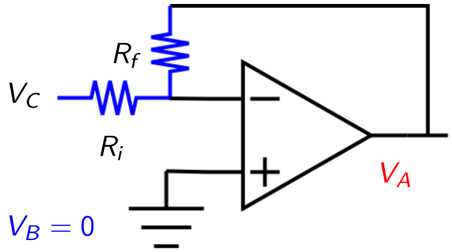
$$V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$$

Finally, we can set V_B to ground.

Finally, we can set V_B to ground.

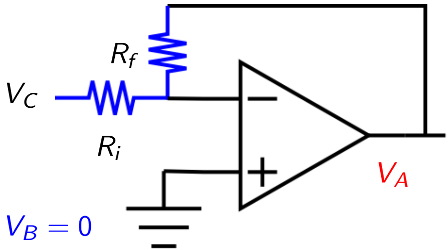


Finally, we can set V_B to ground.



So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

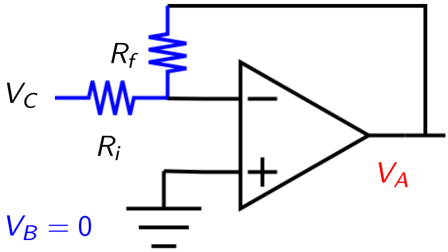
Finally, we can set V_B to ground.



So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$

Finally, we can set V_B to ground.

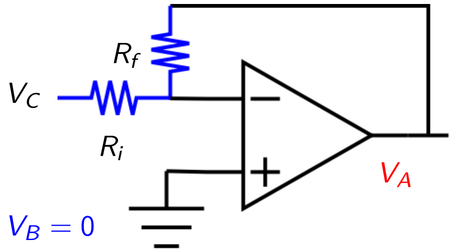


So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$

Then $V_A = -\frac{R_f}{R_i} V_C$

Finally, we can set V_B to ground.

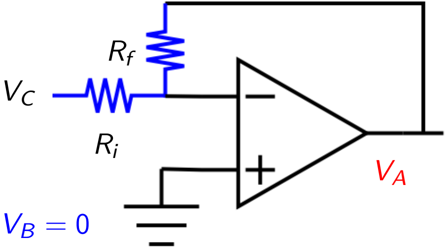


So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$

Then $V_A = -\frac{R_f}{R_i} V_C$ **which is an inverting amplifier!**

Finally, we can set V_B to ground.



So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$

Then $V_A = -\frac{R_f}{R_i} V_C$ **which is an inverting amplifier!**

So the feedback factor, β, is the same as for a non-inverting amplifier.