Electronics
Negative Feedback in Operational Amplifiers

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The equation for the output of an op amp is always true as long as the output is not saturated.
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The equation for the output of an op amp is always true *as long as the output is not saturated*

\[ V_o = A(V_+ - V_-) \]
For negative feedback, make the voltage at $V_-$ be some portion of $V_o$. 

This ratio of $V_-$ to $V_o$ is called the feedback factor, $\beta$. 

$\beta V_o + V_- = V_o$
For negative feedback, make the voltage at $V_-$ be some portion of $V_o$.
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This ratio of $V_-$ to $V_o$ is called the feedback factor, $\beta$. 

\[ \beta \frac{V_-}{V_o} \]
For instance, with just a wire from $V_- \text{ to } V_o$, the feedback factor is 1.
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This is the case for a \textit{voltage follower}.
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This is the case for a voltage follower.
Op amp output with negative feedback
Example: non-inverting amplifier

For feedback
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\[ V_- = \beta V_o \]
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where
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\[ \beta \in [0, 1] \]
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So the op amp equation
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\[ V_- = \beta V_o \]

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\[ V_o = A(V_+ - V_-) \]

becomes
For feedback

\[ V_- = \beta V_o \]

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So the op amp equation

\[ V_o = A (V_+ - V_-) \]

becomes

\[ V_o = A (V_+ - \beta V_o) \]
For feedback

\[ V_- = \beta V_o \]

where

\[ \beta \in [0, 1] \]

So the op amp equation

\[ V_o = A(V_+ - V_-) \]

becomes

\[ V_o = A(V_+ - \beta V_o) \]

\[ V_o = AV^+ - \beta AV_o \]
Op amp output with negative feedback
Example: non-inverting amplifier

\[ V_o + \beta AV_o = AV_+ \]
Op amp output with negative feedback
Example: non-inverting amplifier

\[ V_o + \beta AV_o = AV_+ \]

\[ V_o (1 + \beta A) = AV_+ \]
Op amp output with negative feedback
Example: non-inverting amplifier

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\[ V_o (1 + \beta A) = AV_+ \]
Op amp output with negative feedback

Example: non-inverting amplifier

\[ V_o + \beta AV_o = AV_+ \]

\[ V_o (1 + \beta A) = AV_+ \]

\[ V_o = V_+ \left( \frac{A}{1 + \beta A} \right) \]
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\[ V_o (1 + \beta A) = AV_+ \]

\[ V_o = V_+ \left( \frac{A}{1+\beta A} \right) \]

\[ V_o = V_+ \left( \frac{1}{\frac{1}{A} + \beta} \right) \]
If the gain, $A$, is much greater than 1, then
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and so

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$$V_o = V_+ \left( \frac{1}{\frac{1}{A} + \beta} \right)$$

thus

$$V_o \approx V_+ \left( \frac{1}{\beta} \right)$$
If the gain, $A$, is much greater than 1, then

$$A \gg \beta$$

and so

$$\frac{1}{A} \ll \beta$$

thus

$$V_o = V_+ \left( \frac{1}{\frac{1}{A} + \beta} \right)$$

so $V_o$ only depends on $\beta$. 
Op amp output with negative feedback
Example: non-inverting amplifier

$V_o \approx V_+ \left( \frac{1}{\beta} \right)$
Op amp output

Op amp output with negative feedback
Example: non-inverting amplifier

\[ V_o \approx V_+ \left( \frac{1}{\beta} \right) \]

So for a voltage follower, where \( \beta = 1 \),
Op amp output

Op amp output with negative feedback

Example: non-inverting amplifier

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So for a voltage follower, where \( \beta = 1 \),

\[ V_o \approx V_+ \]

as expected.
Non-inverting amplifier
Non-inverting amplifier
This can be redrawn
This can be redrawn

\[ V_+ \quad V_- \quad V_o \quad R_f \quad R_i \]
This can be redrawn
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\[ \begin{align*}
V_- &amp; \quad V_+ \\
R_f &amp; \quad R_i \\
V_o
\end{align*} \]
$R_f$ and $R_i$ form a voltage divider, so the voltage at $V_-$ is a fraction of $V_o$.
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By definition, this is the feedback factor, $\beta$. 

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Thus
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By definition, this is the feedback factor, $\beta$.

Thus

$$\beta = \frac{R_i}{R_f + R_i}$$
$R_f$ and $R_i$ form a voltage divider, so the voltage at $V_-$ is a fraction of $V_o$.
By definition, this is the feedback factor, $\beta$.
Thus

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Thus

$$\beta = \frac{R_i}{R_f + R_i}$$

so

$$gain = \frac{1}{\beta} = \frac{R_f + R_i}{R_i}$$
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By definition, this is the feedback factor, $\beta$.

Thus

$$\beta = \frac{R_i}{R_f + R_i}$$

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$$gain = \frac{1}{\beta} = \frac{R_f + R_i}{R_i}$$

as expected
This can be redrawn
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\[ V_o = V_A \]

\[ V_B = 1 + \frac{R_f}{R_i} \]

\[ V_i \]
This can be redrawn

\[ \frac{V_o}{V_i} = \frac{V_A}{V_B} = 1 + \frac{R_f}{R_i} \]
If $R_i$ is not grounded, then it looks like this.
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![Diagram of op amp with negative feedback](image)

\[ V_B = V_i - V_C \]
\[ V_A = V_B - V_C \]
\[ V_o = V_A - V_C \]

\[ V_o = V_i \left( 1 + \frac{R_f}{R_i} \right) \]
If $R_i$ is not grounded, then it looks like this.

$V_0$ is $V_A - V_C$
If $R_i$ is not grounded, then it looks like this.

\[ V_0 = V_A - V_C \]
\[ V_i = V_B - V_C \]
If $R_i$ is not grounded, then it looks like this.

$V_0$ is $V_A - V_C$

$V_i$ is $V_B - V_C$

so $\frac{V_o}{V_i} = \frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$
We can redraw the circuit slightly differently.
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Since \( \frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i} \)
We can redraw the circuit slightly differently.

Since $\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}$

This can be rewritten $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$
The derivation goes like this:
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\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}
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V_A - V_C = \left(1 + \frac{R_f}{R_i}\right)(V_B - V_C)
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\[
V_A - V_C = \left(1 + \frac{R_f}{R_i}\right) (V_B - V_C)
\]

\[
V_A - V_C = \left(1 + \frac{R_f}{R_i}\right) V_B - V_C - \frac{R_f}{R_i} V_C
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The derivation goes like this:

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\frac{V_A - V_C}{V_B - V_C} = 1 + \frac{R_f}{R_i}
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\[
V_A - \left(1 + \frac{R_f}{R_i}\right)V_B = -\frac{R_f}{R_i}V_C
\]
Finally, we can set $V_B$ to ground.
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$$V_B = 0$$
Finally, we can set $V_B$ to ground.

So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$
Finally, we can set $V_B$ to ground.

So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$
Finally, we can set $V_B$ to ground.

So if $V_A = \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$

Then $V_A = -\frac{R_f}{R_i} V_C$
Finally, we can set $V_B$ to ground.

So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$

and $V_B = 0$

Then $V_A = -\frac{R_f}{R_i} V_C$ which is an inverting amplifier!
Finally, we can set $V_B$ to ground.

So if $V_A - \left(1 + \frac{R_f}{R_i}\right) V_B = -\frac{R_f}{R_i} V_C$
and $V_B = 0$
Then $V_A = -\frac{R_f}{R_i} V_C$ which is an inverting amplifier!

So the feedback factor, $\beta$, is the same as for a non-inverting amplifier.