Prime Number Indentifier Circuit
PC/CP220 Project Phase II

Terry Sturtevant

Fall 2015

Truth Table

For this particular problem, it would be helpful to create a table of numbers, their binary representations, and indication of their status (i.e. prime, composite, or neither).

number	binary ($a_3a_2a_1a_0$)	p/c/n
0	0000	n
1	0001	n
2	0010	p
3	0011	p
4	0100	c
5	0101	p
6	0110	c
7	0111	p
8	1000	c
9	1001	c
10	1010	c
11	1011	p
12	1100	c
13	1101	p
14	1110	c
15	1111	c

Table 1: Truth Table

In this case, a Karnaugh map can be used to determine simplified sum-of-products logic equations.

		a_1a_0			
		00	01	11	10
a_3a_2	00	0	0	1	1
	01	0	1	1	0
	11	0	1	0	0
	10	0	0	1	0

Table 2: Karnaugh Map Table for *prime*

We can highlight groups of ones in this table:

		a_1a_0			
		00	01	11	10
a_3a_2	00	0	0	1	1
	01	0	1	1	0
	11	0	1	0	0
	10	0	0	1	0

Table 3: Highlighting two groups

The terms given by these groups will be

- $\overline{a_3}\ \overline{a_2}\ a_1$ (a_0 is irrelevant)
- $\overline{a_3}\ a_2\ a_0$ (a_1 is irrelevant)

		$a_1 a_0$			
		00	01	11	10
$a_3 a_2$	00	0	0	1	1
	01	0	1	1	0
	11	0	1	0	0
	10	0	0	1	0

Table 4: Highlighting two other groups

We can highlight two other groups of ones in this table.
Note that you might miss one of the groups *if you forget that the table wraps around at the edges.*
The terms given by these groups will be

- $\overline{a_2}\ a_1\ a_0$ (a_3 is irrelevant)
- $a_2\ \overline{a_1}\ a_0$ (a_3 is irrelevant)

Thus by combining those terms the final equation for the output is

$$prime = \overline{a_3}\ \overline{a_2}\ a_1 + \overline{a_3}\ a_2\ a_0 + \overline{a_2}\ a_1\ a_0 + a_2\ \overline{a_1}\ a_0$$

Actually, you may notice the last two terms can be simplified with an XOR, so we could rewrite the equation as

$$prime = \overline{a_3}\ a_2\ a_0 + \overline{a_3}\ \overline{a_2}\ a_1 + (a_2 \oplus a_1)\, a_0$$

We could also factor $\overline{a_3}$ out of the first two terms to get

$$prime = \overline{a_3}\,(a_2\ a_0 + \overline{a_2}\ a_1) + (a_2 \oplus a_1)\, a_0$$

Testing Logic

Maxima can be used to test the equation. Since maxima doesn't have exclusive or built in, I'll use the sum-of-produucts form, namely:

$$prime = \overline{a_3}\ \overline{a_2}\ a_1 + \overline{a_3}\ a_2\ a_0 + \overline{a_2}\ a_1\ a_0 + a_2\ \overline{a_1}\ a_0$$

```
 maxima
Maxima 5.25.1 http://maxima.sourceforge.net
using Lisp CLISP 2.49 (2010-07-07)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1) t1:(not a3) and (not a2) and a1;
(%o1)                    (not a3) and (not a2) and a1
(%i2) t2:(not a3) and  a2 and a0;
(%o2)                       (not a3) and a2 and a0
(%i3) t3:(not a2) and  a1 and a0;
(%o3)                       (not a2) and a1 and a0
(%i4) t4:a2 and (not a1) and a0;
(%o4)                       a2 and (not a1) and a0
(%i5) prime:t1 or t2 or t3 or t4;
(%o5) ((not a3) and (not a2) and a1) or ((not a3) and a2 and a0)
                    or ((not a2) and a1 and a0) or (a2 and (not a1) and a0)
(%i6) prime, a0=false,a1=false,a2=false,a3=false;
(%o6)                                false
(%i7) prime, a0=true,a1=false,a2=false,a3=false;
(%o7)                                false
(%i8) prime, a0=false,a1=true,a2=false,a3=false;
(%o8)                                true            ←———    2
(%i9) prime, a0=true,a1=true,a2=false,a3=false;
(%o9)                                true            ←———    3
(%i10) prime, a0=false,a1=false,a2=true,a3=false;
(%o10)                               false
(%i11) prime, a0=true,a1=false,a2=true,a3=false;
(%o11)                               true            ←———    5
(%i12) prime, a0=false,a1=true,a2=true,a3=false;
(%o12)                               false
(%i13) prime, a0=true,a1=true,a2=true,a3=false;
(%o13)                               true            ←———    7
(%i14) prime, a0=false,a1=false,a2=false,a3=true;
(%o14)                               false
(%i15) prime, a0=true,a1=false,a2=false,a3=true;
(%o15)                               false
(%i16) prime, a0=false,a1=true,a2=false,a3=true;
(%o16)                               false
(%i17) prime, a0=true,a1=true,a2=false,a3=true;
(%o17)                               true            ←———    11
(%i18) prime, a0=false,a1=false,a2=true,a3=true;
(%o18)                               false
(%i19) prime, a0=true,a1=false,a2=true,a3=true;
(%o19)                               true            ←———    13
(%i20) prime, a0=false,a1=true,a2=true,a3=true;
(%o20)                               false
(%i21) prime, a0=true,a1=true,a2=true,a3=true;
(%o21)                               false
```

The numbers shown are the only ones for which *prime* is true, so the equation is correct.