Number of Segments Circuit
PC/CP220 Project Phase II

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Creating Equations

Truth Table

Table 1 is a truth table for the *Number of Segments Circuit*.

number	$a_3a_2a_1a_0$	number of segments	$b_2b_1b_0$
0	0000	6	110
1	0001	2	010
2	0010	5	101
3	0011	5	101
4	0100	4	100
5	0101	5	101
6	0110	6	110
7	0111	3	011
8	1000	7	111
9	1001	6	110
10	1010	error	000
11	1011	error	000
12	1100	error	000
13	1101	error	000
14	1110	error	000
15	1111	error	000

Table 1: Truth Table

In order to determine the logic equations for the *Number of Segments Circuit*, the only quantities to consider are the binary inputs and outputs. **The fact that the bits are grouped together to represent numbers is irrelevant.**

The number of binary outputs for the circuit is the number of equations for the circuit, since each output implements a specific logic function (i.e. an equation).

The truth table containing only binary quantities is shown in Table 2

a_3	a_2	a_1	a_0	b_2	b_1	b_0
0	0	0	0	1	1	0
0	0	0	1	0	1	0
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	0	0
0	1	0	1	1	0	1
0	1	1	0	1	1	0
0	1	1	1	0	1	1
1	0	0	0	1	1	1
1	0	0	1	1	1	0
1	0	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	0	0	0

Table 2: Truth Table showing only binary inputs and outputs

Each bit of *output* produces an equation. In this case, a Karnaugh map can be used to determine simplified sum-of-products logic equations for each bit of output. After that, each equation can be tested independently using a computer algebra system.

Output b_2

The section of the truth table for output b_2 is shown in Table 3.

a_3	a_2	a_1	a_0	b_2
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Table 3: Truth Table for b_2

This produces the Karnaugh map for bit b_2 shown in Table 4.

b_2		a_1a_0			
		00	01	11	10
a_3a_2	00	1	0	1	1
	01	1	1	0	1
	11	0	0	0	0
	10	1	1	0	0

Table 4: Karnaugh Map Table for b_2

b_2			a_1a_0			
			00	01	11	10
a_3a_2	00		1	0	1	1
	01		1	1	0	1
	11		0	0	0	0
	10		1	1	0	0

Table 5: Karnaugh Map Table for b_2 highlighting two terms

We can highlight two groups of ones in this table. (The ones which are missed will be grouped later.) Note that you might miss one of the groups *if you forget that the table wraps around at the edges.*

The terms given by these groups will be

- $\overline{a_3}\ \overline{a_0}$ (a_2 and a_1 are irrelevant)
- $a_3\ \overline{a_2}\ \overline{a_1}$ (a_0 is irrelevant)

b_2			a_1a_0			
			00	01	11	10
a_3a_2	00		1	0	1	1
	01		1	1	0	1
	11		0	0	0	0
	10		1	1	0	0

Table 6: Karnaugh Map Table for b_2 highlighting two more terms

We can highlight two other groups of ones in this table. (The ones which were grouped earlier aren't highlighted.) The terms given by these groups will be

- $\overline{a_3}\ \overline{a_2}\ a_1$ (a_0 is irrelevant)

  (Note: Since yellow isn't easy to see, this term will be displayed in **bold** instead.)

  $\overline{\mathbf{a_3}}\ \overline{\mathbf{a_2}}\ \mathbf{a_1}$ (a_0 is irrelevant)

- $\overline{a_3}\ a_2\ \overline{a_1}$ (a_0 is irrelevant)

Thus by combining those terms the final equation for the output is

$$b_2 = \overline{a_3}\ \overline{a_0} + a_3\ \overline{a_2}\ \overline{a_1} + \overline{\mathbf{a_3}}\ \overline{\mathbf{a_2}}\ \mathbf{a_1} + \overline{a_3}\ a_2\ \overline{a_1}$$

Output b_1

The section of the truth table for output b_1 is shown in Table 7

a_3	a_2	a_1	a_0	b_1
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Table 7: Truth Table for b_1

This produces the Karnaugh map for bit b_1 shown in Table 8.

b_1		a_1a_0 00	01	11	10
a_3a_2	00	1	1	0	0
	01	0	0	1	1
	11	0	0	0	0
	10	1	1	0	0

Table 8: Karnaugh Map Table for b_1

b_1		a_1a_0 00	01	11	10
a_3a_2	00	1	1	0	0
	01	0	0	1	1
	11	0	0	0	0
	10	1	1	0	0

Table 9: Karnaugh Map Table for b_1 highlighting all terms

This will produce the following equation for b_1.

$$b_1 = \overline{a_2}\ \overline{a_1} + \overline{a_3}\ a_2\ a_1$$

Output b_0

The section of the truth table for output b_0 is shown in Table 10

a_3	a_2	a_1	a_0	b_0
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Table 10: Truth Table for b_0

This produces the Karnaugh map for bit b_0 shown in Table 11.

b_0		a_1a_0 00	01	11	10
a_3a_2	00	0	0	1	1
	01	0	1	1	0
	11	0	0	0	0
	10	1	0	0	0

Table 11: Karnaugh Map Table for b_0 highlighting all terms

This will produce the following equation for b_0.

$$b_0 = \overline{a_3}\ \overline{a_2}\ a_1 + \overline{a_3}\ a_2\ a_0 + a_3\overline{a_2}\ \overline{a_1}\ \overline{a_0}$$

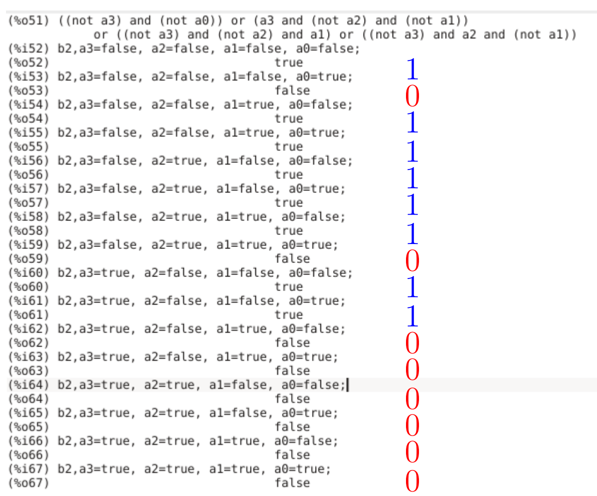
b_0		a_1a_0 00	01	11	10
a_3a_2	00	0	0	1	1
	01	0	1	1	0
	11	0	0	0	0
	10	1	0	0	0

Table 12: Karnaugh Map Table for b_0

Equation Testing

The equations were tested with Maxima.

Output b_2



```
(%o51) ((not a3) and (not a0)) or (a3 and (not a2) and (not a1))
        or ((not a3) and (not a2) and a1) or ((not a3) and a2 and (not a1))
(%i52) b2,a3=false, a2=false, a1=false, a0=false;
(%o52)                          true          1
(%i53) b2,a3=false, a2=false, a1=false, a0=true;
(%o53)                          false         0
(%i54) b2,a3=false, a2=false, a1=true, a0=false;
(%o54)                          true          1
(%i55) b2,a3=false, a2=false, a1=true, a0=true;
(%o55)                          true          1
(%i56) b2,a3=false, a2=true, a1=false, a0=false;
(%o56)                          true          1
(%i57) b2,a3=false, a2=true, a1=false, a0=true;
(%o57)                          true          1
(%i58) b2,a3=false, a2=true, a1=true, a0=false;
(%o58)                          true          1
(%i59) b2,a3=false, a2=true, a1=true, a0=true;
(%o59)                          false         0
(%i60) b2,a3=true, a2=false, a1=false, a0=false;
(%o60)                          true          1
(%i61) b2,a3=true, a2=false, a1=false, a0=true;
(%o61)                          true          1
(%i62) b2,a3=true, a2=false, a1=true, a0=false;
(%o62)                          false         0
(%i63) b2,a3=true, a2=false, a1=true, a0=true;
(%o63)                          false         0
(%i64) b2,a3=true, a2=true, a1=false, a0=false;
(%o64)                          false         0
(%i65) b2,a3=true, a2=true, a1=false, a0=true;
(%o65)                          false         0
(%i66) b2,a3=true, a2=true, a1=true, a0=false;
(%o66)                          false         0
(%i67) b2,a3=true, a2=true, a1=true, a0=true;
(%o67)                          false         0
```

Figure 1: Test of b_2

This matches the truth table, so the equation for b_2 is correct.

Output b_1

```
(%i19) b1:(not a2 and not a1) or (not a3 and a2 and a1);
(%o19)        ((not a2) and (not a1)) or ((not a3) and a2 and a1)
(%i20) b1,a3=false, a2=false, a1=false, a0=false;
(%o20)                             true          1
(%i21) b1,a3=false, a2=false, a1=false, a0=true;
(%o21)                             true          1
(%i22) b1,a3=false, a2=false, a1=true, a0=false;
(%o22)                             false         0
(%i23) b1,a3=false, a2=false, a1=true, a0=true;
(%o23)                             false         0
(%i24) b1,a3=false, a2=true, a1=false, a0=false;
(%o24)                             false         0
(%i25) b1,a3=false, a2=true, a1=false, a0=true;
(%o25)                             false         0
(%i26) b1,a3=false, a2=true, a1=true, a0=false;
(%o26)                             true          1
(%i27) b1,a3=false, a2=true, a1=true, a0=true;
(%o27)                             true          1
(%i28) b1,a3=true, a2=false, a1=false, a0=false;
(%o28)                             true          1
(%i29) b1,a3=true, a2=false, a1=false, a0=true;
(%o29)                             true          1
(%i30) b1,a3=true, a2=false, a1=true, a0=false;
(%o30)                             false         0
(%i31) b1,a3=true, a2=false, a1=true, a0=true;
(%o31)                             false         0
(%i32) b1,a3=true, a2=true, a1=false, a0=false;
(%o32)                             false         0
(%i33) b1,a3=true, a2=true, a1=false, a0=true;
(%o33)                             false         0
(%i34) b1,a3=true, a2=true, a1=true, a0=false;
(%o34)                             false         0
(%i35) b1,a3=true, a2=true, a1=true, a0=true;
(%o35)                             false         0
```

Figure 2: Test of b_1

This matches the truth table, so the equation for b_1 is correct.

Output b_0

```
(%i1) b0:(not a3 and not a2 and a1) or (not a3 and a2 and a0) or (a3 and not a2 and not a1 and not a0);
(%o1) ((not a3) and (not a2) and a1) or ((not a3) and a2 and a0)
                                   or (a3 and (not a2) and (not a1) and (not a0))
(%i2) b0,a3=false, a2=false, a1=false, a0=false;
(%o2)                              false            0
(%i4) b0,a3=false, a2=false, a1=false, a0=true;
(%o4)                              false            0
(%i5) b0,a3=false, a2=false, a1=true, a0=false;
(%o5)                              true             1
(%i6) b0,a3=false, a2=false, a1=true, a0=true;
(%o6)                              true             1
(%i7) b0,a3=false, a2=true, a1=false, a0=false;
(%o7)                              false            0
(%i8) b0,a3=false, a2=true, a1=false, a0=true;
(%o8)                              true             1
(%i9) b0,a3=false, a2=true, a1=true, a0=false;
(%o9)                              false            0
(%i10) b0,a3=false, a2=true, a1=true, a0=true;
(%o10)                             true             1
(%i11) b0,a3=true, a2=false, a1=false, a0=false;
(%o11)                             true             1
(%i12) b0,a3=true, a2=false, a1=false, a0=true;
(%o12)                             false            0
(%i13) b0,a3=true, a2=false, a1=true, a0=false;
(%o13)                             false            0
(%i14) b0,a3=true, a2=false, a1=true, a0=true;
(%o14)                             false            0
(%i15) b0,a3=true, a2=true, a1=false, a0=false;
(%o15)                             false            0
(%i16) b0,a3=true, a2=true, a1=false, a0=true;
(%o16)                             false            0
(%i17) b0,a3=true, a2=true, a1=true, a0=false;
(%o17)                             false            0
(%i18) b0,a3=true, a2=true, a1=true, a0=true;
(%o18)                             false            0
```

Figure 3: Test of b_0

This matches the truth table, so the equation for b_0 is correct.

Summary

The equations for the outputs are:

$$b_2 = \overline{a_3}\ \overline{a_0} + a_3\ \overline{a_2}\ \overline{a_1} + \overline{a_3}\ \overline{a_2}\ a_1 + \overline{a_3}\ a_2\ \overline{a_1}$$

$$b_1 = \overline{a_2}\ \overline{a_1} + \overline{a_3}\ a_2\ a_1$$

$$b_0 = \overline{a_3}\ \overline{a_2}\ a_1 + \overline{a_3}\ a_2\ a_0 + a_3\overline{a_2}\ \overline{a_1}\ \overline{a_0}$$

These equations have been tested and verified to be correct.