1 Problem 23

1.1 Problem 23, 9th edition

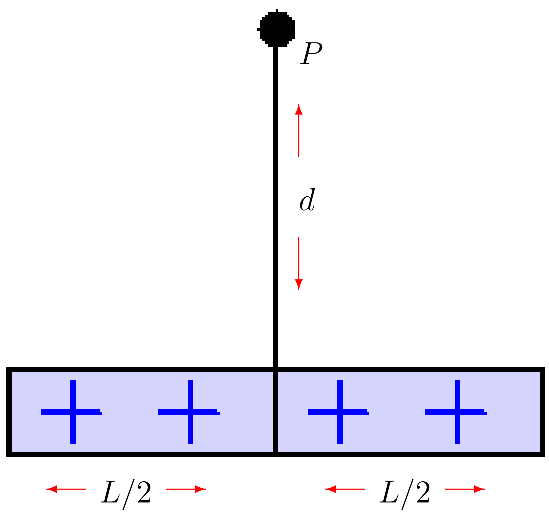

Figure 1: From book, for part a

- Charge density is $\lambda = +3.68pC/m$
- $L = 6cm$
- $d = 8.0cm$
- Assume $V = 0$ at infinity.

We want to show:

(a) What is V at P along the rod's perpendicular bisector?

(b) What is V at P if half of the rod is replaced by a section of equal but opposite charge density?

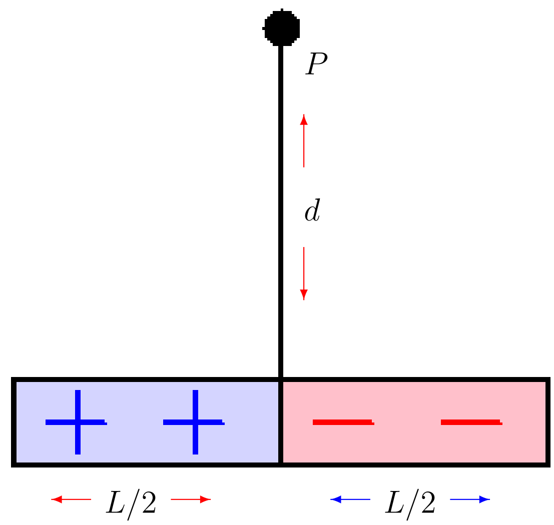

Figure 2: From book, for part b

BEFORE MATH

For problems involving potential, the first thing to do is to figure out which method of calculating it will be easier.

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

$$V = -\int_i^f \vec{E}\cdot\vec{ds}$$

Things to note:

1. For part b, the symmetry means that by the integration of charge formula $V \equiv 0$. (Each place on the left side has a corresponding piece of opposite charge at the same distance on the right side.)

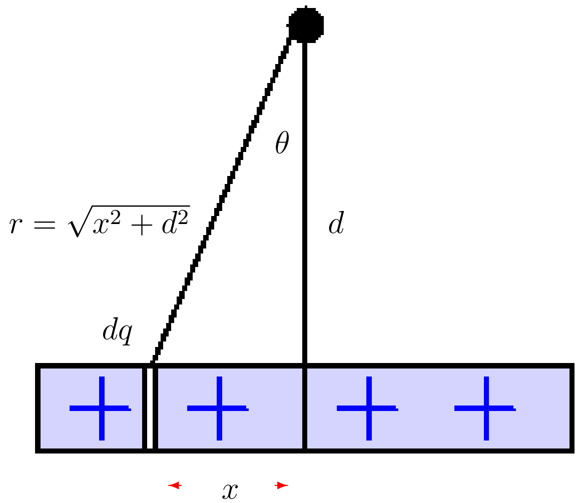

Figure 3: Geometry

CALCULATIONS

VARIABLE SUBSTITUTION

Looking at one side:

$$dq = \lambda \quad dx$$

$$\tan\theta = x/d$$

$$\theta = \arctan\left(x/d\right)$$

$$x = d \quad \tan\theta$$

$$dx = d \quad \sec^2\theta \quad d\theta$$

When $x = 0$, $\theta = 0$
When $x = L/2$, $\theta = \arctan\left(\frac{L}{2d}\right)$

$$V_L = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_0^{L/2} \frac{\lambda \quad dx}{\sqrt{x^2+d^2}}$$

where V_L means *"the potential due to the left half of the charged rod"*. Substituting variables

$$V_L = \frac{1}{4\pi\epsilon_0}\int_0^{L/2} \frac{\lambda \quad dx}{\sqrt{x^2+d^2}} = \frac{1}{4\pi\epsilon_0}\int_0^{\arctan\left(\frac{L}{2d}\right)} \frac{\lambda \quad d \quad \sec^2\theta \quad d\theta}{\sqrt{(d \quad \tan\theta)^2 + d^2}}$$

So

$$V_L = \frac{1}{4\pi\epsilon_0}\int_0^{\arctan\left(\frac{L}{2d}\right)} \frac{\lambda \quad d \quad \sec^2\theta \quad d\theta}{d\sqrt{(\tan\theta)^2+1}}$$

Since

$$1+\tan^2\theta = \sec^2\theta$$

then

$$V_L = \frac{1}{4\pi\epsilon_0}\int_0^{\arctan\left(\frac{L}{2d}\right)} \frac{\lambda \quad d \quad \sec^2\theta \quad d\theta}{d\sqrt{\sec^2\theta}}$$

so

$$V_L = \frac{1}{4\pi\epsilon_0}\int_0^{\arctan\left(\frac{L}{2d}\right)} \frac{\lambda \quad d \quad \sec^2\theta \quad d\theta}{d\sec\theta}$$

and thus

$$V_L = \frac{1}{4\pi\epsilon_0}\int_0^{\arctan\left(\frac{L}{2d}\right)} \frac{\lambda \quad \cancel{d} \quad \sec\theta \quad d\theta}{\cancel{d}}$$

and finally

$$V_L = \frac{\lambda}{4\pi\epsilon_0}\int_0^{\arctan\left(\frac{L}{2d}\right)} \sec\theta \quad d\theta$$

$$V_L = \frac{\lambda}{4\pi\epsilon_0}\left[\ln\left(\sec\theta+\tan\theta\right)\right]_0^{\arctan\left(\frac{L}{2d}\right)}$$

Since $\cos\theta = \frac{d}{\sqrt{x^2+d^2}}$ then $\sec\theta = \frac{\sqrt{x^2+d^2}}{d}$ so when $x = L/2$, $\sec\theta = \frac{\sqrt{L/2^2+d^2}}{d}$

$$V_L = \frac{\lambda}{4\pi\epsilon_0}\left(\left[\ln\left(\frac{\sqrt{L/2^2+d^2}}{d} + \left(\frac{L}{2d}\right)\right)\right] - \left[\ln\left(1+0\right)\right]\right)$$

$$V_L = \frac{\lambda}{4\pi\epsilon_0}\left(\left[\ln\left(\sqrt{\left(\frac{L}{2d}\right)^2+1}+\left(\frac{L}{2d}\right)\right)\right]\right)$$

Remember this is the potential due to one side, so the total is twice this, so

$$V = 2\frac{\lambda}{4\pi\epsilon_0}\left(\left[\ln\left(\sqrt{\left(\frac{L}{2d}\right)^2+1}+\left(\frac{L}{2d}\right)\right)\right]\right)$$

$$V = 2\frac{+3.68pC/m}{4\pi 8.85\times 10^{-12}}\left(\left[\ln\left(\sqrt{\left(\frac{6}{2\times 8}\right)^2+1}+\left(\frac{6}{2\times 8}\right)\right)\right]\right)$$

(Note I didn't put L and d in MKS units. Why not? If they are being divided, the units go away *as long as they are consistent.*)

$$V \approx 2\frac{+3.68pC/m}{4\pi 8.85\times 10^{-12}}\,(0.3667) = 24.3mV$$