1 Problem 22

1.1 Problem 20, 8th edition

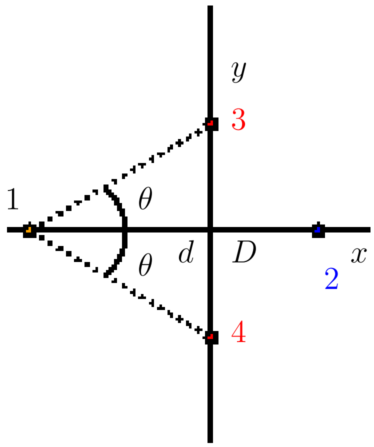

Figure 1: From book

- $\theta = 30.0°$
- $d = 2.00cm$
- $q_2 = +8.00 \times 10^{-19}C$
- $q_3 = q_4 = -1.6 \times 10^{-19}C$

We want to find:

(a) What value of D corresponds to $F_{1_{NET}} = 0$?

(b) If q_3 and q_4 were moved toward the origin symmetrically, would D increase, decrease, or stay the same?

BEFORE MATH

Things to note:

1. By symmetry, $F_{13_y} = -F_{14_y}$, and $F_{12_y} \equiv 0$, so $F_{1_{NET}} \equiv F_{1_x}$.
2. Also, by symmetry, $F_{13_x} = F_{14_x}$.
3. The sign of q_1 only affects the sign of the force, not the magnitude, so we don't need to know the sign of q_1.
4. Since the sign of q_2 is the opposite of the sign of q_3 and q_4, there will be some point where they cancel out. (It would have to be either to the left or the right of all three charges.)

Consequences:

1. If q_3 and q_4 are moved towards the axis, the x components of their forces on q_1 will *increase*, so to balance this with the force from q_2 would require q_2 to get closer to q_1.

   D will have to *decrease* as q_3 and q_4 are moved towards the axis. **This answers (b) above.**

CALCULATIONS

To determine F_{13_x}, we have to resolve into components.

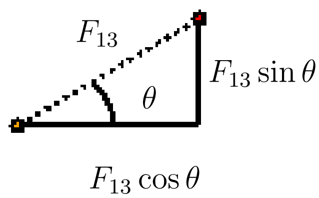

Figure 2: Resolving force into components

By the logic above,

$$F_{1_{NET}} \equiv F_{1_x} = 2F_{13_x} + F_{12}$$

$$F_{13_x} = F_{13}\cos\theta$$

and so

$$F_{1_{NET}} = 2F_{13}\cos\theta + F_{12}$$

Using similar triangles, we can create a new triangle where the base is the distance d, to determine the distances for all of the forces.

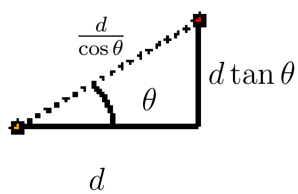

Figure 3: Using similar triangles

$$F_{13} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1q_3}{{r_{13}}^2}$$

so

$$F_{13}\cos\theta = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1q_3}{{r_{13}}^2}\cos\theta$$

From the similar triangle, $r_{13} = \frac{d}{\cos\theta}$ so

$$F_{13}\cos\theta = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_3}{\left(\frac{d}{\cos\theta}\right)^2}\cos\theta$$

giving

$$F_{13}\cos\theta = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_3}{d^2}\cos\theta^3$$

Also,

$$F_{12} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_2}{{r_{12}}^2}$$

and $r_{12} = d + D$ so

$$F_{12} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_2}{(d+D)^2}$$

giving

$$F_{12} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_2}{d^2(1+D/d)^2}$$

So

$$F_{1_{NET}} = 2F_{13}\cos\theta + F_{12} = 2\left(\left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_3}{d^2}\cos^3\theta\right) + \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_2}{d^2}\frac{1}{(1+D/d)^2}$$

which becomes

$$F_{1_{NET}} = \left(\frac{q_1}{4\pi\epsilon_0 d^2}\right)\left(2\left(q_3\cos^3\theta\right) + q_2\frac{1}{(1+D/d)^2}\right)$$

When $F_{1_{NET}} \equiv 0$, then

$$2\left(q_3\cos^3\theta\right) + q_2\frac{1}{(1+D/d)^2} = 0$$

so

$$2\left(q_3\cos^3\theta\right)(1+D/d)^2 = -q_2$$

$$(1+D/d)^2 = \frac{-q_2}{2\left(q_3\cos^3\theta\right)} = \frac{-q_2}{q_3}\frac{1}{2\left(\cos^3\theta\right)}$$

Since $\theta = 30.0°$, $q_2 = +8.00 \times 10^{-19}C$ and $q_3 = q_4 = -1.6 \times 10^{-19}C$ then

$$(1+D/d)^2 = \left(\frac{-8}{-1.6}\right)\frac{1}{2\left(\left(\frac{\sqrt{3}}{2}\right)^3\right)} \approx 5\frac{1}{2\,(0.65)} = 3.85$$

or

$$(1 + D/d) \approx \pm 1.96$$

This gives two solutions, namely

$$1 + D/d \approx 1.96 \rightarrow D/d \approx 0.96$$

and

$$1 + D/d \approx -1.96 \rightarrow D/d \approx -2.96$$

Since we know that a solution should exist for positive values of D, that's the solution we want. Since $d = 2.00cm$, then

$$D/d \approx 0.96 \rightarrow D \approx 0.96 \times 2.00cm = 1.92cm$$

This is the answer to (a) above. (The other solution can't work, since that would make the forces from all of the charges in the same direction, so it wouldn't have a net force of zero.)

CHECK

$$F_{1_{NET}} = 2F_{13}\cos\theta + F_{12} = \left(\frac{q_1}{4\pi\epsilon_0}\right)\left(2\frac{q_3}{d^2}\cos^3\theta + \frac{q_2}{d^2}\frac{1}{(1+D/d)^2}\right)$$

Plug in the values and see....