PC212 Tutorial Problem
Wilfrid Laurier University

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Wilfrid Laurier University

April 29, 2011

Definition

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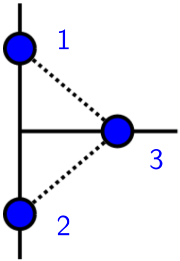

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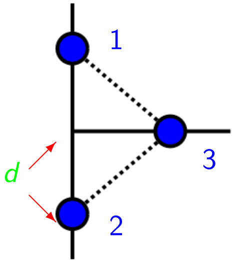

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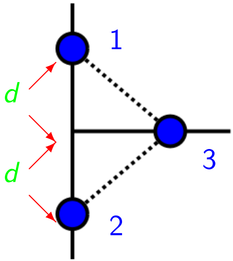

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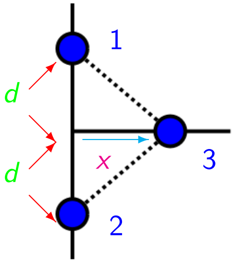

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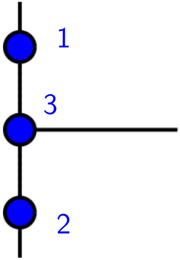

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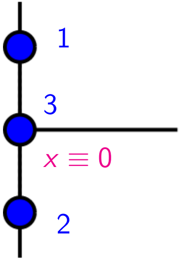

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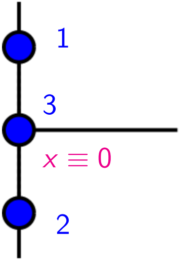

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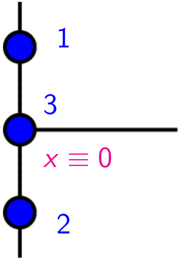

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$$|F_x|_{3,net} = |F_{x31} + F_{x32}| = 2\,|F_{x31}|$$

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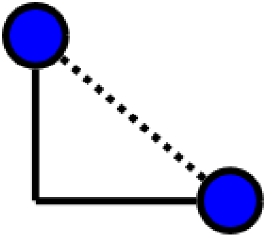

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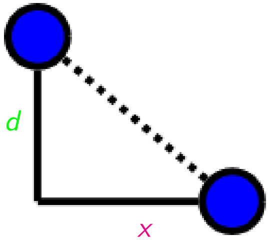

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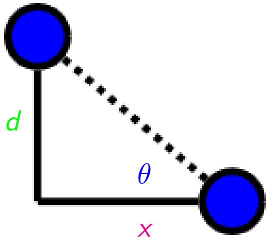

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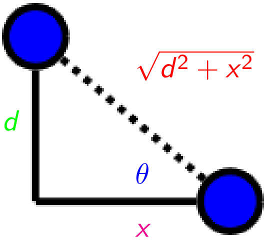

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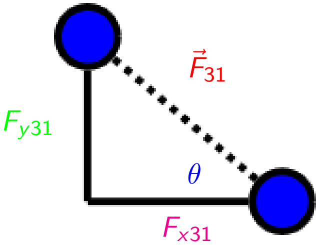

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since none of the other terms are zero.

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