Transient Process for RLC Circuit

Purpose

1. Observe the transient process for RLC circuit
2. Study the transient characteristics for RLC circuit

Theory

For a circuit contained a resistor R, a capacitor C and an inductor L, when the voltage of source changes suddenly from 0 to *E* volts the voltage or current of the circuit can't respond it immediately. It performs a transient process.

1. A simple RC circuit

   As shown in Figure 1, a simple RC circuit is supplied by a source voltage u(t) which is a square signal jumped from 0 to E or vice verse. The circuit equation is described as:

   $$iR + u_c = E \qquad (1)$$

   where u_c is the voltage across the capacitor, and i is the AC current passing through the capacitor and equals to

   $$i = c\frac{du_c}{dt} \qquad (2)$$

   The solution of Equation (1) is:

   $$u_c = E(1 - e^{-\frac{t}{RC}}) \qquad (3)$$

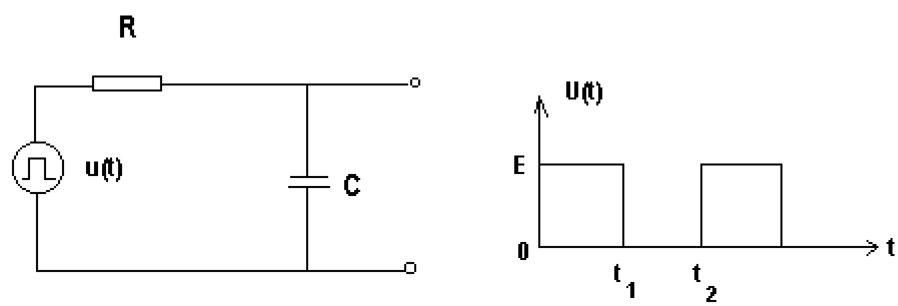

Figure 1 A simple RC circuit and its source voltage

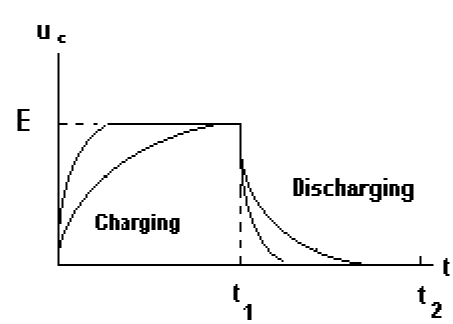

Figure 2 The voltage changed on the capacitor

It tells us that u_c increases exponentially when charging and decreases exponentially when discharging as shown in Figure 2.

2. A series RLC circuit

Figure 3 is a RLC circuit described by the following equation:

$$L\frac{d^2u_c}{dt^2}+R\frac{du_c}{dt}+\frac{1}{c}u_c=0 \qquad (3)$$

The solution of Equation (3) is:

$$u_c=Ee^{-\frac{t}{\tau}}\cos\omega t \qquad (4)$$

where τ is a time constant determined by

$$\tau=\frac{2L}{R} \qquad (5)$$

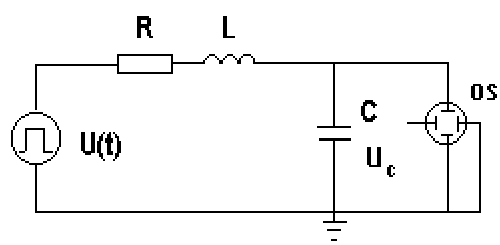

Figure 3 A RLC circuit

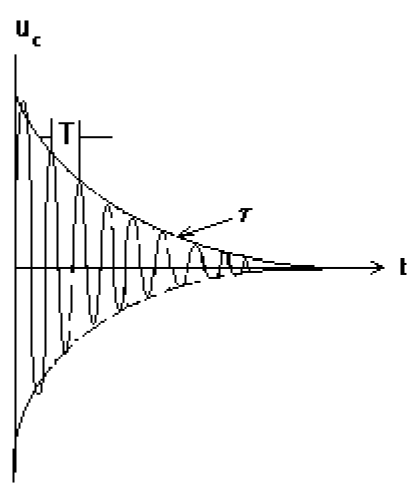

Figure 4 The transient voltage u_c across capacitor for a RLC circuit

In Equation (4), ω is the natural resonant frequency determined by:

$$\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} \qquad (5)$$

Figure 4 shows the transient respond of the series RLC circuit on the capacitor. Figure 5 is a typical waveform observed from the oscilloscope. We can determine τ and ω based on the waveform using the following method. As shown in Figure 5, when $t=t_n$ the amplitude of oscillation u_n can be expressed as:

$$u_n = Ee^{-\frac{t_n}{\tau}} \cos \omega t_n \qquad (6)$$

Assuming that the period of oscillation is T, when $t=t_n+NT$ the amplitude of oscillation u_{n+NT} then becomes:

$$u_{n+NT} = Ee^{-\frac{t_n+NT}{\tau}} \cos \omega (t_n + NT) \qquad (7)$$

where

$$\omega (t_n + NT) = \omega t_n + 2\pi N \qquad (8)$$

Therefore, Equation (7) becomes then

$$u_{n+NT} = Ee^{-\frac{t_n+NT}{\tau}} \cos \omega t_n \qquad (9)$$

Taking the ratio of Equation (9) and (6), we find that:

$$\frac{u_{n+N}}{u_n} = e^{-\frac{NT}{\tau}} \qquad (10)$$

Then the time constant τ can be easily determined as:

$$\tau = \frac{-NT}{\ln(\frac{u_{n+N}}{u_n})} \qquad (11)$$

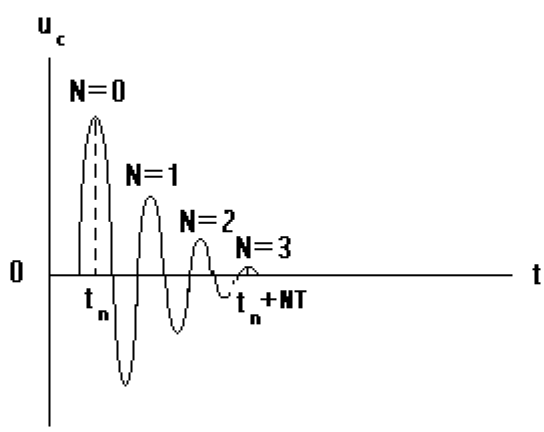

Figure 5 Determining τ and ω from the a waveform

Apparatus

1. Oscilloscope with two channels
2. Square signal generator
3. Inductor
4. Capacitor
5. Resistor box

Procedure

1. A simple RC circuit
   a. Observe charging and discharging for a *RC* circuit
      - Construct the apparatus according to Figure 1
      - Use square signal generator with f=100Hz as u(t) (If f is too high, what will happen?)
      - Take C=0.1μF, and R=300Ω, 1KΩ and 3KΩ respectively
      - Set input as DC for oscilloscope
      - Use oscilloscope to observe both u_c and u_R by two channels shown in Figure 6
      - Sketch $u_c \sim t$ and $u_R \sim t$ for three different resistances

   b. Determine the time constants τ, and compare the experiment values with theoretical ones

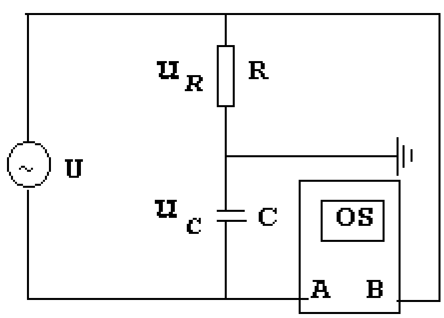

Figure 6 Measuring u_c and u_R

2. A series RLC circuit
   a. Study the transient process for the *RLC* circuit
      - Construct the apparatus according to Figure 3
      - Use square signal generator with f=100Hz as u(t) (If f is too high or too low, what will happen?)
      - Take C=4700pF, L=0.6 or 0.3 H and $R=R_0+R_L+R_M$. R_M-the internal resistance of generator (usually 50Ω), R_L-the impedance of the inductor, R_0-the resistance of the box. Make sure $R^2 <4L/C$, otherwise, see what will happen
      - Sketch the oscillation waveform
      - Calculate τ and ω by using Equation (11) when setting R_0=0
      - Calculate R_L if R_0=0 and R_M=50Ω by using Equation (5)

   b. Increase the resistance until no oscillation can been seen