Steady-State Processes for RLC Circuit and Diodes

**Purpose**
1. Observe the amplitude and phase properties of RLC circuit driven by sinusoidal source
2. Study the sinusoidal characteristics for RLC circuit
3. Measure amplitude characteristics for diodes and Zener Diodes

**Theory**
For a circuit contained a resistor $R$, a capacitor $C$ and an inductor $L$, when a sinusoidal voltage applies on the RLC circuit the amplitude and phase characteristics varied with frequencies of input voltage are particularly important.

1. Amplitude-frequency characteristics for a series circuit
   a. RC circuit
   As shown in Figure 1, a simple RC circuit is supplied by a source with sinusoidal voltage.

   \[
   \dot{U} = \dot{U}_R + \dot{U}_C = iR + i \frac{1}{j\omega C} = i\dot{Z} \tag{1}
   \]

   where

   \[
   \dot{Z} = R + \frac{1}{j\omega C} \tag{2}
   \]

   is the impedance of the series RC circuit. Its absolute value is then

   \[
   Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \tag{3}
   \]

   ![Figure 1 A simple RC circuit with sinusoidal source](image)

   From Equation (1), we find that
\[ \dot{I} = \frac{\dot{U}}{\dot{Z}} = \frac{\dot{U}}{R + \frac{1}{j\omega C}} \]  \hspace{1cm} (4)

The effective value of current is equal to
\[ I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \]  \hspace{1cm} (5)

where \( U \) is the effective value of the total voltage. The effective values on \( R \) and \( C \) are:
\[ U_R = IR = \frac{UR\omega C}{\sqrt{1 + (RC\omega)^2}} \]  \hspace{1cm} (6)
\[ U_C = \frac{I}{\omega C} = \frac{U}{\sqrt{1 + (RC\omega)^2}} \]  \hspace{1cm} (7)

respectively. It is true that \( U_R \) and \( U_C \) are varied with \( \omega \), but \( U_R^2 + U_C^2 \) remains a constant.

b. A series RLC circuit

Figure 3 is a RLC circuit described by the following equation:

\[ \dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C = \dot{I}[R + i(\omega L - \frac{1}{\omega C})] = i\dot{Z} \]  \hspace{1cm} (8)

Similarly, the impedance of RLC circuit is equal to:
\[ Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \]  \hspace{1cm} (9)

and the effective current is:
\[ I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \]  

where \( U \) – effective value of total voltage.

It is easy to conclude that when \( \omega L = \frac{1}{\omega C} \), \( Z \) possesses minimum while \( I \) shows maximum if \( U \) keeps constant. When this frequency has reached, the radian frequency is called resonant one \( \omega_0 \). The current \( I \) varying with frequency is shown in Figure 3 known as resonant curve for series RLC circuit. Generally, a quality factor \( Q \) is used for judging how good the performance for the resonant circuit. It can be proved that:

\[ Q = \omega_0 = \frac{\omega_0}{\Delta \omega} = \frac{\omega_0}{\omega_2 - \omega_1} \]  

where \( \omega_1 \) and \( \omega_2 \) are defined the frequencies when \( I \) decreases to \( I' \) from its maximum \( I_m \) down to 70.7% of \( I_m \). In Figure 3, there are two resonant curves marked with \( Q_1 \) and \( Q_2 \). Compared two of them, we find that \( Q_1 > Q_2 \).

Figure 3  A resonant curve for a series RLC circuit

2. Phase-frequency characteristics
   a. It can be seen that the phase difference between \( U \) and \( I \), or the phasor of \( Z \) for RC circuit satisfies:

\[ \tan \phi = -\frac{1}{\omega RC} \]  

(12)
The phase-frequency characteristics for RC and RLC circuits are shown in Figure 4 and 5 respectively.

3. Measuring phase difference $\phi$ by using an oscilloscope

There are two ways to measure the phase difference between $\dot{U}$ and $\dot{I}$ by using an oscilloscope: Lissajous pattern and two traces.
By using Lissajous pattern, U and I input to A and B channels, as the result, the an ellipse is appeared on screen of oscilloscope as shown in Figure 6, then the phase difference between U and I is:

$$\phi = \arcsin\left(\frac{a}{V}\right) = \arcsin\left(\frac{b}{H}\right) \quad (13)$$

By using two traces way as shown in Figure 7, the phase difference is:

$$\phi = \frac{AB}{AC} \times 360^\circ \quad (14)$$

4. Amplitude characteristics for diodes

An example of a non-ohmic device is the diode. A solid state diode is made by joining together a p-type semiconductor (the anode) to an n-type semiconductor (the cathode). Most devices of this type are packaged in a small, cylindrical epoxy casing with a wire lead protruding from each end. The cathode is usually marked with a painted band near one end.

The I-V characteristic of a diode is governed by the (theoretical) expression:

$$I = I_o\left(e^{V_r/V_T} - 1\right) \quad (15)$$

where $I_0$ is the reverse saturation current and

$$V_T = \frac{kT}{e} \quad (16)$$

is a parameter which is approximately 25 mV at room temperature (300K). In addition, $I_0$ is temperature dependent and is given by:

$$I_o = BT^3 e^{V_g/V_T} \quad (17)$$

where $B$ is a constant, and $V_g$ is the semiconductor energy gap in volts. Equation (15) is known as the ideal diode equation. It does not take into account the bulk resistance of the semiconductor, or the contact resistance between the metallic leads and the p-type and n-type materials.

When a diode is reverse biased, at a sufficient voltage it will undergo breakdown, (which is not as dangerous as it sounds), and as it approaches breakdown its current is given by

$$I = \frac{I_o}{1 - \left(\frac{V}{V_{BR}}\right)^n} \quad (18)$$

where $V_{BR}$ is the reverse breakdown voltage of the diode, and $n$ is a parameter (~ 1) determined by experiment. (Note that for this equation, currents and voltages are considered positive, even though the device is reverse biased.) $V_{BR}, I_0, V_T, I_T$ are
indicated in Figure 8.

![Diagram of I-V characteristics for a diode](image.png)

Figure 8  I-V characteristics for a diode

In this experiment, you shall examine the behavior (the I-V characteristic) of both ohmic and non-ohmic devices as a function of current.

**Apparatus**
1. Oscilloscope with two channels
2. Sinusoidal signal generator
3. Inductor (0.3H or 0.6H)
4. Capacitor
5. Resistor box
6. Diode and Zener Diode

![Diagram of RC circuit](image1.png)  ![Diagram of RLC circuit](image2.png)

Figure 9 Measuring $U_C \sim f$ and $U_R \sim f$  Figure 10 Measuring $I \sim f$ for RLC

For RC circuit

**Procedure**
1. A simple RC circuit (Optional)
   - Construct the apparatus according to Figure 9
   - Use sinusoidal signal generator with $f=100\text{Hz} \sim 200\text{kHz}$ as $u(t)$
   - Take $c=0.1\mu F$, and $R=400\Omega$
   - Measure $U_C \sim f$ and $U_R \sim f$
   - Sketch $U_C \sim f$ and $U_R \sim f$
2. Measure amplitude characteristics curve $I \sim f$ for RLC series circuit
   - Construct the apparatus according to Figure 10
• Take $c=0.1\,\mu\text{F}$, and $R=50\,\Omega$ and $L=0.3$ or $0.6\text{H}$
• Use sinusoidal signal generator with an appropriate range for $f$ as $u(t)$
• Sketch curve $I \sim f$, and then Calculate $Q$

3. Measure phase characteristics curve $\Delta\phi \sim f$ for RLC series circuit

Figure 11  Measuring phase characteristics curve $\Delta\phi \sim f$ for RLC

• Construct the apparatus according to Figure 11
• Take $C=0.1\,\mu\text{F}$, $L=0.3\text{ H (or 0.6H)}$ and $R=400\,\Omega$
• Measure phase values $\Delta\phi$ between $U$ and $I$ for different $f$, and then draw the curve for $\Delta\phi \sim f$.

4. Measure $I-V$ for a Zener diode

Figure 12 Measuring $I \sim V$ for a diode

• Construct the apparatus according to Figure 12
• Use sinusoidal signal generator as $u(t)$, take $R=1\,\text{K}\Omega$
• Sketch curve $I \sim V$ for a Zener diode
• Record the values of $I_0$, $I_{\text{Fmax}}$, and $V_{BR}$ for the diodes to be used, where $I_0$ is the reverse saturation current, typically nanoamps. $I_{\text{Fmax}}$ is the maximum forward current, typically up to 100 mA or so. $V_{BR}$ is the reverse breakdown voltage, typically 50 or more volts for a regular diode, and less for a Zener diode.
• Compare your results to the manufacturer's specifications.