Electronics
Internal resistance of a voltage source

Terry Sturtevant

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- function generators

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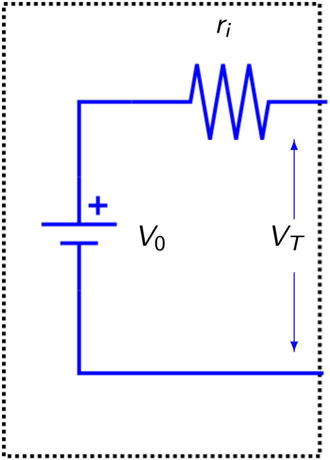
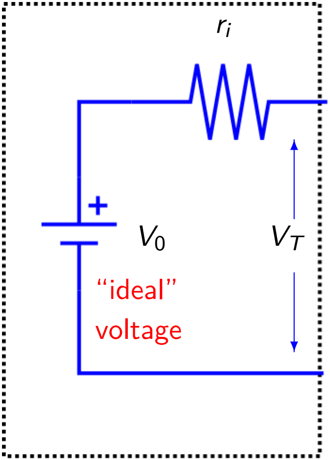
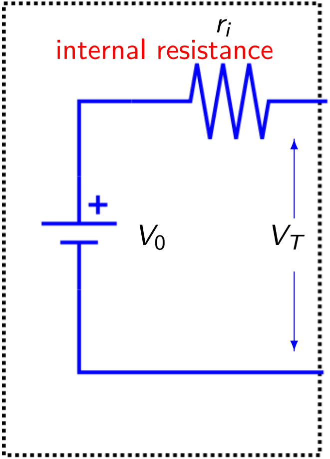
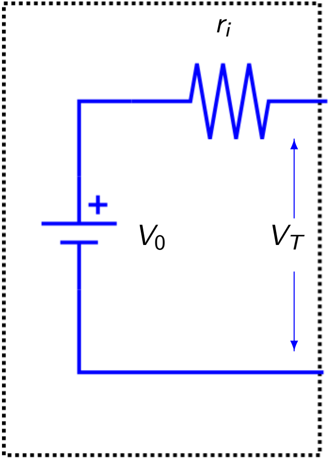
This can be represented as a resistance in series with the *ideal* voltage output of the device.

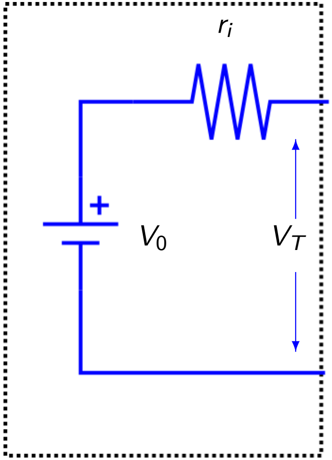

r_i
+
V_0
V_T


r_i
+
V_0
V_T
"ideal"
voltage


r_i
internal resistance
+
V_0
V_T



terminal voltage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The potential difference between the terminals of a battery, V_T, decreases as the current supplied by the battery increases.

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This is due to the potential drop which appears across the battery's **internal resistance** r_i.

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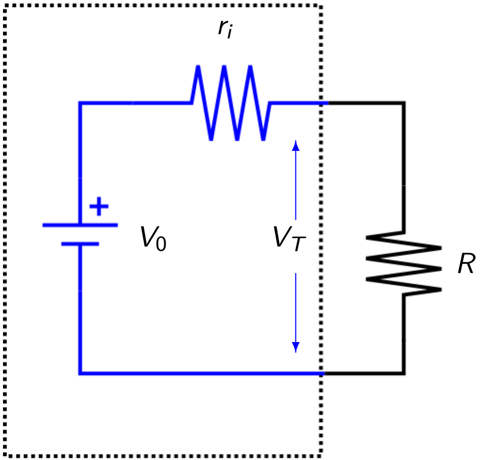
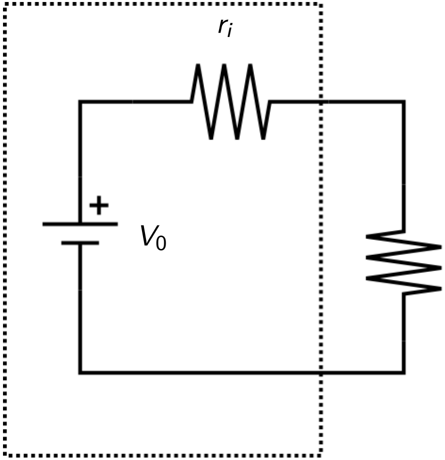
This is due to the potential drop which appears across the battery's **internal resistance** r_i.

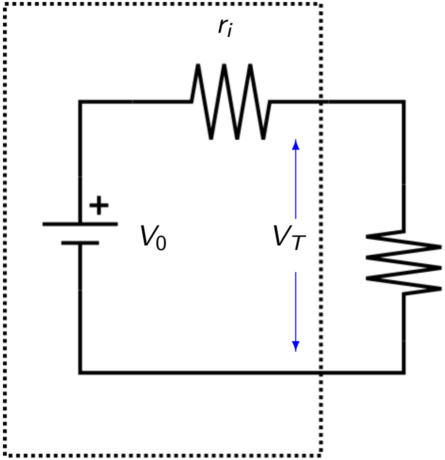
Thus the maximum potential difference is attained only with *zero* output current.

We can see the effect of this by watching how the measured voltage changes if we put a resistance across the terminals of the device.


r_i
V_0
V_T
R

Notice that what we've done is to create a voltage divider with r_i and R.


r_i
+
V_0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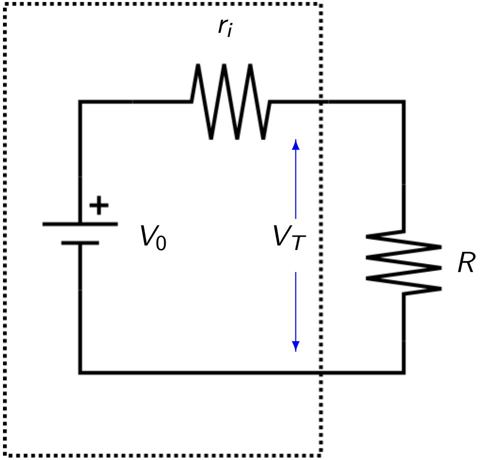
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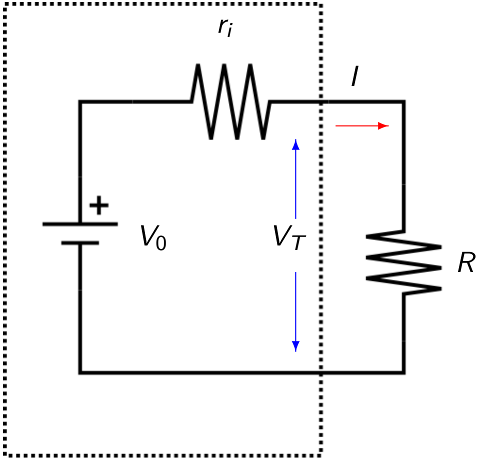
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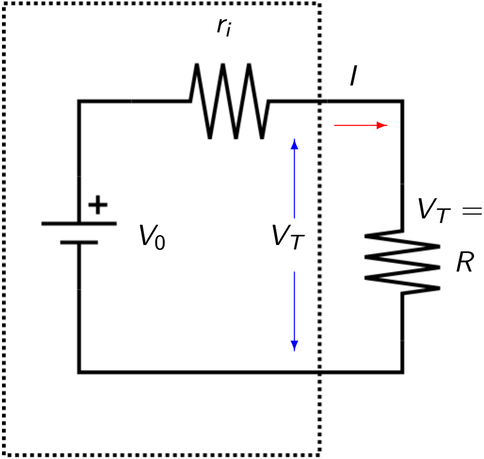

$$V_T = V_0 \left(\frac{R}{R+r_i} \right)$$

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$$r_i = \left(\frac{V_0}{V_T} - 1 \right) R$$

So if we can measure the voltage with no load; (i.e. essentially V_0), and then the voltage V_T with some resistance R, then we can determine r_i.

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Note that the bigger the drop with load, the bigger r_i is, which makes sense.

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Note that the bigger the drop with load, the bigger r_i is, which makes sense.

Question: If R is chosen to equal r_i, what will be the value of V_T?

Graphical analysis

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To graph this, it makes sense to linearize the equation:

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If we plot

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If we plot $\frac{1}{V_T}$

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$$\frac{1}{V_T} = \frac{1}{V_0} + \frac{r_i}{V_0}\frac{1}{R}$$

If we plot $\frac{1}{V_T}$ versus $\frac{1}{R}$

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$$\frac{1}{V_T} = \frac{1}{V_0} + \frac{r_i}{V_0}\frac{1}{R}$$

If we plot $\frac{1}{V_T}$ versus $\frac{1}{R}$ then we will get a straight line with a slope of $\frac{r_i}{V_0}$ and a y-intercept of $\frac{1}{V_0}$.
We can get r_i by dividing the slope by the y-intercept.

Here's the setup:

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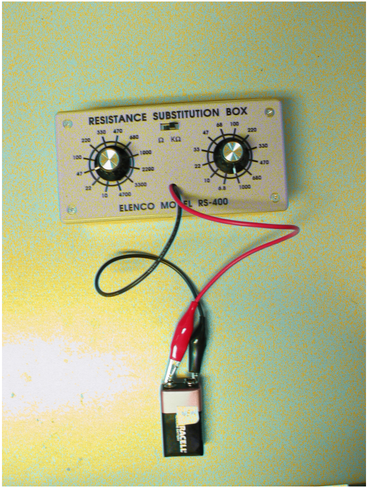

Here are the data collected for a new 9 volt battery.

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27

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∞	9.27
1000000	9.27

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∞	9.27
1000000	9.27
680000	9.27

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20
1000	9.17

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20
1000	9.17
680	9.14

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20
1000	9.17
680	9.14
330	9.08

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20
1000	9.17
680	9.14
330	9.08
220	9.01

External resistance R (Ω)	Terminal voltage V_T (V)
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20
1000	9.17
680	9.14
330	9.08
220	9.01
100	8.80

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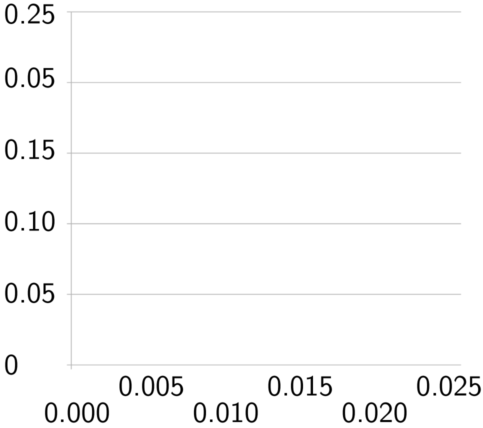
∞	9.27
1000000	9.27
680000	9.27
$\vdots$	$\vdots$
6800	9.24
3300	9.22
2200	9.20
1000	9.17
680	9.14
330	9.08
220	9.01
100	8.80
47	8.40

The graph of $\frac{1}{V_T}$ versus $\frac{1}{R}$ looks like this: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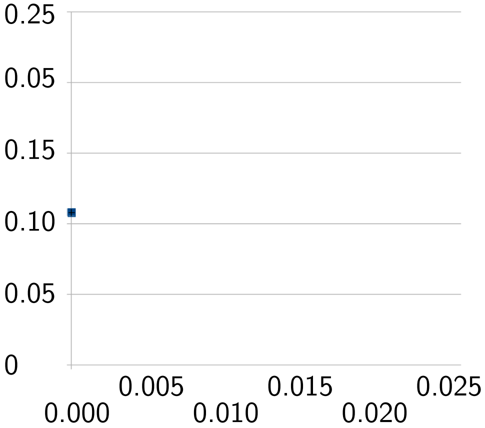
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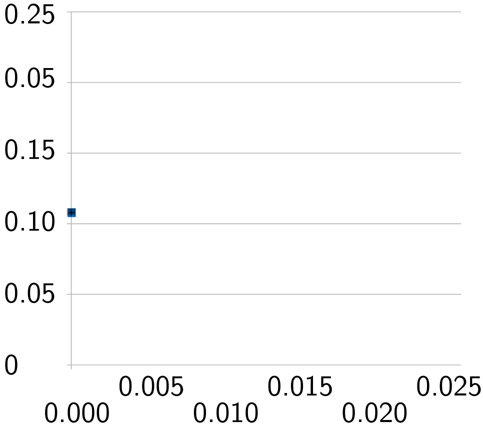
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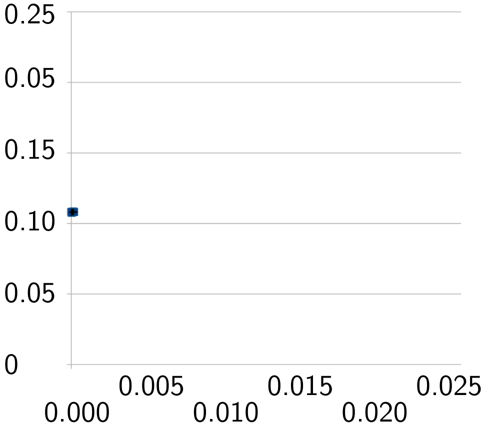
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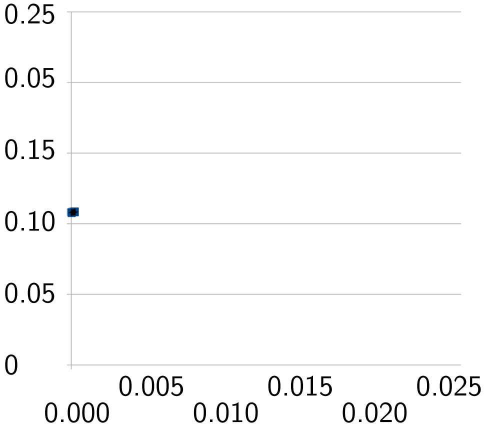
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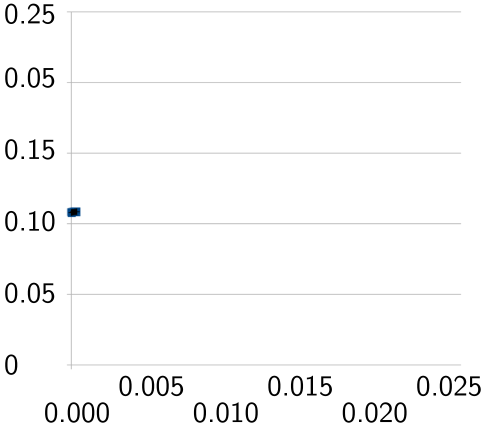
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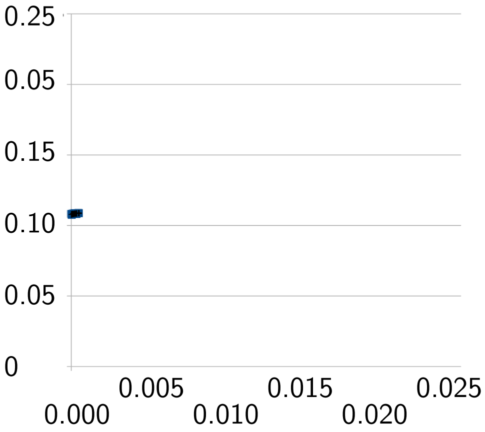
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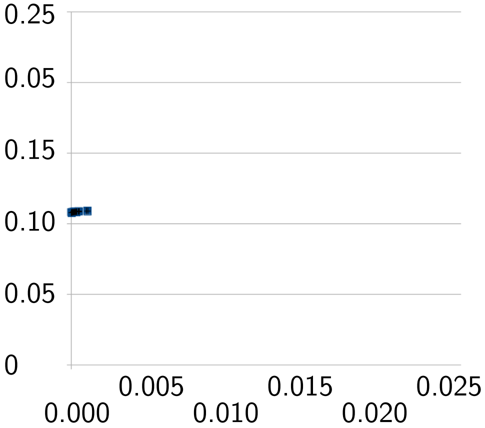
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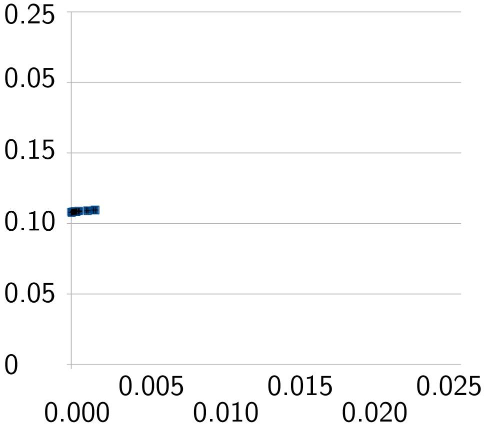
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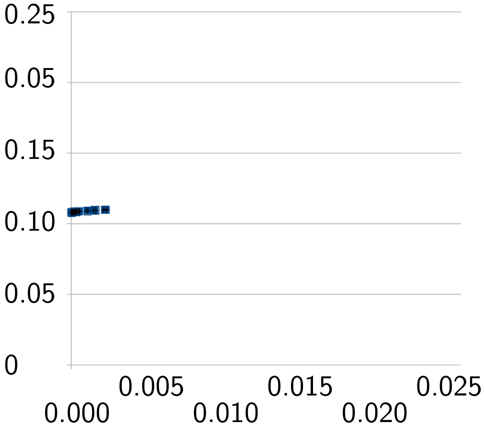
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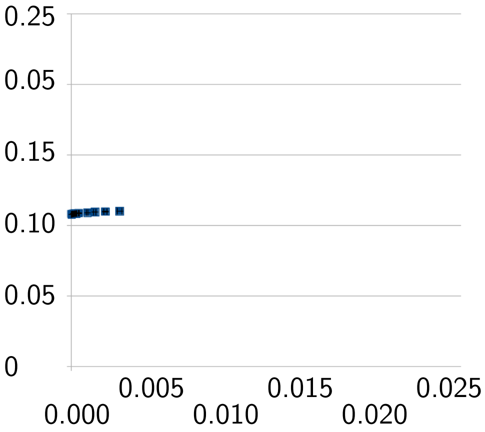
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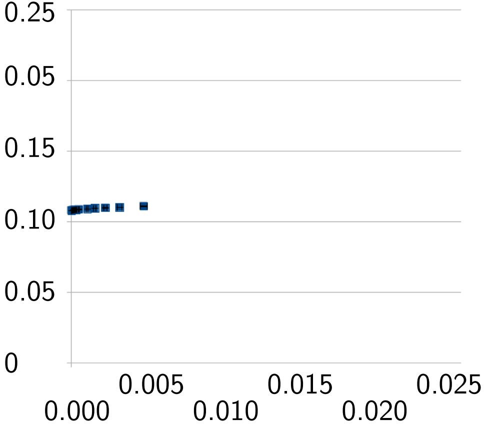
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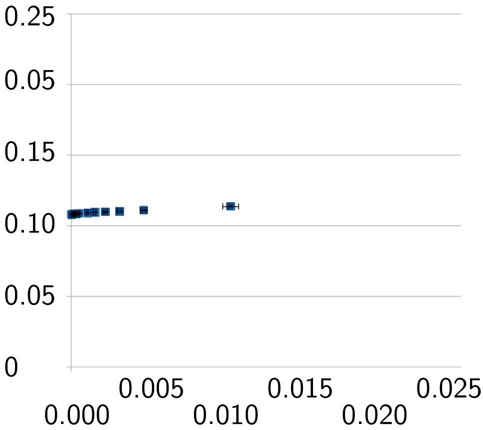
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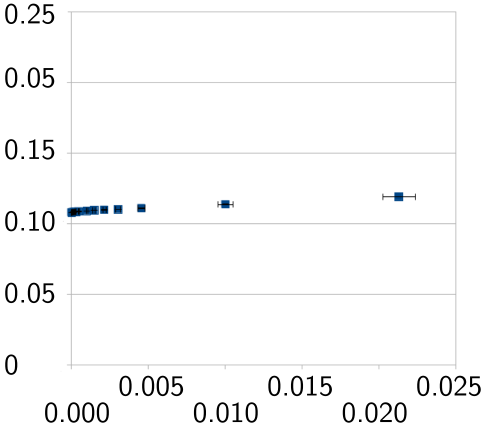
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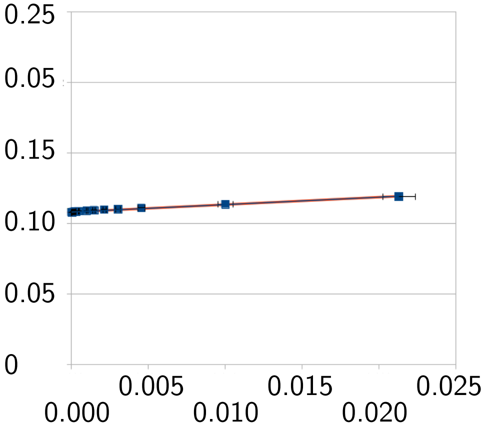
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A least squares fit to the data gives:

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$$V_0 = 9.246 \pm 0.006 V$$

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$$I_{max} = 1.89 \pm 0.05A$$

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$$V_0 = 9.246 \pm 0.006\,V$$

$$I_{max} = 1.89 \pm 0.05A$$

$$r_i = 4.8 \pm 0.1\Omega$$

Here are the data collected for a *used* 9 volt battery.

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1000000	7.32

External resistance R (Ω)	Terminal voltage V_T (V)
∞	7.32
1000000	7.32
680000	7.32

External resistance R (Ω)	Terminal voltage V_T (V)
∞	7.32
1000000	7.32
680000	7.32
$\vdots$	$\vdots$
6800	7.11

External resistance R (Ω)	Terminal voltage V_T (V)
∞	7.32
1000000	7.32
680000	7.32
$\vdots$	$\vdots$
6800	7.11
3300	7.01

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∞	7.32
1000000	7.32
680000	7.32
$\vdots$	$\vdots$
6800	7.11
3300	7.01
2200	6.93

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∞	7.32
1000000	7.32
680000	7.32
$\vdots$	$\vdots$
6800	7.11
3300	7.01
2200	6.93
1000	6.71

External resistance R (Ω)	Terminal voltage V_T (V)
∞	7.32
1000000	7.32
680000	7.32
$\vdots$	$\vdots$
6800	7.11
3300	7.01
2200	6.93
1000	6.71
680	6.56

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$\vdots$	$\vdots$
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2200	6.93
1000	6.71
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220	5.72

External resistance R (Ω)	Terminal voltage V_T (V)
∞	7.32
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$\vdots$	$\vdots$
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3300	7.01
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1000	6.71
680	6.56
330	6.10
220	5.72
100	4.71

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1000000	7.32
680000	7.32
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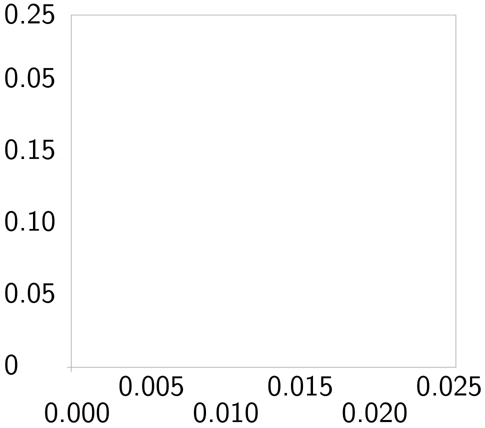
3300	7.01
2200	6.93
1000	6.71
680	6.56
330	6.10
220	5.72
100	4.71
47	3.55

The graph of $\frac{1}{V_T}$ versus $\frac{1}{R}$ looks like this: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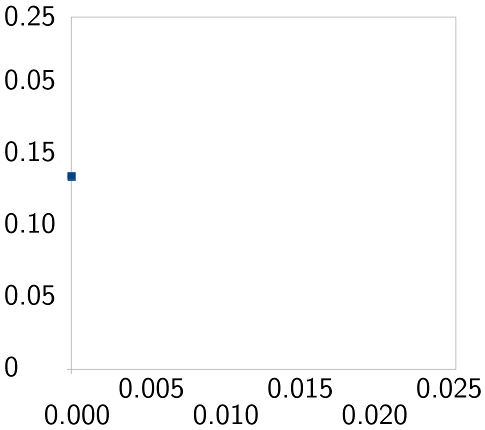
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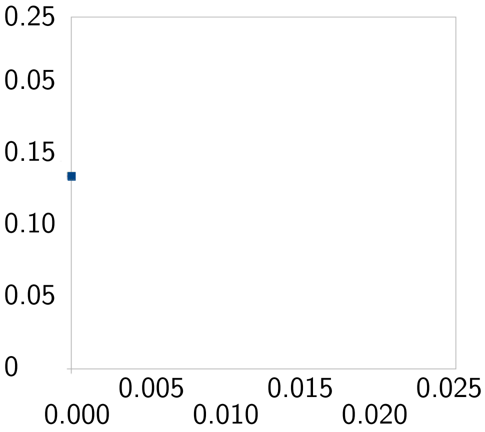
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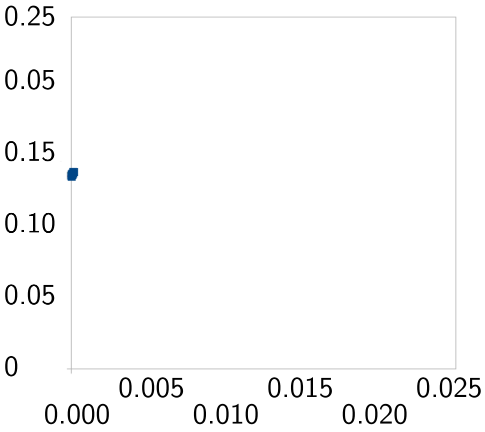
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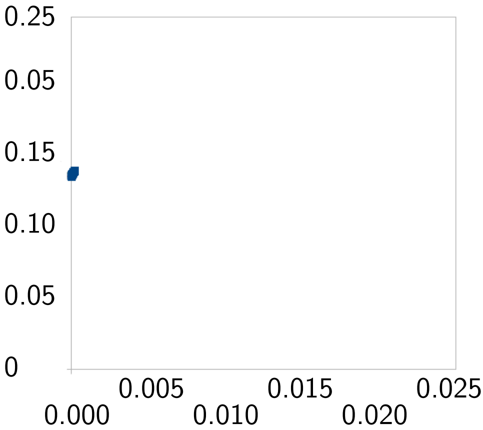
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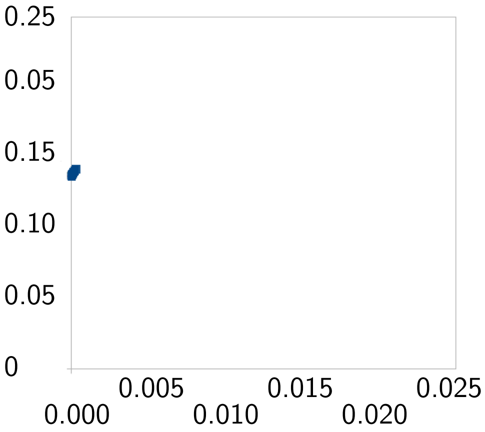
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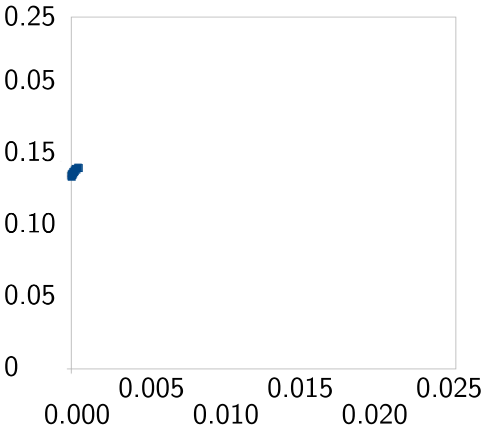
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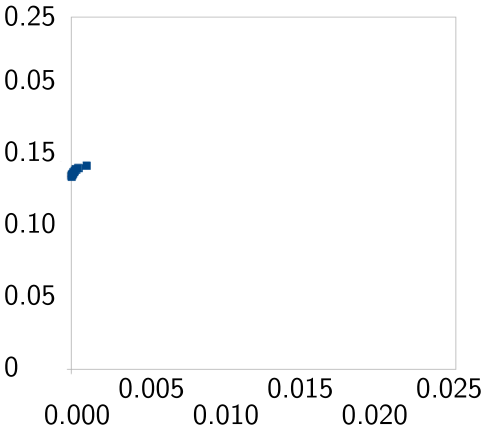
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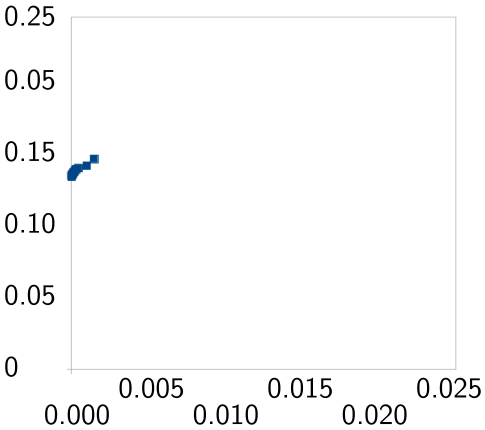
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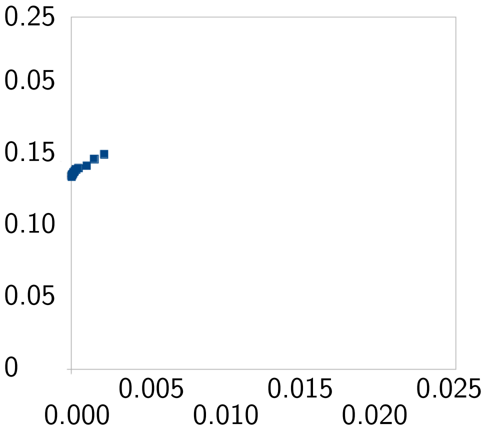
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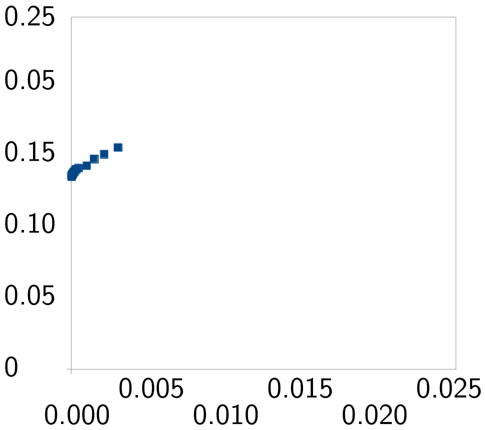
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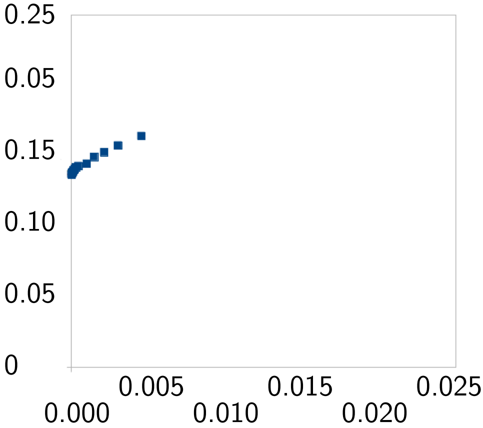
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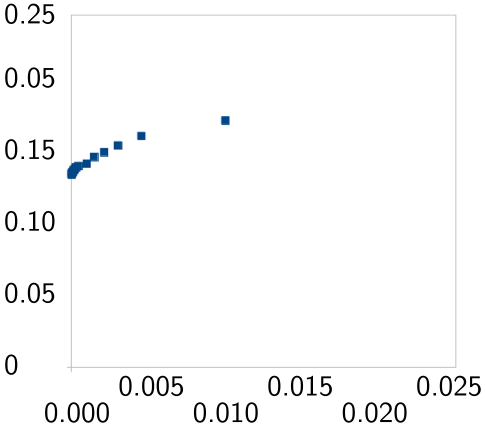
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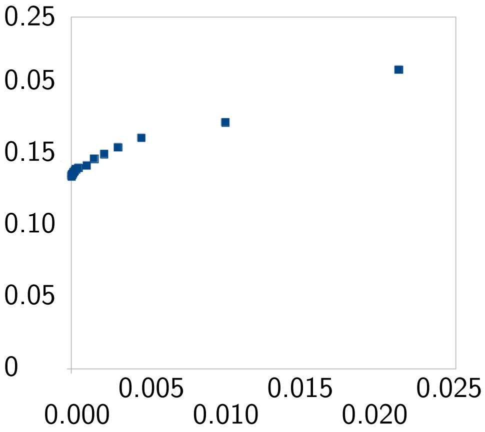
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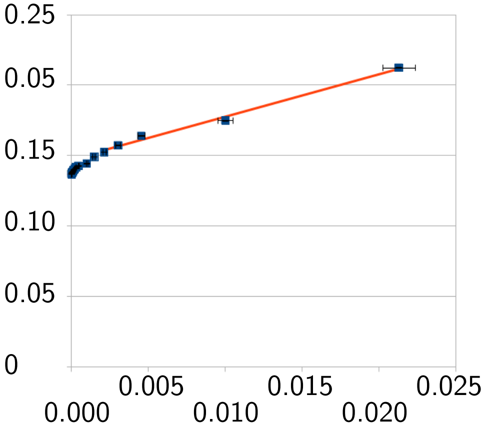
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The steeper slope of this graph compared to the last one indicates that the internal resistance is much higher; in other words, the output voltage changes much more as current increases compared to the new battery.

A least squares fit was produced, as before, but only the last five points were used due to the curve at the left of the graph.

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$$V_0 = 6.78 \pm 0.08\,V$$

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$$V_0 = 6.78 \pm 0.08 V$$

$$I_{max} = 0.33 \pm 0.02 A$$

A least squares fit was produced, as before, but only the last five points were used due to the curve at the left of the graph.

$$V_0 = 6.78 \pm 0.08 V$$

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$$r_i = 20 \pm 1 \Omega$$

A least squares fit was produced, as before, but only the last five points were used due to the curve at the left of the graph.

$$V_0 = 6.78 \pm 0.08 V$$

$$I_{max} = 0.33 \pm 0.02 A$$

$$r_i = 20 \pm 1 \Omega$$

(Using all of the data points gives slightly different results, but ones that are still much different than those for the new battery.)

Parameter	New battery	Old battery
$V_0(V)$	9.246 ± 0.006	6.78 ± 0.08
$I_{max}(A)$	1.89 ± 0.05	0.33 ± 0.02
$r_i(\Omega)$	4.8 ± 0.1	20 ± 1

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$V_0(V)$	9.246 ± 0.006	6.78 ± 0.08
$I_{max}(A)$	1.89 ± 0.05	0.33 ± 0.02
$r_i(\Omega)$	4.8 ± 0.1	20 ± 1

Even though the nominal battery voltage is only 27% lower, the internal resistance is 5 times as high, and the maximum current available is only 1/6 of what it is for the new battery.