PC132 Acceleration of Bodies in Free Fall
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Purpose

This experiment was an expansion of the *"Measuring 'g' "* experiment from PC131. Instead of using a fixed height, fall times were recorded for several heights to determine the acceleration due to gravity. Both a steel ball and a ping pong ball were used to see if the effects of air resistance could be observed.

Introduction

The equation for an object in free fall under gravity is

$$h = \frac{1}{2}gt^2$$

where

- h is the height
- t is the fall time
- g is the acceleration due to gravity

Procedure

The procedure was similar to the *"Measuring 'g' "* lab in the PC131 lab manual [1], with the following changes:

- There was only one timer, who was the person dropping the ball.
- Instead of using a single height, various heights were used so that the dependence of time with height could be examined.

Results

Raw Data

Five trials were performed for each ball at each height. Table 1 is a sample of the data for one height.

- All times are in seconds.
- The precision measure of the stopwatch was 0.01s.
- The uncertianty in the height was 2cm, due to variation in release position.

Height of 1.10 m		
Trial #	steel	ping pong
1	0.51	0.42
2	0.40	0.52
3	0.46	0.47
4	0.39	0.45
5	0.44	0.44
average	0.44	0.46
σ	0.049	0.038
α	0.022	0.017
$\Delta\bar{t}$	0.02	0.02

Table 1: One Height Data for Both Balls

Modified Data

Because each set of data involved 5 trials, only the averaged data have been included here.

h	Δh	t	Δt
m		s	
1.10	0.02	0.44	0.02
1.90	0.02	0.61	0.02
3.16	0.02	0.83	0.04
4.19	0.02	0.94	0.02
5.28	0.02	1.12	0.03

Table 2: Data for Steel Ball

h	Δh	t	Δt
m		s	
1.10	0.02	0.46	0.02
1.90	0.02	0.61	0.02
3.16	0.02	0.87	0.04
4.19	0.02	1.05	0.02
5.28	0.02	1.26	0.03

Table 3: Data for Ping Pong Ball

Following are the results for one possible linearization of the data. (A graph is given for each ball in order to avoid clutter.) The slope and y-intercept were determined by a least squares fit, and the uncertainties in the slope and y-intercept were given by the standard errors since the line of best fit does not cross all of the error bars.

Note: Since the graph is logarithmic, it's not really possible to talk about units for the slope and y-intercept.

Linearization

$$\ln h = 2\ln t + \ln\left(\frac{1}{2}g\right)$$

$\ln h$	$\Delta(\ln h)$ $\frac{\Delta h}{h}$	$\ln t$	$\Delta(\ln t)$ $\frac{\Delta t}{t}$
0.10	0.02	-0.82	0.05
0.64	0.01	-0.50	0.03
1.15	0.01	-0.19	0.05
1.433	0.005	-0.07	0.02
1.664	0.004	0.11	0.03

Table 4: Linearization in $\ln h$ data for steel ball

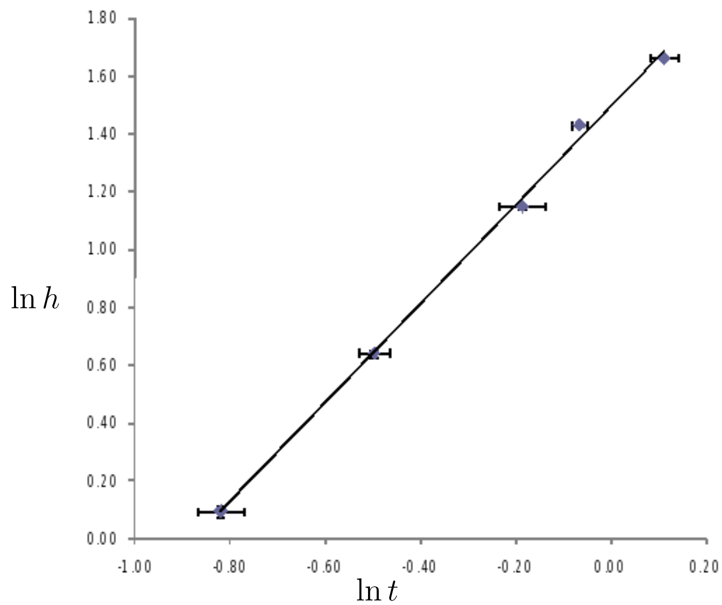

Figure 1: Steel ball linearization

$\ln h$	$\Delta(\ln h)$	$\ln t$	$\Delta(\ln t)$
	$\frac{\Delta h}{h}$		$\frac{\Delta t}{t}$
0.10	0.02	-0.78	0.04
0.64	0.01	-0.50	0.04
1.15	0.01	-0.13	0.05
1.433	0.005	-0.04	0.02
1.664	0.004	0.23	0.02

Table 5: Linearization in $\ln h$ data for ping pong ball

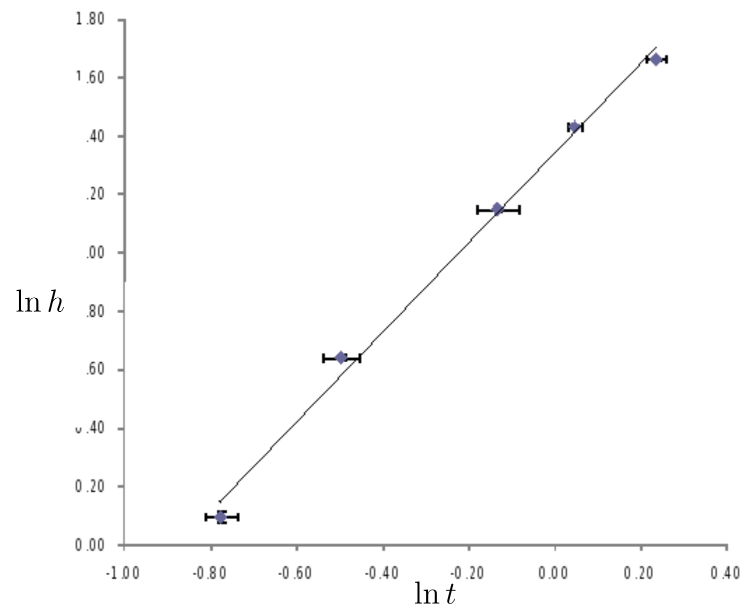

Figure 2: Ping pong ball linearization

Linearization details

$$\ln h = 2\ln t + \ln\left(\frac{1}{2}g\right)$$

$$\Delta\left(\ln h\right) = \frac{1}{h}\Delta h = \frac{\Delta h}{h}$$

Similarly,

$$\Delta\left(\ln t\right) = \frac{\Delta t}{t}$$

$$y - intercept = \ln\left(\frac{1}{2}g\right)$$

$$g = 2 \times e^{y-intercept}$$

$$\Delta g = 2 \times e^{y-intercept} \times \Delta y - intercept$$

The slope has an expected value of 2.

Results for the steel ball (from least squares fit)

$$Slope = 1.71 \pm 0.05$$

$$y - intercept = 1.5 \pm 0.02$$

$$g_s = 9.0 \pm 0.2 \quad m/s^2$$

Results for the ping pong ball (from least squares fit)

$$Slope = 1.54 \pm 0.07$$

$$y - intercept = 1.35 \pm 0.03$$

$$g_p = 7.7 \pm 0.2 \quad m/s^2$$

Sample Calculations

For the steel ball,

$$y - intercept = 1.5 \pm 0.02$$

$$g = 2 \times e^{y-intercept} \approx 2 \times e^{1.5} \approx 2 \times 4.48 \approx 8.96 \quad m/s^2$$

$$\Delta g = 2 \times e^{y-intercept} \times \Delta y - intercept \approx 8.96 \times 0.02 \approx 0.18 \quad m/s^2$$

Thus

$$g_s = 9.0 \pm 0.2 \quad m/s^2$$

Visualization of results

Figure 3 summarizes the results, including an indication of the accepted value for g.

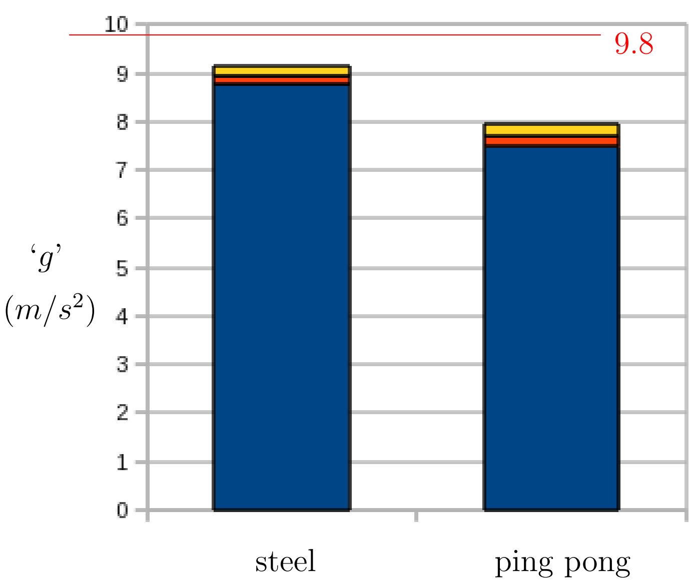

Figure 3: Comparing steel ball to ping pong ball

Discussion

Choice of linearization

While there were a few possible linearizations of the data, this one was chosen because it gave smaller uncertainties for the values of g calculated, and because it gave a value of g for the steel ball which was closest to the accepted value.

Slope not equal to 2

For the chosen linearization, we should get a slope of 2. The actual slopes of 1.71 ± 0.05 for the steel ball and 1.54 ± 0.07 for the ping pong ball did not equal 2 within experimental uncertainty, producing differences of $\left|\frac{2.0-1.71}{2.0}\right| \times 100 = 15\%$ and $\left|\frac{2.0-1.54}{2.0}\right| \times 100 = 23\%$ respectively. This suggests there may be some systematic error in the experiment. A systematic error in the height is unlikely; at least one large enough to have an effect of this size. However, a systematic error in the time due to a reaction time delay is quite possible.

Uncertainties from the graphs

Each graph fits the case of large scatter which suggests that having a more precise measurement of either time or height would not have improved the results. On the other hand, because of the variations from linearity, (indicated by the slope not being 2 within experimental uncertainty), a more accurate measurement of time, (i.e. one which reduces reaction time error), might have a significant effect on the results.

Comparing the two different types of ball

For this linearization, the value for g was significantly lower for the ping pong ball than for the steel ball. Specifically:

Comparing the values of g for the steel and ping pong ball for the chosen linearization we get

$$\left|\frac{9.0-7.7}{9.0}\right| \times 100 = 14\%$$

Comparing to the expected value with no friction

Comparing the steel ball to the expected value with no friction, there is a

$$\left|\frac{9.8-9.0}{9.8}\right| \times 100 = 8\%$$

difference for the chosen linearization.

Conclusions

Here are several conclusions which can be drawn from this analysis:

- For the chosen linearization, the value for g for the ping pong ball was significantly lower than for the steel ball, showing air resistance to be significant. Specifically, $g_s = 9.0 \pm 0.2 \quad m/s^2$ and $g_p = 7.7 \pm 0.2 \quad m/s^2$ which do not agree. There is a 14% difference.

- For the chosen linearization, even the g value for the steel ball was not within experimental uncertainty of the accepted value of $9.8 m/s^2$, which suggests that even the steel ball was significantly affected by air resistance. The value of $g_s = 9.0 \pm 0.2 \quad m/s^2$ was 8% below the accepted value.

References

[1] Terry Sturtevant. PC131 Lab Manual. From PC131 web page, 2010.