Uncertainty Calculation Sensitivity
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4. The higher in the list a quantity is, the greater its contribution to the total uncertainty.

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Note that in the first equation, all of the Δz terms are gone, and in the second, all of the Δw terms are gone.

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Note that *until you plug values into these equations, you can't tell which uncertainty contribution is larger.*

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In order to improve this experiment, it would be more important to try and reduce Δz than it would be to try and reduce Δw.

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