Uncertainties in Measurements
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It is the difference between the nominal value and the maximum or minimum value.

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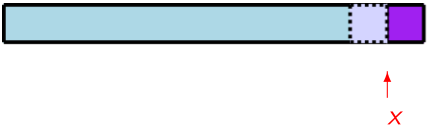

The nominal value of x is here. (i.e. the value without considering uncertainties)

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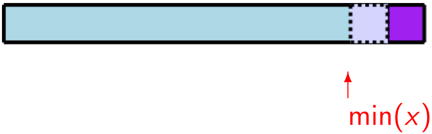
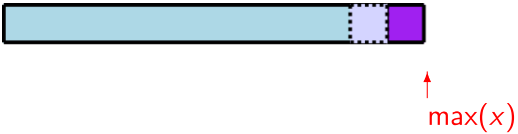


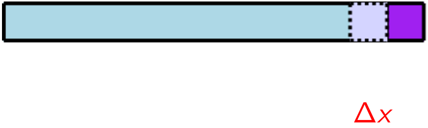
The minimum value of x is here. (i.e. the value with the uncertainty subtracted)

Graphically,



The maximum value of x is here. (i.e. the value with the uncertainty added)

Graphically,



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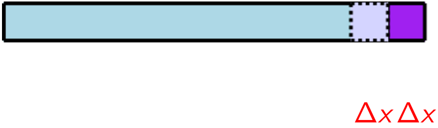

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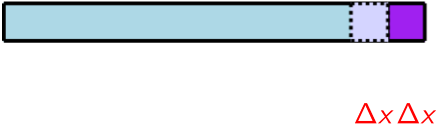

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(In other words, it doesn't make sense to have an extremely *precise* measure of the *imprecision* in a value!)

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   In that case, we calculate the *percent difference* between the two values.