Uncertainties by Inspection Wilfrid Laurier University

Terry Sturtevant

Wilfrid Laurier University

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Functions of a single variable Functions of multiple variables

Overview

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Functions of a single variable Functions of multiple variables

Overview

Whenever quantities are measured, they have *uncertainties* in them.

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Overview

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When calculations are performed involving these quantities, then the **results** have uncertainty as well.

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Overview

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When calculations are performed involving these quantities, then the **results** have uncertainty as well.

The uncertainty reflects the range of possible *calculated* values based on the range of possible *data* values.

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Functions of a single variable Functions of multiple variables

Overview

Whenever quantities are measured, they have *uncertainties* in them.

When calculations are performed involving these quantities, then the **results** have uncertainty as well.

The uncertainty reflects the range of possible *calculated* values based on the range of possible *data* values.

It is the difference between the nominal value and the maximum or minimum value.

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Functions of a single variable Functions of multiple variables

Example

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Functions of a single variable Functions of multiple variables

Example

Suppose we have a measurement of 2.0 ± 0.3 cm.

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Example

Suppose we have a measurement of 2.0 \pm 0.3 cm. The nominal value is 2.0cm.

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Example

Suppose we have a measurement of 2.0 ± 0.3 cm. The nominal value is 2.0cm.

The maximum value it can have is 2.0 + 0.3 cm.

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Example

Suppose we have a measurement of 2.0 \pm 0.3 cm.

The nominal value is 2.0cm.

The maximum value it can have is 2.0 + 0.3 cm.

The uncertainty is the difference between this maximum value and the nominal value.

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Example

Suppose we have a measurement of 2.0 \pm 0.3 cm.

- The nominal value is 2.0cm.
- The maximum value it can have is 2.0 + 0.3 cm.
- The uncertainty is the difference between this maximum value and the nominal value.
- The minimum value it can have is 2.0 0.3 cm.

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Example

Suppose we have a measurement of 2.0 \pm 0.3 cm.

- The nominal value is 2.0cm.
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The uncertainty is the difference between this maximum value and the nominal value.

- The minimum value it can have is 2.0 0.3 cm.
- The uncertainty is the difference between the nominal value and this minimum value.

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Overview Functions of a single variable Summary of Uncertainty Principles Functions of multiple variables

Thus if we want to find the uncertainty in a function, f(x), we can say

 $\Delta f(x) \approx f_{max} - f$

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Overview Functions of a single variable Summary of Uncertainty Principles Functions of multiple variables

Thus if we want to find the uncertainty in a function, f(x), we can say

 $\Delta f(x) \approx f_{max} - f$

or

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Overview Functions of a single variable Summary of Uncertainty Principles Functions of multiple variables

Thus if we want to find the uncertainty in a function, f(x), we can say

 $\Delta f(x) \approx f_{max} - f$ or $\Delta f(x) \approx f - f_{min}$

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$$\Delta f(x) \approx f_{max} - f$$

or

$$\Delta f(x) \approx f - f_{min}$$

where

f is the *nominal* value of the function; i.e. ignoring uncertainties,

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$$\Delta f(x) \approx f_{max} - f$$

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f is the *nominal* value of the function; i.e. ignoring uncertainties,

 f_{max} is the same function with x replaced by either $x + \Delta x$ or $x - \Delta x$; whichever makes f bigger,

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$$\Delta f(x) \approx f_{max} - f$$

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 f_{min} is the same function with x replaced by either $x + \Delta x$ or $x - \Delta x$; whichever makes f smaller.

(The approximately equals sign is to reflect the fact that these two values may not be quite the same, depending on the function f.)

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Functions of a single variable Functions of multiple variables

Example

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Functions of a single variable Functions of multiple variables

Example

Suppose $f(x) = x^2 + 5$.

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Example

Suppose $f(x) = x^2 + 5$.

If x is positive, and greater than 1, then replacing x by $x + \Delta x$ will make f a maximum.

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Example

Suppose $f(x) = x^2 + 5$.

If x is positive, and greater than 1, then replacing x by $x + \Delta x$ will make f a maximum.

So $f_{max} = f(x + \Delta x) = (x + \Delta x)^2 + 5$.

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The uncertainty is the difference between this maximum value and the nominal value.

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The uncertainty is the difference between this maximum value and the nominal value.

$$\Delta f(x) \approx f_{max} - f = f(x + \Delta x) - f(x) = \left((x + \Delta x)^2 + 5 \right) - (x^2 + 5)$$

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If x is positive, and greater than 1, then replacing x by $x - \Delta x$ will make f a minimum.

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If x is positive, and greater than 1, then replacing x by $x - \Delta x$ will make f a minimum.

So $f_{min} = f(x - \Delta x) = (x - \Delta x)^2 + 5$.

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If x is positive, and greater than 1, then replacing x by $x - \Delta x$ will make f a minimum.

So $f_{min} = f(x - \Delta x) = (x - \Delta x)^2 + 5$.

The uncertainty is the difference between the nominal value and this minimum value.

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If x is positive, and greater than 1, then replacing x by $x - \Delta x$ will make f a minimum.

So
$$f_{min} = f(x - \Delta x) = (x - \Delta x)^2 + 5$$
.

The uncertainty is the difference between the nominal value and this minimum value.

$$\Delta f(x) \approx f - f_{min} = f(x) - f(x - \Delta x) = (x^2 + 5) - ((x - \Delta x)^2 + 5)$$

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$$\Delta f(x) \approx f - f_{min} = f(x) - f(x - \Delta x) = (x^2 + 5) - ((x - \Delta x)^2 + 5)$$

Note that this value will be slightly different than the value given by $f_{max} - f$.

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Functions of a single variable Functions of multiple variables

Another example

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Functions of a single variable Functions of multiple variables

Another example

Suppose
$$g(t) = \frac{1}{\sqrt{t}}$$
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Another example

Suppose $g(t) = \frac{1}{\sqrt{t}}$.

If t is positive, then replacing t by $t - \Delta t$ will make g a maximum.

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Another example

Suppose $g(t) = \frac{1}{\sqrt{t}}$.

If t is positive, then replacing t by $t - \Delta t$ will make g a maximum.

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If t is positive, then replacing t by $t - \Delta t$ will make g a maximum.

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The uncertainty is the difference between this maximum value and the nominal value.

$$\Delta g(t) pprox g_{max} - g = g(t - \Delta t) - g(t) = \left(rac{1}{\sqrt{(t - \Delta t)}}
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Functions of a single variable Functions of multiple variables

Numerical example

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Functions of a single variable Functions of multiple variables

Numerical example

Suppose we measure the diameter of a marble, d, with an uncertainty Δd .

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Numerical example

Suppose we measure the diameter of a marble, d, with an uncertainty Δd .

The marble volume, V, will be given by:

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Numerical example

Suppose we measure the diameter of a marble, d, with an uncertainty Δd .

The marble volume, V, will be given by:

$$V = \frac{4}{3}\pi \left(\frac{d}{2}\right)^3$$

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Suppose we measure the diameter of a marble, d, with an uncertainty Δd .

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 $\Delta V pprox V(d + \Delta d) - V(d)$

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Suppose we measure the diameter of a marble, d, with an uncertainty Δd .

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$$egin{split} V &= rac{4}{3} \pi igg(rac{d}{2}igg)^3 \ \Delta V &pprox V(d+\Delta d) - V(d) \ \Delta V &pprox rac{4}{3} \pi igg(rac{d+\Delta d}{2}igg)^3 - rac{4}{3} \pi igg(rac{d}{2}igg)^3 \end{split}$$

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Suppose we measure the diameter of a marble, d, with an uncertainty Δd .

The marble volume, V, will be given by:

$$V = \frac{4}{3}\pi \left(\frac{d}{2}\right)^{3}$$
$$\Delta V \approx V(d + \Delta d) - V(d)$$
$$\Delta V \approx \frac{4}{3}\pi \left(\frac{d + \Delta d}{2}\right)^{3} - \frac{4}{3}\pi \left(\frac{d}{2}\right)^{3}$$

If we have a value of $d=1.0\pm0.1~{
m cm}$, then $\Delta V=0.173~{
m cm}^3$ by this method.

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The marble volume, V, will be given by:

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$$\Delta V \approx \frac{4}{3}\pi \left(\frac{d + \Delta d}{2}\right)^3 - \frac{4}{3}\pi \left(\frac{d}{2}\right)^3$$

If we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.173$ cm³ by this method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm³ as the value to be quoted.

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Functions of multiple variables

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Functions of a single variable Functions of multiple variables

Functions of multiple variables

Suppose we have a function of two variables:

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Functions of a single variable Functions of multiple variables

Functions of multiple variables

Suppose we have a function of two variables:

$$h(w,z) = \frac{\sqrt{w}}{z^2}$$

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Suppose we have a function of two variables:

$$h(w,z) = \frac{\sqrt{w}}{z^2}$$

We want to replace *each* quantity with the appropriate value in order to maximize the total.

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Suppose we have a function of two variables:

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If w and z are both positive,

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Suppose we have a function of two variables:

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If w and z are both positive,

Replacing w by $w + \Delta w$ will make h bigger;

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Suppose we have a function of two variables:

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Suppose we have a function of two variables:

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We want to replace *each* quantity with the appropriate value in order to maximize the total.

If w and z are both positive,

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So
$$h_{max} = \frac{\sqrt{(w+\Delta w)}}{(z-\Delta z)^2}$$

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Suppose we have a function of two variables:

$$h(w,z) = \frac{\sqrt{w}}{z^2}$$

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So
$$h_{max} = \frac{\sqrt{(w + \Delta w)}}{(z - \Delta z)^2}$$

$$\Delta h pprox h_{max} - h = rac{\sqrt{(w + \Delta w)}}{(z - \Delta z)^2} - rac{\sqrt{w}}{z^2}$$

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Summary of Uncertainty Principles

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Summary of Uncertainty Principles

All measurements have uncertainties.

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Summary of Uncertainty Principles

All measurements have uncertainties.

Because of this, all calculated results have uncertainties.

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Summary of Uncertainty Principles

All measurements have uncertainties.

Because of this, all calculated results have uncertainties.

The uncertainty in a quantity is the difference between the nominal value and the maximum or minimum value.

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