Uncertainty Calculations - Addition
Wilfrid Laurier University

Terry Sturtevant

Wilfrid Laurier University

May 9, 2013

Calculations with uncertainties

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Following is a discussion of **addition**.

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Following is a discussion of **addition**.

For the following examples, the values of $x = 2 \pm 1$ and $y = 32.0 \pm 0.2$ will be used.

Addition

Addition

Addition with uncertainties

Addition - Example

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

  $\rightarrow$ z can be as *small* as $1 + 31.8 = 32.8$

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

  $\rightarrow z$ can be as *small* as $1 + 31.8 = 32.8$

  since x can be as *small* as 1 and y can be as *small* as 31.8

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

  $\rightarrow z$ can be as *small* as $1 + 31.8 = 32.8$

  since x can be as *small* as 1 and y can be as *small* as 31.8

  $\rightarrow z$ can be as *big* as $3 + 32.2 = 35.2$

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

  $\rightarrow z$ can be as *small* as $1 + 31.8 = 32.8$

  since x can be as *small* as 1 and y can be as *small* as 31.8

  $\rightarrow z$ can be as *big* as $3 + 32.2 = 35.2$

  since x can be as *big* as 3 and y can be as *big* as 32.2

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

  $\rightarrow z$ can be as *small* as $1 + 31.8 = 32.8$

  since x can be as *small* as 1 and y can be as *small* as 31.8

  $\rightarrow z$ can be as *big* as $3 + 32.2 = 35.2$

  since x can be as *big* as 3 and y can be as *big* as 32.2
- The *nominal* value of z is

Addition - Example

If we add these numbers,

- $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

  $\rightarrow z$ can be as *small* as $1 + 31.8 = 32.8$

  since x can be as *small* as 1 and y can be as *small* as 31.8

  $\rightarrow z$ can be as *big* as $3 + 32.2 = 35.2$

  since x can be as *big* as 3 and y can be as *big* as 32.2

- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$

To summarize,

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$
- So we can say $z = 34.0 \pm 1.2$

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$
- So we can say $z = 34.0 \pm 1.2$

  since $1.2 = 34.0 - 32.8 = 35.2 - 34.0$

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$
- So we can say $z = 34.0 \pm 1.2$

  since $1.2 = 34.0 - 32.8 = 35.2 - 34.0$
- We can see that $\Delta z = 1.2 = 1 + 0.2 = \Delta x + \Delta y$

To summarize,

- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$
- So we can say $z = 34.0 \pm 1.2$

  since $1.2 = 34.0 - 32.8 = 35.2 - 34.0$
- We can see that $\Delta z = 1.2 = 1 + 0.2 = \Delta x + \Delta y$
- So in general, $\Delta (x + y) = \Delta x + \Delta y$

To summarize,

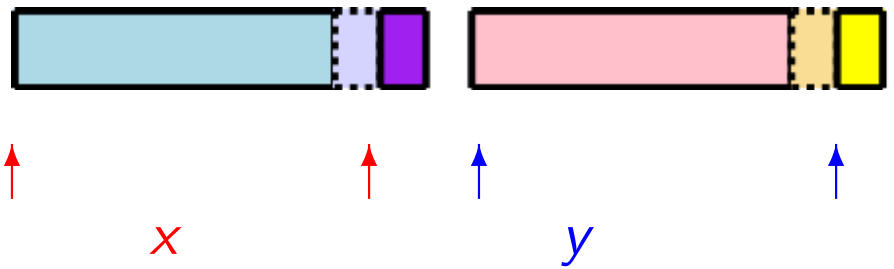
- z can be as *small* as $1 + 31.8 = 32.8$
- z can be as *big* as $3 + 32.2 = 35.2$
- The *nominal* value of z is

  $z = 2 + 32.0 = 34.0$
- So we can say $z = 34.0 \pm 1.2$

  since $1.2 = 34.0 - 32.8 = 35.2 - 34.0$
- We can see that $\Delta z = 1.2 = 1 + 0.2 = \Delta x + \Delta y$
- So in general, $\Delta (x + y) = \Delta x + \Delta y$

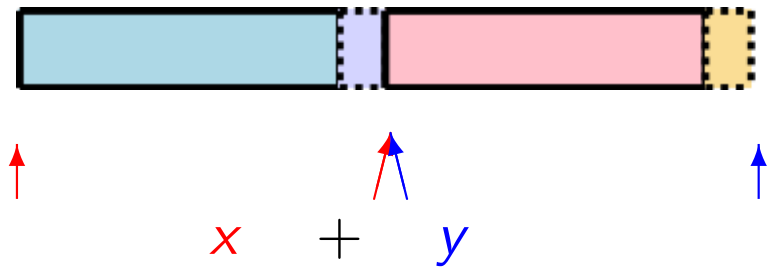
  When adding numbers, we add uncertainties.

Graphically,



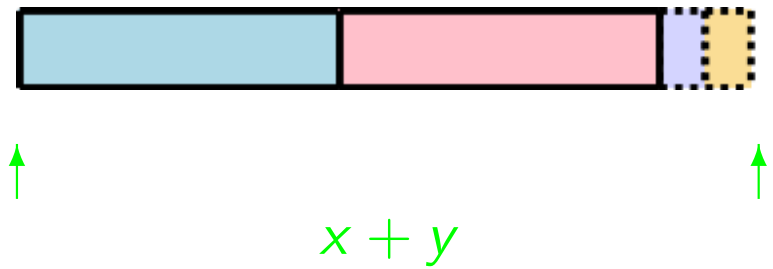
- $(x \pm \Delta x) + (y \pm \Delta y)$

Graphically,



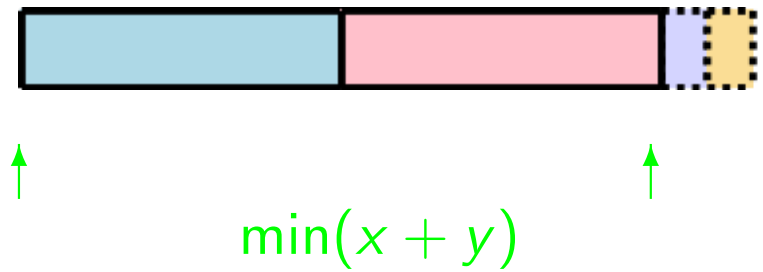
- This is the nominal value of $x + y$.
  (i.e. the nominal value of x plus the nominal value of y)

Graphically,



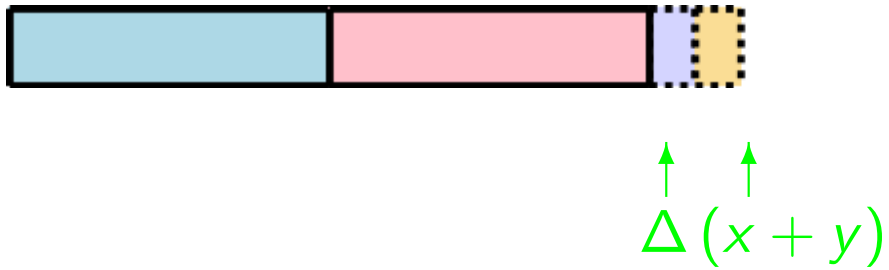
- This is the nominal value of $x + y$, redrawn.

Graphically,



- This is the minimum value of $x + y$.
  (i.e. the minimum value of x plus the minimum value of y)

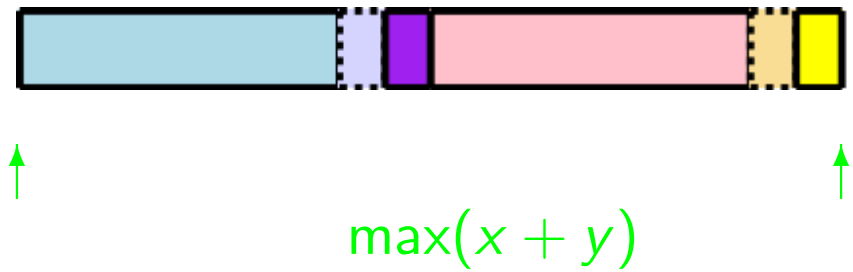
Graphically,



- This is $\Delta(x+y)$.

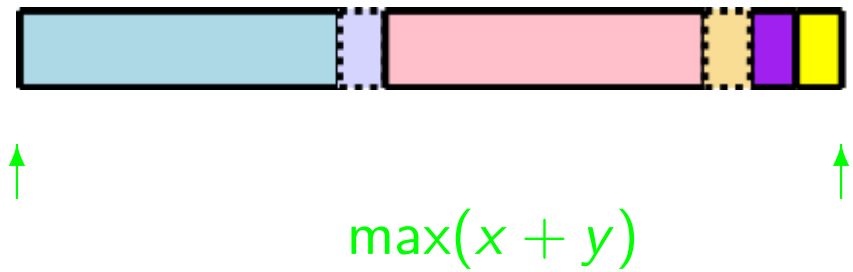
  (i.e. the nominal value of $x+y$ minus the minimum value of $x+y$)

Graphically,



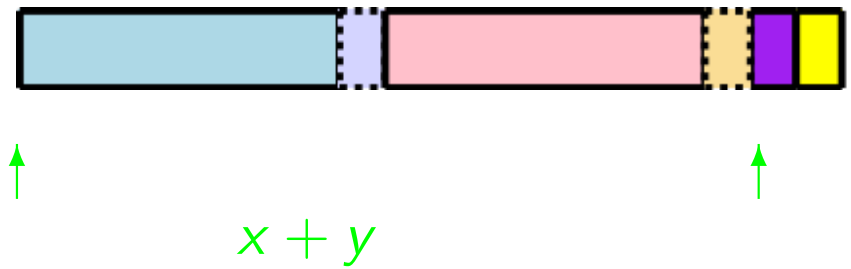
- This is the maximum value of $x + y$.
  (i.e. the maximum value of x plus the maximum value of y)

Graphically,



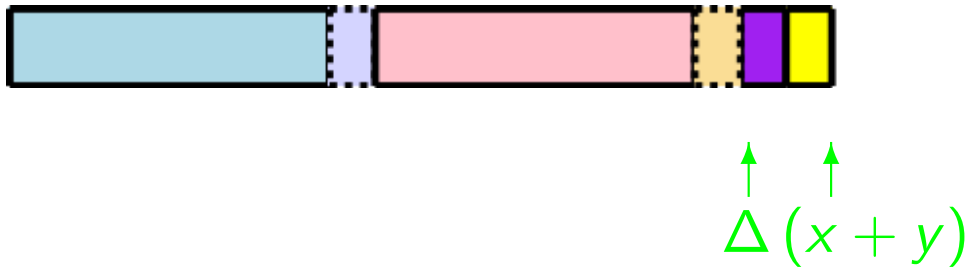
- This is the maximum value of $x + y$, redrawn.

Graphically,



- This is the nominal value of $x + y$.

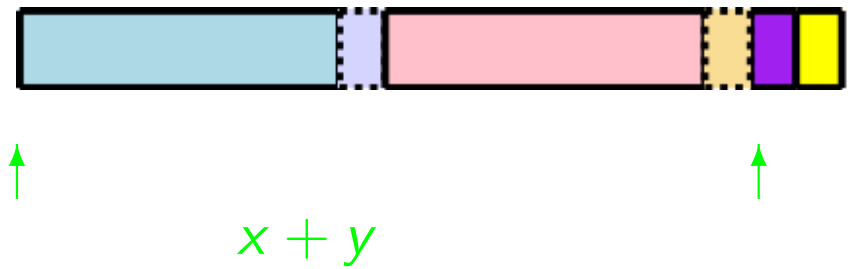
Graphically,



- This is $\Delta(x+y)$.

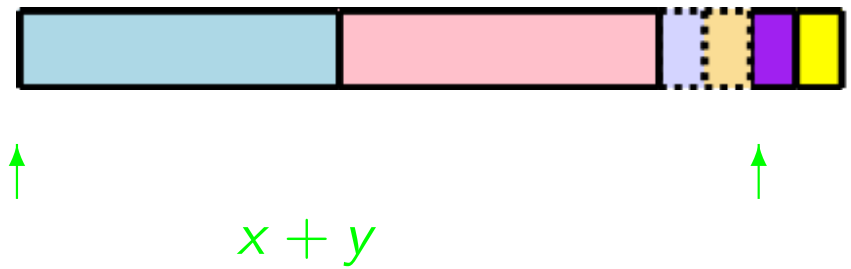
  (i.e. the maximum value of $x+y$ minus the nominal value of $x+y$)

Graphically,



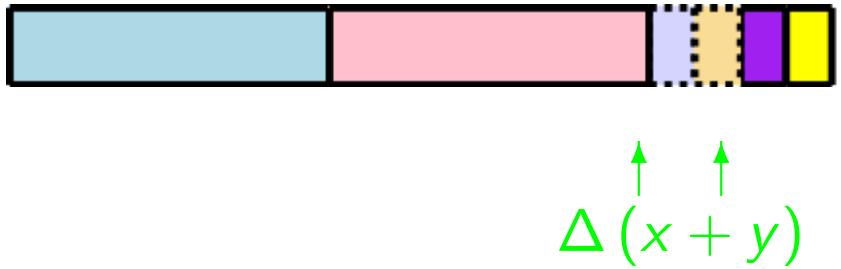
- This is the nominal value of $x + y$.

Graphically,



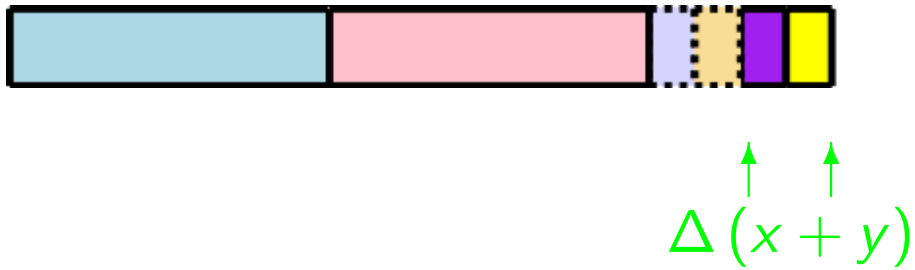
- This is the nominal value of $x + y$, redrawn.

Graphically,



- This is $\Delta(x+y)$.

Graphically,



- This is $\Delta(x+y)$.

Recap

Recap

1. When adding numbers, we add the *absolute* uncertainties.

Recap

1. When adding numbers, we add the *absolute* uncertainties.

   For example,

$$(2 \pm 1) + (32.0 \pm 0.2) = (2 + 32.0) \pm (1 + 0.2) = 34.0 \pm 1.2$$

Recap

1. When adding numbers, we add the *absolute* uncertainties.

   For example,

$$(2 \pm 1) + (32.0 \pm 0.2) = (2 + 32.0) \pm (1 + 0.2) = 34.0 \pm 1.2$$

2. Uncertainties in final results are usually expressed to one significant figure, so the above result becomes

$$(2 \pm 1) + (32.0 \pm 0.2) = 34.0 \pm 1.2 = 34 \pm 1$$