Uncertainty Calculations - Addition Wilfrid Laurier University

Terry Sturtevant

Wilfrid Laurier University

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Calculations with uncertainties

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Addition

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

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Addition

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Following is a discussion of **addition**.

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Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Following is a discussion of **addition**.

For the following examples, the values of $x = 2 \pm 1$ and $y = 32.0 \pm 0.2$ will be used.

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Addition with uncertainties

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Addition - Example

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Addition - Example

If we add these numbers,

•
$$z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$$

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Addition

Addition - Example

If we add these numbers,

• $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

 \rightarrow z can be as *small* as 1 + 31.8 = 32.8

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since x can be as small as 1 and y can be as small as 31.8

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If we add these numbers,

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 \rightarrow z can be as *small* as 1 + 31.8 = 32.8

since x can be as *small* as 1 and y can be as *small* as 31.8 $\rightarrow z$ can be as *big* as 3 + 32.2 = 35.2

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DQ P

If we add these numbers,

• $z = (x = 2 \pm 1) + (y = 32.0 \pm 0.2)$

 \rightarrow z can be as *small* as 1 + 31.8 = 32.8 since x can be as *small* as 1 and y can be as *small* as 31.8 \rightarrow z can be as *big* as 3 + 32.2 = 35.2 since x can be as *big* as 3 and y can be as *big* as 32.2

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• The *nominal* value of z is

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z = 2 + 32.0 = 34.0

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• z can be as small as 1 + 31.8 = 32.8

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- z can be as *small* as 1 + 31.8 = 32.8
- z can be as *big* as 3 + 32.2 = 35.2

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- z can be as *small* as 1 + 31.8 = 32.8
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DQC2

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DQC2

- z can be as small as 1 + 31.8 = 32.8
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z = 2 + 32.0 = 34.0

• So we can say $z = 34.0 \pm 1.2$

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SQR

Addition

To summarize,

- z can be as small as 1 + 31.8 = 32.8
- z can be as *big* as 3 + 32.2 = 35.2
- The nominal value of z is

z = 2 + 32.0 = 34.0

• So we can say $z=34.0\pm1.2$

since 1.2 = 34.0 - 32.8 = 35.2 - 34.0

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SQR

- z can be as small as 1 + 31.8 = 32.8
- z can be as *big* as 3 + 32.2 = 35.2
- The nominal value of z is

z = 2 + 32.0 = 34.0

• We can see that $\Delta z = 1.2 = 1 + 0.2 = \Delta x + \Delta y$

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SQR

Addition

To summarize,

- z can be as small as 1 + 31.8 = 32.8
- z can be as *big* as 3 + 32.2 = 35.2
- The nominal value of z is

z = 2 + 32.0 = 34.0

- So we can say z = 34.0 ± 1.2
 since 1.2 = 34.0 32.8 = 35.2 34.0
- We can see that $\Delta z = 1.2 = 1 + 0.2 = \Delta x + \Delta y$
- So in general, $\Delta(x + y) = \Delta x + \Delta y$

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Addition

To summarize,

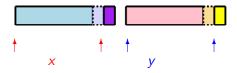
- z can be as small as 1 + 31.8 = 32.8
- z can be as *big* as 3 + 32.2 = 35.2
- The nominal value of z is

z = 2 + 32.0 = 34.0

- So we can say z = 34.0 ± 1.2
 since 1.2 = 34.0 32.8 = 35.2 34.0
- We can see that $\Delta z = 1.2 = 1 + 0.2 = \Delta x + \Delta y$
- So in general, $\Delta (x + y) = \Delta x + \Delta y$ When adding numbers, we add uncertainties.

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Calculations with Uncertainties Recap	ddition
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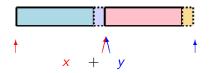
•
$$(x \pm \Delta x) + (y \pm \Delta y)$$

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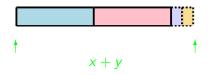


- This is the nominal value of x + y.
 - (i.e. the nominal value of x plus the nominal value of y)

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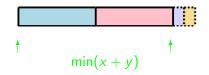


• This is the nominal value of x + y, redrawn.

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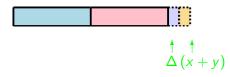
• This is the minimum value of x + y.

(i.e. the minimum value of x plus the minimum value of y)

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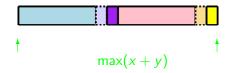


This is Δ (x + y).
 (i.e. the nominal value of x + y minus the minimum value of x + y)

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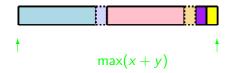
• This is the maximum value of x + y.

(i.e. the maximum value of x plus the maximum value of y)

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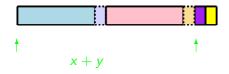


• This is the maximum value of x + y, redrawn.

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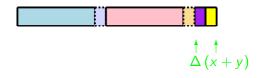


• This is the nominal value of x + y.

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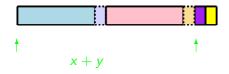


This is Δ (x + y).
 (i.e. the maximum value of x + y minus the nominal value of x + y)

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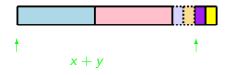


• This is the nominal value of x + y.

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• This is the nominal value of x + y, redrawn.

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Addition

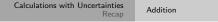
Graphically,



• This is $\Delta(x+y)$.

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• This is $\Delta(x+y)$.

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Recap

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Recap

1 When adding numbers, we add the *absolute* uncertainties.

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Recap

When adding numbers, we add the *absolute* uncertainties.
 For example,

 $(2 \pm 1) + (32.0 \pm 0.2) = (2 + 32.0) \pm (1 + 0.2) = 34.0 \pm 1.2$

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Recap

When adding numbers, we add the *absolute* uncertainties.
 For example,

 $(2 \pm 1) + (32.0 \pm 0.2) = (2 + 32.0) \pm (1 + 0.2) = 34.0 \pm 1.2$

② Uncertainties in final results are usually expressed to one significant figure, so the above result becomes

 $(2 \pm 1) + (32.0 \pm 0.2) = 34.0 \pm 1.2 = 34 \pm 1$

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