Wilfrid Laurier University
PC141 Lab Manual

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Chapter 1

Lab Manual Layout

1.1 This is a Reference

Just as the theory you learn in courses will be required again in later courses, the skills you learn in the lab will be required in later lab courses. *Save this manual as a reference: you will be expected to be able to do anything in it in later lab courses.* That is part of why the manual has been made to fit in a binder; it can be combined with later manuals to form a reference library.

1.2 Parts of the Manual

The lab manual is divided mainly into four parts: *background information*, *lab exercises*, *experiments*, and *appendices*.

New definitions are usually presented **like this** and words or phrases to be highlighted are *emphasized like this*.

1.3 Lab Exercises and Experiment Descriptions

Each experiment description is divided into several parts:

- **Purpose**

  The specific objectives of the experiment are given. These may be in terms of *theories to be tested* (see Theory below) or in terms of *skills* *to be developed* (see Introduction below).

- **Introduction**

  This section should explain or mention new measuring techniques or equipment to be used, data analysis methods to be incorporated, or other *skills* to be developed in this experiment. *Knowledge is cumulative; what you learn in one lab you will be assumed to know and use subsequently, in this course and beyond. As well, proficiency comes with practice; the only way to become comfortable with a new skill is to take every opportunity to use it. If you get someone else (such as your lab partner) to do something which you don't like doing, you will never be able to do it better, and will get more intimidated by it as time goes on.*

- **Theory**

  (Note: lab exercises and experiment descriptions will make slightly different use of the "theory" section. In a lab exercise, "theory" will refer to explanations and derivations of the *techniques* being taught. In experiment descriptions, "theory" will refer to the *physics* behind an experiment.)

  A physical theory is often expressed as a mathematical relationship between measurable quantities. Testing a theory involves trying to determine whether such a mathematical relationship *may* exist. All measurements have **uncertainties** associated with them, so we can only say whether or not any difference between our results and those given by the relationship (theory) can be accounted for by the *known* uncertainties or not. (There may be other factors affecting the results which were not accounted for.) We cannot conclude that a theory is "true" or "false", only whether our experiment "agrees" with or "supports" it. Experimentation in general is an **iterative** process; one sets up an experiment, performs it and takes measurements, analyzes the results, refines the experiment, and the process repeats. No experiment is ever "perfect", although it may at some point be "good enough", meaning that it demonstrates what was required *within experimental uncertainty.* The theory section for an experiment should give any mathematical relationship(s) pertinent to that experiment, along with any definitions, etc. which may be needed. *You don't have to understand a theory in depth to test it; inasmuch as it is a mathematical* *relationship between measurable quantities, all you need to understand is* how *to measure the quantities in question and* how *they are related.* This is why whether or not you understand the theory is irrelevant in the lab. (In fact, you may at times find the experiment helps you understand the theory, whether you do the lab before or after you cover the material in class.)

Consider the example of Table 1.1

qwertys	*poiuyts*
1	1
2	3
3	5

Table 1.1: Relationship between qwertys and poiuyts

If you were asked, "Does $p = q^2$?", you would say no. It is no different to be asked "Is the following theorem correct?"

Lab Practice 1 *In a first year physics laboratory, poiuyts always vary as the square of qwertys.*

As long as you can measure qwertys and poiuyts (or can calculate them from other things you *can* measure), then you can answer the question, even without knowing *why* there should be such a relationship.

Sometimes there is a *disadvantage* to knowing too much about what to expect. It is easy to overlook unexpected data because it is not "right"; (meaning it doesn't give you the result you expected.)

The data are always right!

If your data[1] are giving you a result you don't like, that is a message that either you have made a mistake or there is more going on than you have accounted for.

[1] Dat*um* is the singular term. Dat*a* is the plural term.

- **Procedure**

  This tells you what you are required to do to perform the experiment. Unless you are told otherwise, these instructions are to be followed precisely. If there are any changes necessary, you will be informed in the lab.

  Questions will need to be handed in; *tasks* will be checked off in the lab.

  Included in this section are three important subsections:

  – Preparation (including pre-lab questions and tasks)

    The amount of time spent in a lab can vary greatly depending on what has been done ahead of time. This section attempts to minimize wasted time in the lab .

  – Experimentation or Investigation

    (including in-lab questions and tasks)

    For some lab exercises, there won't be an "experiment" as such, but there will be things to be done in the lab. The in-lab questions can usually only be answered while you have access to the lab equipment, but the answers will be important for further calculations and interpretation. For computer labs, instead of questions there will often be tasks, consisting of points to be demonstrated while you are in the lab.

  – Analysis (usually for labs) or Follow-up (usually for exercises)

    (including post-lab questions and tasks)

    Many of the calculations for a lab can be done afterward, provided you understand clearly what you are doing in the lab, record all of the necessary data, and answer all of the in-lab questions. The post-lab questions summarize the important points which must be addressed in the lab report.

    Since most of the exercises are developing skills, the results can usually be applied immediately to labs either already begun or upcoming.

- **Bonus**

  Most experiments will have a bonus question allowing you to take on further challenges to develop more understanding, either of data analysis concepts or of the underlying theory. (Bonus questions will usually be worth an extra 5% or so on the lab if they are done correctly.)

1.4 Templates

Each experiment, and many exercises, include a **template**. This is to help you ensure that you are not missing any data when you leave the lab. Since you will perform many calculations outside the lab, you'll need to make sure you have everything you need before you leave.

1.4.1 Table format in templates and lab reports

The templates are set up to help you consistently record information. For that reason, the tables are very "generic". It would be more concise to create tables that are specific to each experiment, but that would not be as helpful for your education. *When you write a report, you should set up tables which are concise and experiment-specific, even if they look different than the ones in the template.*

Don't automatically set up tables in your lab reports like the ones in the templates.

1.4.2 Template tables

- The first table contains information which you should record *every* time you do an experiment, and looks like this:

  My name:
  My student number:
  My partner's name:
  My other partner's name:
  My lab section:
  My lab demonstrator:
  Today's date:

1.4.3 Before the lab

- For any quantities to be *calculated*, fill in the equations in Table 1.2.

quantity	symbol	equation	uncertainty

Table 1.2: Calculated quantities

- For any *constants* to be used, either in calculations or to be compared with results, look up values and fill in Table 1.3.

quantity	symbol	value	uncertainty	units

Table 1.3: Given (ie. non-measured) quantities (ie. constants)

1.4.4 In the lab

- For any *new measuring instruments*, (ie. ones you have not used previously), fill in the information in Table A.1 or Table A.2.
- For any quantity measured *only once*, fill in Table 1.4. *There may be quantities which you do not need in your calculations, but which you would like to record for completeness. For that purpose there is a section in the table under the heading* Not in equations*.*
- If there is an effective uncertainty for the quantity in Table 1.4, then give specific information in Table 1.5. *Always include at least one source of systematic error, even if the bound yo give is small enough to make*

quantity	symbol	measuring instrument	value	effective uncertainty	units
mass 1	m_1				
mass 2					
mass	M				
Not in equations					
temperature	T				

Table 1.4: Quantities measured only once

it insignificant. This is so that you can show you understand how it would affect the results if it were big enough.

- Sometimes there may be several parts to an experiment, in which case it may help to keep things straight by separating them in the table, as in Table 1.5.

symbol	factor	bound	units	s/r
For Part 1				
For Part 2				

Table 1.5: Experimental factors responsible for effective uncertainties

- For any quantity where measurement is *repeated*, fill in Table 1.6. *In this case, there is no place for an effective uncertainty, since that will be determined by statistical analysis.*

quantity	symbol	measuring instrument	units
time	t	stopwatch	
For Station 1			
angle 1	θ	protractor	
For Station 2			
angle 2			
Not in equations			

Table 1.6: Repeated measurement quantities and instruments used

- For repeated measurements, there will be experiment-specific tables, such as Table 1.7.

1.4.5 Spreadsheet Templates

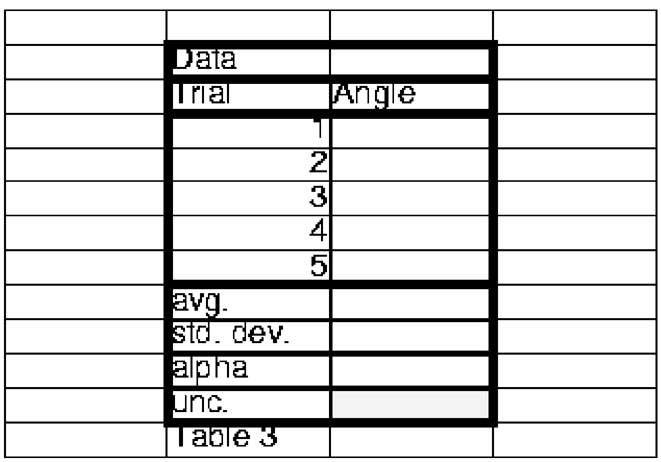
For some labs and exercises, there will be spreadsheets set up to potentially help you do your calculations. *For experiment-specific tables, you may choose to print and bring the spreadsheet template instead. This will make the most sense if you are going to use the spreadsheet to do your calculations.*

Even when there is a spreadsheet, there will still be information in the lab to record in the manual template, which does not have corresponding sections in the spreadsheet.

Trial #	Angle
1	
2	
3	
4	
5	

Table 1.7: Example of experiment-specific table

- Figure 1.1 is an example of how the corresponding experiment-specific table might look in the spreadsheet. The extra cells at the bottom are for calculation results.



	Data		
	Trial	Angle	
	1		
	2		
	3		
	4		
	5		
	avg.		
	std. dev.		
	alpha		
	unc.		
	Table 3		

Figure 1.1: Example of spreadsheet table

Chapter 2

Goals for PC141 Labs

If you're going to be a cook, you have to read some cookbooks, but eventually you need to get into the kitchen and cook.

If you're going to be an artist, you can study art and go to galleries, but eventually you're going to need to go into a studio.

If you're going to become a programmer, you can read about computers, study operating systems and programming languages, but eventually you have to sit down and program.

If you're going to become a scientist, you can read about science, go to lectures and watch videos about science, but eventually you need to go into a lab and do some research.

In the lecture part of this course, you'll learn a lot of physics. But it's only in the lab that you can learn about how to do research, which is ultimately what science is all about.

The labs and exercises in this course are to teach you about how to collect, analyze, and interpret data, and how to report your results so that they can be useful to other researchers.

The labs and exercises are chosen to teach research principles, rather than to illustrate specific physics concepts. This means, (among other things), that the "theory" behind any particular lab may not be covered in great detail in class. What you need will be covered briefly in the lab (or the manual).

Chapter 3

Instructions for PC141 Labs

Students will be divided into sections, each of which will be supervised by a lab supervisor and a demonstrator. This lab supervisor should be informed of any reason for absence, such as illness, as soon as possible. (If the student knows of a potential absence in advance, then the lab supervisor should be informed in advance.) A student should provide a doctor's certificate for absence due to illness. Missed labs will normally have to be made up, and usually this will be scheduled as soon as possible after the lab which was missed *while the equipment is still set up for the experiment in question.*

It is up to the student to read over any theory for each experiment and understand the procedures and do any required preparation before the laboratory session begins. This may at times require more time outside the lab than the time spent in the lab.

Students are normally expected to complete all the experiments assigned to them, and to submit a written report of your experimental work, including raw data, as required.

You will be informed by the lab instructor of the location for submission of your reports during your first laboratory period. This report will usually be graded and returned to you by the next session. The demonstrator who marked a particular lab will be identified, and any questions about marking should first be directed to that demonstrator. *Such questions must be directed to the marker within one week of the lab being returned to the student if any additional marks are requested.*

Unless otherwise stated, all labs and exercises will count toward your lab mark, although they may not all carry equal weight. If you have questions about this talk to the lab supervisor.

3.1 Expectations

As a student in university, there are certain things expected of you. Some of them are as follows:

- You are expected to *come to the lab prepared*. This means first of all that you will ensure that you have all of the information you need to do the labs, *including answers to the pre-lab questions*. After you have been told what lab you will be doing, you should read it ahead and be clear on what it requires. You should bring the lab manual, lecture notes, etc. with you to every lab. (*Of course you will be on time* so you do not miss important information and instructions.)

- You are expected to *be organized* This includes recording raw data with sufficient information so that you can understand it, keeping proper backups of data, reports, etc., hanging on to previous reports, and so on. It also means starting work early so there is enough time to clarify points, write up your report and hand it in on time.

- You are expected to *be adaptable and use common sense*. In labs it is often necessary to change certain details (eg. component values or procedures) at lab time from what is written in the manual. You should be alert to changes, and think rationally about those changes and react accordingly.

- You are expected to *value the time of instructors and lab demonstrators*. This means that you make use of the lab time when it is scheduled, and try to make it as productive as possible. This means NOT arriving late or leaving early and then seeking help at other times for what you missed.

- You are expected to *act on feedback from instructors, markers, etc.* If you get something wrong, find out how to do it right and do so.

- You are expected to *use all of the resources at your disposal*. This includes everything in the lab manual, textbooks for other related courses, the library, etc.

- You are expected to *collect your own data*. This means that you perform experiments with your partner and *no one else*. If, *due to an* *emergency*, you are *forced* to use someone else's data, you *must* explain why you did so and explain *whose* data you used. Otherwise, you are committing *plagiarism*.

- You are expected to *do your own work*. This means that you prepare your reports with *no one else*. If you ask someone else for advice about something in the lab, make sure that anything you write down is based on your *own* understanding. If you are basically regurgitating someone else's ideas, even in your own words, you are committing *plagiarism*. (See the next point.)

- You are expected to *understand your own report*. If you discuss ideas with other people, even your partner, do not use those ideas in your report unless you have adopted them yourself. You are responsible for all of the information in your report.

- You are expected to *be professional* about your work. This means meeting deadlines, understanding and meeting requirements for labs, reports, etc. This means *doing what should be done*, rather than what you think you can get away with. This means proofreading reports for spelling, grammar, etc. before handing them in.

- You are expected to *actively participate* in your own education. This means that in the lab, you do not leave tasks to your partner because you do not understand them. This means that you try and learn *how* and *why* to do something, rather than merely finding out the *result* of doing something.

3.2 Workload

Even though the labs are each only worth part of your course mark, the amount of work involved is probably disproportionately higher than for assignments, etc. Since most of the "hands-on" portion of your education will occur in the labs, this should not be surprising. (*Note: skipping lectures or labs to study for tests is a very bad idea.* Good time management is a much better idea.)

3.3 Administration

1. Students will be required to have a binder to contain all lab manual sections and all lab reports which have been returned. (A 3 hole punch will be in the lab.)

2. Templates will be used in each experiment as follows:

   (a) The template must be checked and initialed by the demonstrator before students leave the lab.

   (b) No more than 3 people can use one set of data. If equipment is tight groups will have to split up. (ie. Only as many people as fit the designated places for names on a template may use the same data.)

   (c) Part of the lab mark will be for the template.

   (d) The template *must* be included with lab handed in. penalty will be incurred if it is missing.) It must be the original, not a photocopy.

   (e) If a student misses a lab, and if space permits (decided by the lab supervisor) the student may do the lab in another section the same week without penalty. (However the due date is still for the student's own section.) In that case the section recorded on the template should be where the experiment was *done*, not where the student normally belongs.

3. In-lab tasks must be checked off before the end of the lab, and answers to in-lab questions must be handed in at the end of the lab. Students are to make notes about question answers and keep them in their binders so that the points raised can be discussed in their reports. Marks for answers to questions will be added to marks for the lab. For people who have missed the lab without a doctor's note and have not made up the lab, these marks will be forfeit. The points raised in the answers will still be expected to be addressed in the lab report.

4. Labs handed in after the due date incur a late penalty according to the lateness of the submission. After the reports for an experiment have been returned, any late reports submitted for that experiment cannot receive a grade higher than the lowest mark from that lab section for the reports which were submitted on time.

No labs will be accepted after the last day of classes.

3.4 Plagiarism

5. Plagiarism includes the following:
    - Identical or nearly identical wording in any block of text.
    - Identical formatting of lists, calculations, derivations, etc. which *suggests* a file was copied.
6. You will get one warning the first time plagiarism is suspected. After this any suspected plagiarism will be forwarded directly to the course instructor. With the warning you will get a zero on the relevant section(s) of the lab report. If you wish to appeal this, you will have to discuss it with the lab supervisor and the course instructor.
7. If there is a suspected case of plagiarism involving a lab report of yours, it does not matter whether yours is the original or the copy. The sanctions are the same.

3.5 Calculation of marks

8. The precise weightings of labs, exercises, and anything else will be discussed later in the lab manual.
9. The weighting of individual labs and exercises may depend on the quality of the work; ie. if you do better work on some things they will count more toward your final grade. Details will be discussed in the lab.
10. There may be a lab test at the end of term.

Chapter 4

How To Prepare for a Lab

> "The theory section for an experiment should give any mathematical relationship(s) pertinent to that experiment, along with any definitions, etc. which may be needed. *You don't have to understand a theory in depth to test it; inasmuch as it is a mathematical relationship between measurable quantities all you need to understand is* how *to measure the quantities in question and* how *they are related.* This is why whether or not you understand the theory is irrelevant in the lab. (In fact, you may at times find the experiment helps you understand the theory, whether you do the lab before or after you cover the material in class.)"

1. *Check* the web page after noon on the Friday before the lab to make sure of what you need to bring, hand in, etc. (It is a good idea to check the web page the day of your lab, in case there are any last minute corrections to the instructions.)

2. *Read over* the lab write–up to determine what the physics is behind it. (Even without understanding the physics in detail, you can do all of the following steps.)

3. *Answer* all of the pre-lab questions and do all of the pre-lab tasks and bring the answers with you to the lab.

4. *Examine* the spreadsheet and/or template for the lab (if either of them exists) to be sure that you understand what all of the quantities, symbols, etc. mean. (If there is a spreadsheet, you can prepare any or all of the formulas before the lab to simplify analysis later.)

5. *Enter* any constants into the appropriate table(s) in the template.
6. *Highlight* all of the in-lab questions and tasks so you can be sure to answer them all in the lab.
7. *Check* the web page the day of the lab in case there are any last minute changes or corrections to previous instructions.
8. *Arrive* on time, prepared. Bring all previous labs, calculator, and anything else which might be of use. (If the theory is in your textbook, maybe it would be good to bring your textbook to the lab!)

Chapter 5

Plagiarism

5.1 Plagiarism vs. Copyright Violation

These two concepts are related, but may get confused. Both involve unethical re-use of one person's work by another person, but they are different because the victim is different in each case.

Copyright is the right of an author to control over the publication or distribution of his or her own work. A violation of copyright is, in effect, a crime against the *producer of the work*, since adequate credit and/or payment is not given.

Plagiarism is the presentation of someone else's work as one's own, and thus the crime is against the *reader or recipient of the work* who is being deceived about its source.

Putting these two together suggests that there is a great deal of overlap, since trying to pass off someone else's work *without that person's permission* as one's own is both plagiarism and a violation of copyright. However, copying someone else's work without permission, *even while admitting who produced it*, is still a violation of copyright. Conversely, presenting someone else's work as your own, *even with that person's permission*,is still plagiarism.

5.2 Plagiarism Within the University

The Wilfrid Laurier University calendar says: " plagiarism ... *is the unacknowledged presentation, in whole or in part, of the work of others as one's* *own, whether in written, oral or other form, in an examination, report, assignment, thesis or dissertation "*

A search of the university web site for the word "plagiarism" turns up several things, among them the following:

- "Of course, under no circumstance is it acceptable to directly use an author's words (or a variation with only a few words of a sentence changed) without giving that author credit; this is plagiarism!!!" (Psychology 229)

- "plagiarism, which includes but is not limited to: the unacknowledged presentation, in whole or in part, of the work of others as one's own; the failure to acknowledge the substantive contributions of academic colleagues, including students, or others; the use of unpublished material of other researchers or authors, including students or staff, without their permission;" (Faculty Association Collective Agreement)

- "DO NOT COPY DOWN A SECTION FROM YOUR SOURCE VERBATIM OR WITH VERY MINOR CHANGES. This is PLAGIARISM and can lead to severe penalties. Obviously,no instructor can catch all offenders but, to paraphrase the great Clint Eastwood, "What you need to ask yourself is 'Do I feel lucky today?' " (Contemporary Studies 100 Notes on Quotes)

- "Some people seem to think that if they use someone else's work, but make slight changes in wording, then all they need to do is make reference to the "other" work in the standard way, i.e., (Smith, 1985), and there is no plagiarism involved. This is not true. You must either use direct quotes (with full references, including page numbers) or completely rephrase things in your own words (and even here you must fully reference the original source of the idea(s))." (Psychology 306, quote from *Making sense in psychology and the life sciences: A student's guide to writing and style* , by Margot Northey and Brian Timney (Toronto: Oxford University Press, 1986, pp. 32-33).)

- "Any student who has been caught submitting material that is not properly referenced, where appropriate, or submits material that is copied from another source (either a text or another student's lab), will be subject to the penalties outlined in the Student Calendar." (Geography 100)

- "Paraphrasing means restating a passage of a text in your own words, that is, rewording the ideas of someone else. In such a case, proper reference to the author must be given, or it is plagiarism. Copying a passage verbatim (not paraphrased) also constitutes plagiarism if it is not placed in quotes and is not referenced. Plagiarism is the appropriation or imitation of the language, ideas, and thoughts of another author, and the representation of these as one's own." (Biology 100)

5.3 How to Avoid Plagiarism

Plagiarism is a serious offense, and will be treated that way, but often students are unclear about what it is. The above quotes should help, but here are some more guidelines:

- If you use the same data as anyone else, this should be clearly documented in your report, WHETHER THE DATA ARE YOURS OR THEIRS.
- If you copy any file, even if you modify it, it is plagiarism unless you clearly document it. (This does not mean you can copy whatever you like as long as it's documented; you still are expected to do your own work. However at least you're not plagiarizing if you document your sources properly.)
- You are responsible for anything in your report; if you answer a question about your report with, "I don't know, my partner did that part", you are guilty of plagiarism, because you are passing off your partner's work as your own.
- The purpose for working together is to help each other learn. If collaboration is done in order for one or more people to avoid having to learn and/or work, then it is very likely going to involve plagiarism, (and is a no-no for pedagogical reasons anyway.)
- If you *give* your data, files, etc. to anyone else and they plagiarize it *you* are in trouble as well, because you are aiding their attempt to

cheat. *Do not give out data, files, or anything else* without express permission from the lab supervisor. This includes giving others your work to "look at"; if you give it to them, for whatever reason, and they copy it, *you* have a problem.

- If you want to talk over ideas with others, *do not* write while you are discussing; if everyone is on their own when they write up their reports, then the group discussion should not be a problem. However, as in a previous point, do not use group consensus as justification for what you write; discussion with anyone else should be to help you sort out your thoughts, not to get the "right answers" for you to parrot.

Look at the following from the writing centre, "How to Use Sources and Avoid Plagiarism"

$< http : //www.wlu.ca/forms/745/How_{t}o_{U}se_{S}ources_{A}PA.pdf >$

Chapter 6

Lab Reports

A lab report is *personal*, in the sense that it explains what *you* did in the lab and summarizes *your* results, as opposed to an assignment which generally answers a question of some sort. On an assignment, there is (usually) a "right answer", and finding it is the main part of the exercise. In a lab report, rather than determining an "answer", you may be asked to *test* something. (Note that no experiment can ever *prove* anything; it can only provide evidence for or against; just like in mathematics finding a single case in which a theorem holds true does not prove it, although a single case in which it does not hold refutes the theorem. A **law** in physics is simply a theorem which has been tested countless times without evidence of a case in which it does not hold.) The point of the lab report, when testing a theorem or law, is to explain whether or not you were successful, and to give reasons why or why not. In the case where you are to produce an "answer", (such as a value for g), your answer is likely to be different from that of anyone else; your job is to describe how you arrived at yours and why it is reasonable under the circumstances.

6.1 Format of a Lab Report

The format of the report should be as follows:

6.1.1 Title

The title should be more specific than what is given in the manual; it should reflect some specifics of the experiment.

6.1.2 Purpose

The specific purpose of the experiment should be briefly stated. (Note that this is not the same as the goals of the whole set of labs; while the labs as a group are to teach data analysis techniques, etc., the specific purpose of one experiment may indeed be to determine a value for g, for instance.) *Usually, the purpose of each experiment will be given in the lab manual. However, it will be very general. As in the title, you should try and be a bit more specific.*

There should always be both **qualitative** and **quantitative** goals for a lab.

Qualitative

This would include things like *"see if the effects of friction can be observed"*. In order to achieve this, however, specific quantitative analyses will need to be performed.

Quantitative

In a scientific experiment, there will always be numerical results produced which are compared with each other or to other values. It is based on the results of these comparrisons that the qualitative interpretations will be made.

6.1.3 Introduction

In general, in this course, you will not have to write an Introduction section.

An introduction contains two things: *theory* for the experiment and *rationale* for the experiment.

Theory

Background and theoretical details should go here. Normally, detailed derivations of mathematical relationships should not be included, but references must be listed. All statements, equations, and 'accepted' values must be justified by either specifying the reference(s) or by derivation if the equation(s) cannot be found in a reference.

Rationale

This describes why the experiment is being done, which may include references to previous research, or a discussion of why the results are important in a broader context.

6.1.4 Procedure

The procedure used *should not be described* unless you deviate from that outlined in the manual, or unless some procedural problem occurred, which must be mentioned. A reference to the appropriate chapter(s) of the lab manual is sufficient most of the time.

Ideally, someone reading your report and having access to the lab manual should be able to *reproduce* your results, within reasonable limits. (Later on we will discuss what "reasonable limits" are.) If you have made a mistake in doing the experiment, then your report should make it possible for someone else to do the experiment *without* making the same mistake. For this reason, lab reports are required to contain **raw data**, (which will be discussed later), and **explanatory notes**.

Explanatory notes are recorded to

- explain any changes to the procedure from that recorded in the lab manual,
- draw attention to measurements of parameters, values of constants, etc. used in calculations, and
- clarify any points about what was done which may otherwise be ambiguous.

Although the procedure need not be included, your report should be clear enough that the reader does not need the manual to understand your write–up.

(If you actually need to describe completely how the experiment was done, then it would be better to call it a "Methods" section, to be consistent with scientific papers.)

6.1.5 Experimental Results

There are two main components to this section; *raw data* and *calculations.*

Raw Data

In this part, the reporting should be done part by part with the numbering and titling of the parts arranged in the same order as they appear in the manual.

The *raw data* are provided so that someone can work from the actual numbers you wrote down originally before doing calculations. Often mistakes in calculation can be recognized and corrected after the fact by looking at the raw data.

In this section:

- Measurements and the names and precision measures of all instruments used should be recorded; in tabular form where applicable.
- If the realistic uncertainty in any quantity is bigger than the precision measure of the instrument involved, then the cause of the uncertainty and a bound on its value should be given.
- Comments, implicitly or explicitly asked for regarding data, or experimental factors should be noted here. This will include the answering of any given *in-lab* questions.

Calculations

There should be a clear path for a reader from raw data to the final results presented in a lab report. In this section of the report:

- Data which is modified from the original should be recorded here; in tabular form where applicable.
- Uncertainties should be calculated for all results, unless otherwise specified. The measurement uncertainties used in the calculations should be those listed as realistic in the raw data section.
- Calculations of quantities and comparisons with known relationships should be given. If, however, the calculations are repetitive, only one sample calculation, shown in detail, need be given. Error analysis should appear here as well.

- Any required graphs would appear in this part. (More instruction about how graphs should be presented will be given later.)
- For any graph, a table should be given which has columns for the data (including uncertainties) which are actually plotted on the graph.
- Comments, implicitly or explicitly asked for regarding calculations, observations or graphs, should be made here.

Sample calculations may be required in a particular order or not. If the order is not specified, it makes sense to do them in the order in which the calculations would be done in the experiment. If the same data can be carried through the whole set of calculations, that would be a good choice to illustrate what is happening.

Printing out a spreadsheet with formulas shown does not count as showing your calculations; the reader does not have to be familiar with spreadsheet syntax to make sense of results.

Post-lab questions should not be answered ina numbered list; rather the answers should be integrated in to the *Discussion* and *Conclusion* sections based on where they would be most appropriate.

6.1.6 Discussion

This section is where you explain the significance of what you have determined and outline the *reasonable limits* which you place on your results. (This is what separates a scientific report from an advertisement.) It should outline the major sources of random and systematic error in an experiment. *Your emphasis should be on those which are most significant, and on which you can easily place a numerical value. Wherever possible, you should try to suggest evidence as to why these may have affected your results, and include recommendations for how their effects may be minimized.* This can be accompanied by suggested improvements to the experiment.

Two extremes in tone of the discussion should be avoided: the first is the "sales pitch" or advertisement mentioned above, and the other is the "apology" or disclaimer (*" I wouldn't trust these results if I were you; they're probably hogwash."*) *Avoid whining* about the equipment, the time, etc. Your job is to explain briefly what factors most influenced your results, not to absolve yourself of responsibility for what you got, but to suggest changes or improvements for someone attempting the same experiment in the future. Emphasis should be placed on improving the experiment by changed *technique*, (which may be somewhat under your control), rather than by changed *equipment*, (which may not).

*M*any of the in-lab questions are directed to things which ought to be discussed here. Like the post-lab questions, don't answer them in a list, but integrate them into the text.

This section is usually worth a large part of the mark for a lab so be prepared to spend enough time thinking to do a reasonable job of it.

You must discuss at least one source of systematic error in your report, even if you reject it as insignificant, in order to indicate how it would affect the results.

6.1.7 Conclusions

Just as there are always both **qualitative** and **quantitative** goals for a lab, there should always be both **qualitative** and **quantitative** *conclusions* from a lab.

Qualitative

This would include things like *"see if the effects of friction can be observed"*. In order to achieve this, however, specific quantitative analyses will need to be performed.

Quantitative

In a scientific experiment, there will always be numerical results produced which are compared with each other or to other values. It is based on the results of these comparrisons that the qualitative interpretations will be made.

General comments regarding the nature of results and the validity of relationships used would be given in this section. Keep in mind that these comments can be made with certainty based on the results of error calculations.

The results of the different exercises should be commented on individually. *Your conclusions should refer to your original purpose; eg. if you set out to determine a value for g then your conclusions should include your calculated value of g and a comparison of your value with what you would expect.*

While you may not have as much to say in this section, what you say should be clear and concise.

6.1.8 References

If an 'accepted' value is used in your report, then the value should be footnoted and the reference given in standard form. Any references used for the theory should be listed here as well.

6.2 Final Remarks

Reports should be clear, concise, and easy to read. Messy, unorganized papers never fail to insult the reader (normally the marker) and your grade will reflect this. A professional report, in quality and detail, is at least as important as careful experimental technique and analysis.

Lab reports should usually be typed so that everything is neat and organized. Be sure to spell check and watch for mistakes due to using words which are correctly spelled but inappropriate.

6.3 Note on Lab Exercises

Lab *exercises* are different than lab reports, and so the format of the write-up is different. Generally exercises will be shorter, and they will not include either a *Discussion* or a *Conclusion* section.

Computer lab exercises may require little or even no report, but will have points which must be demonstrated in the lab.

Chapter 7

Uncertain Results

7.1 The most important part of a lab

The "Discussion of Errors" (or Uncertainties) section of a lab report is where you outline the *reasonable limits* which you place on your results. If you have done a professional job of your research, you should be prepared to defend your results. In other words, you should expect anyone else to get results which agree with yours; if not, you suspect theirs. In this context, you want to discuss sources of error which you have reason to believe are significant.

7.1.1 Operations with Uncertainties

When numbers, some or all of which are approximate, are combined by addition, subtraction, multiplication, or division, the uncertainty in the results due to the uncertainties in the data is given by the *range of possible calculated values based on the range of possible data values.*

> *Remember:* Since uncertainties are an indication of the imprecise nature of a quantity, uncertainties are usually only expressed to one decimal place. (In other words, it doesn't make sense to have an extremely *precise* measure of the *imprecision* in a value!)

For instance, if we have two numbers with uncertainties, such as $x = 2 \pm 1$ and $y = 32.0 + 0.2$, then what that means is that x can be as small as 1 or as big as 3, while y can be as small as 31.8 or as big as 32.2 so adding them can give a result $x + y$ which can be as small as 32.8 or as big as 35.2, so that the uncertainty in the answer is the sum of the two uncertainties. If we call the uncertainties in x and y Δx and Δy, then we can illustrate as follows:

Adding

$$\begin{array}{rcccccc} & x & \pm & \Delta x & = & 2 & \pm & 1 \\ + & y & \pm & \Delta y & = & 32.0 & \pm & 0.2 \\ & (x+y) & \pm & ? & = & 34.0 & \pm & 1.2 \\ \hline & & & & = & (x+y) & \pm & (\Delta x + \Delta y) \end{array}$$

Thus

$$\Delta(x+y) = \Delta x + \Delta y \tag{7.1}$$

Thus $x+y$ can be between 32.8 and 35.2, as above. (Note that we should actually express this result as 34 ± 1 to keep the correct number of significant figures.)

Remember: Uncertainties are usually only expressed to one decimal place, and quantities are written with the last digit being the uncertain one.

Subtracting

If we subtract two numbers, the same sort of thing happens.

$$\begin{array}{rcccccc} & x & \pm & \Delta x & = & 45.3 & \pm & 0.4 \\ - & y & \pm & \Delta y & = & -18.7 & \pm & 0.3 \\ & (x-y) & \pm & ? & = & 26.6 & \pm & 0.7 \\ \hline & & & & = & (x-y) & \pm & (\Delta x + \Delta y) \end{array}$$

Thus

$$\Delta(x-y) = \Delta x + \Delta y \tag{7.2}$$

Note that we still *add* the uncertainties, even though we *subtract* the quantities.

Multiplying

Multiplication and division are a little different. If a block of wood is found to have a mass of 1.00 ± 0.03 kg and a volume of 0.020 ± 0.001 m^3, then the **nominal** value of the density is $\frac{1.00\text{kg}}{0.020\text{m}^3} = 50.0\text{kg/m}^3$ and the uncertainty in its density may be determined as follows:

The mass given above indicates the mass is known to be *greater than or equal to* 0.97 kg, while the volume is known to be *less than or equal to* 0.021 m^3. Thus, the *minimum* density of the block is given by $\frac{0.97\text{kg}}{0.021\text{m}^3} = 46.2\text{kg/m}^3$. Similarly, the mass is known to be *less than or equal to* 1.03 kg, while the volume is known to be *greater than or equal to* 0.019 m^3. Thus, the *maximum* density of the block is given by $\frac{1.03\text{kg}}{0.019\text{m}^3} = 54.2\text{kg/m}^3$.

Notice that the above calculations do not give a symmetric range of uncertainties about the nominal value. This complicates matters, but if uncertainties are small compared to the quantities involved, the range is approximately symmetric and may be estimated as follows:

$$\begin{array}{cccccc}
 & x \pm \Delta x & = & 1.23 \pm 0.01 & = & 1.23 \\
 & & & & \pm & (0.01/1.23 \times 100\%) \\
\times & y \pm \Delta y & = & \times 7.1 \pm 0.2 & = & \times 7.1 \\
 & & & & \pm & (0.2/7.1 \times 100\%) \\
 & (x \times y) \pm ? & = & 8.733 \pm ? & \approx & 8.733 \\
 & & & & \pm & ((0.01/1.23 + 0.2/7.1) \times 100\%) \\
 & & & & \approx & 8.733 \\
 & & & & \pm & ((0.01/1.23 + 0.2/7.1) \times 8.733) \\
 & & & & \approx & 8.733 \\
 & & & & \pm & 0.317 \\
\hline
 & & & & \approx & (x \times y) \\
 & & & & \pm & \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \times y)
\end{array}$$

Thus

$$\Delta(x \times y) \approx \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \times y) \tag{7.3}$$

So rather than adding *absolute* uncertainties, we add *relative* or *percent* uncertainties. (To the correct number of significant figures, the above result would be

$$x \times y \approx 8.7 \pm 0.3$$

with one figure of uncertainty and the last digit of the result being the uncertain one.)

If you're a purist, or if the uncertainties are not small, then the uncertainty in the density can then be estimated in two obvious ways;

1. the *greater of the two* differences between the maximum and minimum and the accepted values

2. (or the maximum and minimum values can both be quoted, which is more precise, but can be cumbersome if subsequent calculations are necessary.)

(In the previous example, the first method would give an uncertainty of 4.2 kg/m^3.)

Dividing

Division is similar to multiplication, as subtraction was similar to addition.

$$\begin{array}{rcccl} x \pm \Delta x & = & 7.6 \pm 0.8 & = & 7.6 \pm (0.8/7.6 \times 100\%) \\ \div y \pm \Delta y & = & \div 2.5 \pm 0.1 & = & \div 2.5 \pm (0.1/2.5 \times 100\%) \\ (x \div y) \pm ? & = & 3.04 \pm ? & \approx & 3.04 \pm ((0.8/7.6 + 0.1/2.5) \times 100\%) \\ & & & \approx & 3.04 \pm ((0.8/7.6 + 0.1/2.5) \times 3.04) \\ & & & \approx & 3.04 \pm 0.4416 \\ \hline & & & = & (x \div y) \pm \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \div y) \end{array}$$

Thus

$$\Delta(x \div y) \approx \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \div y) \tag{7.4}$$

(To the correct number of significant figures, the above result would be

$$x \div y \approx 3.0 \pm 0.4$$

with one figure of uncertainty and the last digit of the result being the uncertain one.)

Determining Uncertainties in Functions Algebraically

Consider a function as shown in Figure 7.1. If we want the know the uncertainty in $f(x)$ at a point x, what we mean is that we want to know *the difference between* $f(x + \Delta x)$ *and* $f(x)$.

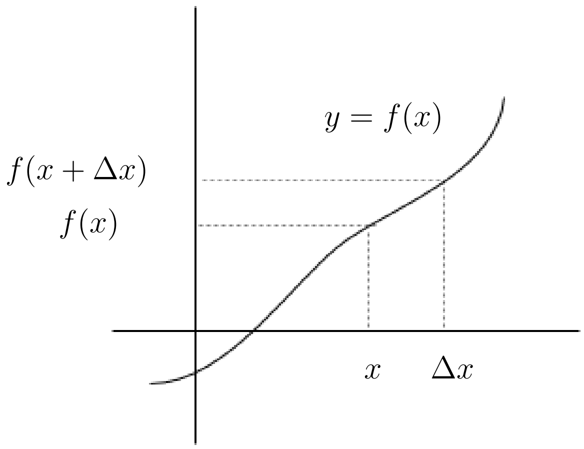


Figure 7.1: Uncertainty in a Function of x

If we take a closer look at the function, like in Figure 7.2, we can see that if Δx is small, then the difference between the function and its tangent line will be small. We can then say that

$$f(x) + f'(x) \times \Delta x \approx f(x + \Delta x)$$

or

$$\Delta f(x) \approx f'(x) \times \Delta x$$

For a function with a negative slope, the result would be similar, but the sign would change, so we write the rule with absolute value bars like this

$$\Delta f(x) \approx |f'(x)|\, \Delta x \tag{7.5}$$

to give an uncertainty which is positive.[1] Remember that uncertainties are usually rounded to one significant figure, so this approximation is generally valid.

[1]Now our use of the Δ symbol for uncertainties should make sense; in this example it has been used as in calculus to indicate "a small change in", but for experimental quantities, "small changes" are the result of uncertainties.

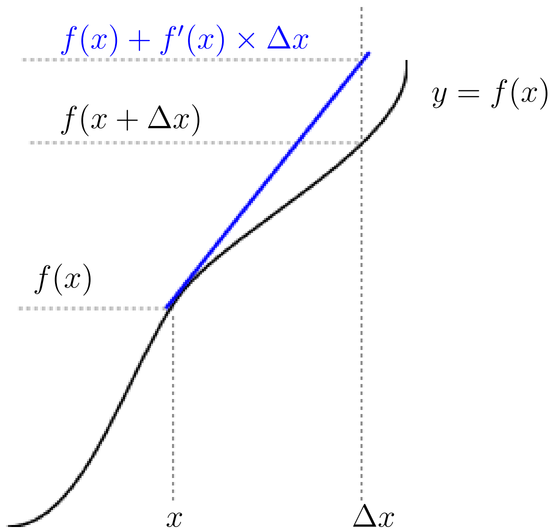


Figure 7.2: Closer View of Figure 7.1

Example: Marble volume Here is an example. Suppose we measure the diameter of a marble, d, with an uncertainty Δd, then quantities such as the volume derived from d will also have an uncertainty. Since

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

then

$$V' = 2\pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{2}d^2$$

and so

$$\Delta V \approx \left|\frac{\pi}{2}d^2\right| \Delta d$$

If we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.157$ cm^3 by this method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3.

Determining Uncertainties in Functions by Inspection

> *Note:* In the following section and elsewhere in the manual, the notation Δx is used to mean *"the uncertainty in x"*.

When we have a measurement of 2.0 ± 0.3 *cm*, this means that the *maximum* value it can have is $2.0 + 0.3cm$. The uncertainty is the difference between this maximum value and the *nominal* value (ie. the one with no uncertainty). We could also say that the *minimum* value it can have is $2.0 - 0.3$ *cm*, and the uncertainty is the difference between the nominal value and this maximum value. Thus if we want to find the uncertainty in a function, $f(x)$, we can say that

$$\Delta f(x) \approx f_{max} - f \tag{7.6}$$

or

$$\Delta f(x) \approx f - f_{min} \tag{7.7}$$

where f_{max} is the same function with x replaced by *either* $x + \Delta x$ or $x - \Delta x$; whichever makes f bigger, and f_{min} is the same function with x replaced by *either* $x + \Delta x$ or $x - \Delta x$; whichever makes f *smaller.* (The approximately equals sign is to reflect the fact that these two values may not be quite the same, depending on the function f.) For instance, if

$$f(x) = x^2 + 5$$

then clearly, if x is positive, then replacing x by $x + \Delta x$ will make f a maximum.

$$f_{max} = f(x + \Delta x) = (x + \Delta x)^2 + 5$$

and so

$$\Delta f(x) \approx f_{max} - f = f(x + \Delta x) - f(x) = \big((x + \Delta x)^2 + 5\big) - \big(x^2 + 5\big)$$

On the other hand, if we wanted to find the uncertainty in

$$g(t) = \frac{1}{\sqrt{t}}$$

then, if t is positive, then replacing t by $t - \Delta t$ will make g a maximum.

$$g_{max} = g(t - \Delta t) = \frac{1}{\sqrt{(t - \Delta t)}}$$

and so

$$\Delta g(t) \approx g_{max} - g = g(t - \Delta t) - g(t) = \left(\frac{1}{\sqrt{(t-\Delta t)}}\right) - \left(\frac{1}{\sqrt{t}}\right)$$

If we had a function of two variables,

$$h(w, z) = \frac{\sqrt{w}}{z^2}$$

then we want to replace *each* quantity with the appropriate value in order to maximize the total, so if w and z are both positive,

$$h_{max} = \frac{\sqrt{(w + \Delta w)}}{(z - \Delta z)^2}$$

and thus

$$\Delta h \approx h_{max} - h = \frac{\sqrt{(w + \Delta w)}}{(z - \Delta z)^2} - \frac{\sqrt{w}}{z^2}$$

Notice in each of these cases, it was necessary to restrict the range of the variable in order to determine whether the uncertainty should be added or subtracted in order to maximize the result. In an experiment, usually your data will automatically be restricted in certain ways. (For instance, masses are always positive.)

Example:Marble volume Using the above example of the volume of a marble,

$$\Delta V \approx V(d + \Delta d) - V(d)$$

Since

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

then

$$\Delta V \approx \frac{4}{3}\pi\left(\frac{d + \Delta d}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

If we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.173$ cm^3 by this method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3 as the value to be quoted.

Determining Uncertainties by Trial and Error

For a function $f(x, y)$, the uncertainty in f will be given by the *biggest* of

$$|f(x + \Delta x, y + \Delta y) - f(x, y)|$$

or

$$|f(x - \Delta x, y + \Delta y) - f(x, y)|$$

or

$$|f(x + \Delta x, y - \Delta y) - f(x, y)|$$

or

$$|f(x - \Delta x, y - \Delta y) - f(x, y)|$$

Note that for each variable with an uncertainty, the number of possibilities doubles. In most cases, common sense will tell you which one is going to be the important one, but things like the sign of numbers involved, etc. will matter a lot! For example, if you are adding two positive quantities, then the first or fourth above will give the same (correct) answer. However, if one quantity is negative, then the second and third will be correct.

> The advantage of knowing this method is that it always works. Sometimes it may be easier to go through this approach than to do all of the algebra needed for a complicated expression, especially if common sense makes it easy to see which combination of signs gives the correct answer.

Determining Uncertainties Algebraically

To summarize, the uncertainty in results can *usually* be calculated as in the following examples (if the percentage uncertainties in the data are small):

$$\begin{array}{cc}
(a) & \Delta(A + B) = (\Delta A + \Delta B) \\
(b) & \Delta(A - B) = (\Delta A + \Delta B) \\
(c) & \Delta(A \times B) \approx |AB| \left(\left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta B}{B}\right|\right) \\
(d) & \Delta(\frac{A}{B}) \approx \left|\frac{A}{B}\right| \left(\left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta B}{B}\right|\right) \\
(e) & \Delta f(A \pm \Delta A) \approx |f'(A)|\, \Delta A
\end{array}$$

Note that the first two rules above *always* hold true.

To put it another way, when *adding* or *subtracting*, you *add absolute* uncertainties. When *multiplying* or *dividing*, you *add percent* or *relative* uncertainties. Note that for the last rule above that angles and their uncertainties must be in *radians* for the differentiation to be correct! (In the examples above, absolute value signs were omitted since all positive quantities were used.) (Some specific uncertainty results can be found in *Appendix I*.)

Remember that a quantity and its uncertainty should always have the same units, so you can check units when calculating uncertainties to avoid mistakes.

Two important corrollaries: constants and powers The above rules can be used to derive the results for two very common situations;

- multiplying a quantity with an uncertainty by a constant
- raising a quantity with an uncertainty to a power

In the first case, a constant can be thought of as a number with *no* uncertainty. The product rule above is

$$\Delta(A \times B) \approx |AB| \left(\left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta B}{B} \right| \right)$$

If A is a constant, then $\Delta A = 0$, so

$$\Delta(A \times B) \approx |AB| \left(\left| \frac{\cancel{\Delta A}}{A} \right| + \left| \frac{\Delta B}{B} \right| \right) = |A\cancel{B}| \left(\left| \frac{\Delta B}{\cancel{B}} \right| \right) = |A \Delta B| = |A| \, \Delta B$$

In the second case, the product rule is:

$$\Delta f(A \pm \Delta A) \approx |f'(A)| \, \Delta A$$

and so if

$$f(A) = A^n$$

then

$$f'(A) = nA^{n-1}$$

and so

$$\Delta(A \pm \Delta A)^n \approx \left| nA^{n-1} \right| \Delta A$$

Example: Marble volume If we have a value of $d = 1.0 \pm 0.1$ cm, as used previously, then $\Delta V = 0.157$ cm^3 by this method.

Mathematically, this result and the previous one are equal if $\Delta d << d$. You can derive this using the **binomial approximation**, which simply means multiplying it out and discarding and terms with two or more Δ terms multiplied together; for instance $\Delta A \Delta B \approx 0$

Choosing Algebra or Inspection

Since uncertainties are usually only expressed to one decimal place, then small differences given by different methods of calculation, (ie. inspection or algebra), do not matter.

Example: Marble volume Using the previous example of the marble, if we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.173$ cm^3 by the inspection method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3 as the value to be quoted. By the algebraic method, $\Delta V = 0.157$ cm^3. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3, which is the same as that given by the previous method. So in this example a 10% uncertainty in d was still small enough to give the same result (to one significant figure) by both methods.

Sensitivity of Total Uncertainty to Individual Uncertainties

When you discuss sources of uncertainty in an experiment, it is important to recognize which ones contributed most to the uncertainty in the final result. In order to determine this, proceed as follows:

1. Write out the equation for the uncertainty in the result, using whichever method you prefer.

2. For each of the quantities in the equation which have an uncertainty, calculate the uncertainty in the result *which you get if all of the other uncertainties are zero.*

3. Arrange the quantities in descending order based on the size of the uncertainties calculated. The higher in the list a quantity is, the greater it's contribution to the total uncertainty.

The sizes of these uncertainties should tell you which factors need to be considered, remembering that only quantities contributing 10% or more to the total uncertainty matter. For example, from before we had a function of two variables,

$$h(w,z) = \frac{\sqrt{w}}{z^2}$$

so by inspection, its uncertainty is given by

$$\Delta h \approx h_{max} - h = \frac{\sqrt{(w+\Delta w)}}{(z-\Delta z)^2} - \frac{\sqrt{w}}{z^2}$$

So we can compute

$$\Delta h_w \approx \frac{\sqrt{(w+\Delta w)}}{z^2} - \frac{\sqrt{w}}{z^2}$$

and

$$\Delta h_z \approx \frac{\sqrt{w}}{(z-\Delta z)^2} - \frac{\sqrt{w}}{z^2}$$

Note that in the first equation, all of the Δz terms are gone, and in the second, all of the Δw terms are gone. By the algebraic method,

$$\Delta h \approx \frac{\sqrt{w}}{z^2}\left(\frac{\Delta w}{2w} + \frac{2\Delta z}{z}\right)$$

and so

$$\Delta h_w \approx \frac{\sqrt{w}}{z^2}\left(\frac{\Delta w}{2w}\right)$$

and

$$\Delta h_z \approx \frac{\sqrt{w}}{z^2}\left(\frac{2\Delta z}{z}\right)$$

Note that *until you plug values into these equations, you can't tell which uncertainty contribution is larger.*

In the above example, if we use values of $w = 1.00 \pm 0.01$ and $z = 2.00 \pm 0.02$, then the proportional uncertainties in both w and z are the same, 1%. However, using either inspection or the algebraic method, $\Delta h = 0.006$, and $\Delta h_w = 0.001$ while $\Delta h_z = 0.005$; in other words, the uncertainty in the *result* due to Δz is five times the uncertainty due to Δw! (As you get more used to uncertainty calculations, you should realize this is because z is raised to a higher power than w, and so its uncertainty counts for more.) In order to improve this experiment, it would be more important to try and reduce Δz than it would be to try and reduce Δw.

Simplifying Uncertainties

Uncertainty calculations can get quite involved if there are several quantities involved. However, since uncertainties are usually only carried to one or two significant figures at most, there is little value in carrying uncertainties through calculations if they do not contribute significantly to the total.

> You do not need to carry uncertainties through if they do not contribute more than 10% of the total uncertainty, since uncertainties are usually only expressed to one decimal place. (However, be sure to give bounds for these uncertainties when you do this.)

Note that this shows a difference between doing calculations by hand versus using a spreadsheet. If you are doing calculations by hand, it makes sense to drop insignificant uncertainties like this.

> If you're using a spreadsheet in order to allow you to change the data and recalculate, it may be worth carrying all uncertainties through in case some of them may be more significant for different data.

7.1.2 Uncertainties and Final Results

When an experiment is performed, it is crucial to determine whether or not the results *make sense*. In other words, do any calculated quantities fall within a "reasonable" range?

The reason for doing calculations with uncertainties is so that uncertainties in *final answers* can be obtained. If, for instance, a physical constant was measured, the calculated uncertainty determines the range around the calculated value in which one would expect to find the "theoretical" value. *If* the theoretical value falls within this range, then we say that our results *agree* with the theory *within our experimental uncertainty.*

For instance, if we perform an experiment and get a value for the acceleration due to gravity of $g = 9.5 \pm 0.5\text{m/s}^2$ then we can say that we say that our values agrees with the accepted value of $g = 9.8\text{m/s}^2$ *within our experimental uncertainty.*

If we have two values to compare, such as initial and final momentum to determine whether momentum was conserved, then we see if the ranges given by the two uncertainties overlap. In other words, if there is a value or range of values common to both, then they agree within experimental uncertainty.

So if an experiment gives us a value of $p_i = 51.2 \pm 0.7$ kg-m/s and $p_f = 50.8 \pm 0.5$ kg-m/s, then we would say the values agree within experimental uncertainty since the range from 50.5 kg-m/s $\rightarrow$ 51.3 kg-m/s is common to both. Since what we were studying was the conservation of momentum, then we would say that in this case momentum was conserved within experimental uncertainty. Note that if both uncertainties were 0.1 kg-m/s, then our results would *not* agree and we would say that momentum was *not* conserved within experimental uncertainty.

Mathematically, if two quantities a and b, with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if

$$|a - b| \leq \Delta a + \Delta b \tag{7.8}$$

A constant given with no uncertainty given can usually be assumed to have an uncertainty of zero.

If we need to compare 3 or more values this becomes more complex.

If two quantities agree within experimental error, this means that the discrepancy between experiment and theory can be readily accounted for on the basis of measurement uncertainties which are known. If the theoretical value does not fall within this range, then we say that our results *do not agree* with the theory within experimental uncertainty. In this situation, we cannot account for the discrepancy on the basis of measurement uncertainties alone, and so some other factors must be responsible.

If two numbers do not agree within experimental error, then the *percentage difference* between the experimental and theoretical values must be calculated as follows:

$$Percent\ Difference = \left| \frac{theoretical - experimental}{theoretical} \right| \times 100\% \tag{7.9}$$

Remember: Only calculate the percent difference if your results do <u>not</u> agree within experimental error.

In our example above, we would *not* calculate the percentage difference between our calculated value for the acceleration due to gravity of $g = 9.5 \pm 0.5\text{m/s}^2$ and the accepted value of $g = 9.8\text{m/s}^2$ since they agree within our experimental uncertainty.

Often instead of comparing an experimental value to a theoretical one, we are asked to test a law such as the Conservation of Energy. In this case, what we must do is to compare the initial and final energies of the system in the manner just outlined.[2] If the values agree, then we can say that energy was conserved, and if the values don't agree then it wasn't. In that case we would calculate the percentage difference as follows:

$$Percent\ Difference = \left| \frac{initial - final}{initial} \right| \times 100\% \tag{7.10}$$

Significant Figures in Final Results

Always express final answers with absolute uncertainties rather than percent uncertainties. Also, always quote final answers with one significant digit of uncertainty, and round the answers so that the least significant digit quoted is the uncertain one. This follows the same rule for significant figures in measured values.

> Even though you want to round off your final answers to the right number of decimal places, don't round off in the middle of calculations since this will introduce errors of its own.

7.1.3 Discussion of Uncertainties

In an experiment, with each quantity measured, it is necessary to consider all of the possible sources of error *in that quantity*, so that a realistic uncertainty can be stated for that measurement. The "Discussion of Uncertainties" (or "Discussion of Errors") is the section of the lab report where this process can be explained.

[2] There is another possibility which you may consider. Suppose you compare the *change in energy* to its expected value of zero. In that case, *any* non-zero change would result in infinite percent difference, which is mathematically correct but not terribly meaningful physically.

Discussions of sources of error should always be made as concrete as possible. That means they should be use specific numerical values and relate to specific experimental quantities. For instance, if you are going to speak about possible air currents affecting the path of the ball in the *"Measuring "g""* experiment, you must reduce it to a finite change in either the fall *time* or the *height*.

Relative Size of Uncertainties

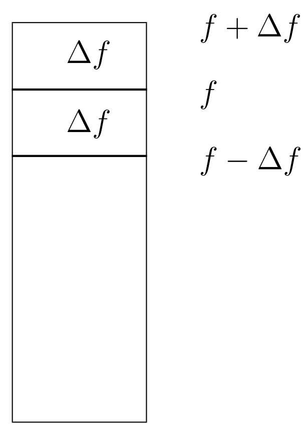


Figure 7.3: Relative Size of Quantity and its Uncertainty

The uncertainties which matter most in an experiment are those which contribute most to the uncertainty in the final result. Consider Figure 7.4, which may be seen as a magnification of one of the bands in Figure 7.3.

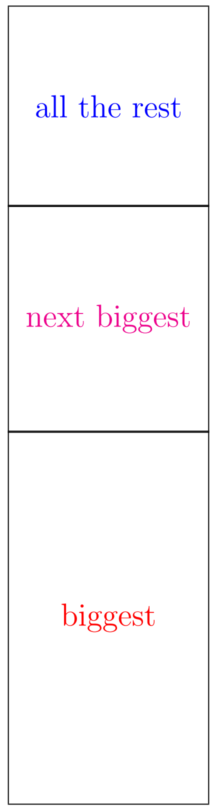


Figure 7.4: Contributions of Various Sources to Total Uncertainty

If the big rectangle represents the uncertainty in the final result, and the smaller rectangles inside represent contributions to the total from various sources, then one source contributes almost half of the total uncertainty in the result. The first two sources contribute about 75% of the total, so that all of the other sources *combined* only contribute about 25%. If we want to improve the experiment, we should try to address the factors contributing most. Similarly, in discussing our uncertainties, the biggest ones deserve most attention. In fact, since uncertainties are rounded to one decimal place, any uncertainty contributing less than 10% to the final uncertainty is basically irrelevant. The only reason to discuss such uncertainties is to explain why they are not significant.

Types of Errors

There are 3 major "categories" of sources of errors, in order of importance;

1. **Measurable uncertainties**-these are usually the biggest. The precision measure of each instrument used must always be recorded with every measurement. If "pre-measured" quantities are used, (such as standard masses), then there will usually be uncertainties given for these as well. If physical constants are given they may have uncertainties given for them, (such as the variation in the acceleration due to gravity by height above sea level, latitude, etc.) Where the realistic uncertainty in a quantity comes from any of these, (which will often be the case), you do not usually need to refer to them in your discussion. However, if there are any which contribute *greatly* to the uncertainty in your results, you should discuss them. For example, when you measure the mass of an object with a balance, then if the precision measure is the uncertainty used in your calculations, you don't need to discuss it, *unless* it is one of the *biggest* uncertainties in your calculations. *Keep in mind that without these values being given, it is impossible to tell whether any of the following sources of error are significant or not.*

2. **Bounded uncertainties**-these are things *which you observed*, and *have put limits on* and *usually* are much smaller than those in the group above. (Remember that since uncertainties are ultimately rounded to one significant digit, any which contribute less than 10% to the total uncertainty can be ignored.) Since you have observed them, you can give some estimate of how much effect they may have. For instance, suppose you measure the length of a table with a metre stick, and notice that the ends of the table are not exactly smooth and straight. If you can find a way to measure the variation in the length of the table due to this, then you can incorporate this into your uncertainty (if it is big enough) and discuss it.

A *plausible* error is one which can be tested. If you cannot figure out how to test for an error, it is not worth discussing. (Putting a bound on an error implies some method of testing for its existence, even if you are not able to do it at the present time.)

3. **Blatant filler**-these are things you may be tempted to throw in to sound more impressive. *Don't!!! If you did not observe them, don't discuss them.* If you suggest the gravitational pull of Jupiter is affecting your results, you'd better be prepared to show evidence (such as getting consistently different results at different times of day as the Earth rotates and so changes the angle of Jupiter's pull.) Do you even *know* in which direction the pull of Jupiter would be???

If you are going to discuss a source of uncertainty, then you must either have included it in your calculations, or given some reasonable bounds on its size. If you haven't done either of those, forget it!

You must discuss at least one source of systematic error in your report, even if you reject it as insignificant, in order to indicate how it would affect the results.

Reducing Errors

Whenever errors are discussed, you should suggest how they may be reduced or eliminated. There is a "hierarchy" of improvements which should be evident in your discussion. The following list starts with the best ideas, and progresses to less useful ones.

1. Be smart in the first place. You should never suggest you may have done something wrong in the lab; a professional who recognizes a mistake goes back and fixes it before producing a report. If you find yourself making a mistake which would seem likely to be repeated by other people, you may want to mention it in your report so that instructions may be clarified for the future.

2. Repeat the measurements once or twice *to check for consistency*. Repetition is a very *good* thing to do if your data are inconsistent or scattered. If certain values appear to be *incorrect*, you may want to repeat

them to make sure. If this seems to be true, and you feel a measurement was wrong, you should still include it in your report but explain why it was not used in your calculations. (This is probably similar to the previous one; *if* you think your data may be messed up, you should try to repeat it *before* you write your report, so this is not something you should be *suggesting* in your own report, although you should explain that you did it if you felt it was necessary.)

3. Change technique. It may be that a different way of doing things, using the *same equipment*, could (potentially) improve your results. If so, this should be explained.

   One example of this which may sound odd at first is to try and increase *the error and see what change is produced. For instance, if you neglected the mass of something in an experiment, you could increase that mass and then repeat the experiment. If the results do not change, then it is unlikely that the original mass had a significant effect.*

   Question: How big a change in the quantity in question (such as the mass just mentioned) should you try? Explain.

4. Make more *types* of observations. In some cases, monitoring certain things during the experiment may ensure they do not affect the results. This may be relevant in the case of "bounded uncertainties" above. It should be possible with equipment available in the lab. (For instance, if you are measuring the speed of sound, and the expected value is given at 25° C, then you might explain a discrepancy by the temperature being different. However, in this case, if you think the temperature may have affected your results, then you should check a thermometer to get the actual temperature during the experiment to suggest whether or not that was likely to have caused an effect.)

5. Repeat the measurements *to average the results.* While it is always good to repeat measurements, there is a law of diminishing returns. (In other words, repeating measurements a few times will give you a lot of information about how consistent your results are; repeating them many *more* times will not tell you as much. That is why the standard deviation of the mean decreases as $1/\sqrt{n}$, where n is the number of measurements; as n gets bigger, the change happens more slowly.) In fact, depending on the uncertainties involved, repetition at some point

is of no value. (That is when the standard deviation of the mean gets smaller than the uncertainty in the individual measurements. At that point you cannot improve without using a more precise instrument, no matter how many times you repeat the experiment.)

6. Change equipment; this is a *last resort*. Since this in essence means doing a different experiment, it is least desirable, and least relevant. Your goal is to produce the best results possible *with the equipment available*.

Ridiculous Errors

Certain errors crop up from time to time in peoples' reports without any justification. The point of your discussion is to *support* your results, placing *reasonable* bounds on them, not to absolve yourself of responsibility for them. Would you want to hire people who did not have faith in their own research? Including errors merely to "pad" your report is not good; one realistic source of error with justification is better than a page full of meaningless ones. Following are some commonly occurring meaningless ones.

- *"..human error..."*

  This is the *most irritating* statement you can make; you should have read over the instructions beforehand until you knew what was required, and then performed the experiment to the best of your ability. If you didn't you were being unprofessional and are wasting the reader's time. After doing your calculations, you should be able to tell from your results if they make sense. If not, you should go back and correct your errors. (Note something like reaction time does *not* fall into this category, because it is well-defined and can easily be measured. Vague, undefined errors are the big no-no.)

- *"..parallax..."*

  Parallax is the error you get from looking at a scale like a speedometer or a clock from the side; the position of the hands will appear different depending on your angle. With just about any scale I've seen, I'd be hard pressed to get an error of more than $5 \rightarrow 10\%$ from parallax (and the latter very rarely). Even that would only occur if I was deliberately *trying* to observe off-axis. Unless there is some reason that you cannot eliminate it, don't ascribe any significant error to it.

- *"..component values may not have been as stated..."*

  Usually people say this about masses, etc. I'm tempted to say *"Well, DUH! "* but I won't. *Of course* if given values are incorrect then calculations will be in error, but unless you have *evidence* for a specific value being wrong, (which should include some bounds on *how* wrong it could be), then it is just wild speculation. (You may allow *reasonable* uncertainties for these given values if you justify them.) Of course, suggesting equipment was damaged or broken is in this same category. If you understand what is going on, you should be able to tell if the equipment is functioning correctly. If it isn't, you should fix it or replace it (unless it's not working because you are not using it correctly; in that case, see "human error" above.) If it's possible you have broken it, you should bring this to the attention of the lab demonstrator, and be *very sure* you know how to use it properly before trying again with new equipment.

A Note on Human Errors

By now you are probably wondering why human error is so bad, even though humans have to make judgments in experiments, which will certainly contribute to uncertainties in the results. The problem is vague unspecified "human error" which is more of a disclaimer than a real thoughtful explanation. *If* you had to judge the time when an object stopped moving, for instance, you *can* discuss the judgment required, but in that case you should be able to determine concrete bounds for the uncertainties introduced, rather than suggesting some vague idea that your results may be meaningless.

> A rule of thumb to follow in deciding whether a particular type of "human error" is valid is this; if it is something which *you* may have done wrong, that is not valid. If it is a limitation which *anyone* would have doing the experiment, then it is OK, provided you bound it. (But don't call it "human error"; be specific about what judgment is involved.)

Chapter 8

Exercise on Estimation, Order of Magnitude Calculations, and Bounding

8.1 Estimation

Lots of experiments involve quantities which must be estimated. (For instance, before you *measure* anything, it's good to be able to *estimate* the result you expect, so you can determine what sort of instrument or method you'll need to perform the measurement.) Some estimates may be better than others, but what really matters is that you have a fair idea about how far off your estimate *could* be.

8.1.1 Bounding

Bounding a quantity is forming an estimate of how far off it could be; an *upper bound* is a bound above the expected value, and a *lower bound* is a bound below the expected value.

Picking Realistic Bounds

It's often easy to come up with reasonable bounds for a quantity by using similar known quantities which are pretty clearly above or below. For instance, if you are estimating a person's height, then you can compare with known heights of family members or friends. If you have to estimate the mass of an object, you can compare it to objects with which you are familiar.

Range of Possible Values for a Quantity

The **range** of values for a quantity is the difference between its upper and lower bounds.

8.1.2 Familiar Comparison

If you're trying to estimate something, and it's similar to something you know, then you can probably make a pretty good estimate by comparing. In other words, if you can establish an upper and a lower bound, then you can estimate something in between.

Illustration of Comparison

To illustrate, we'll try to make a few simple estimates. The first question we want to answer is: *How tall am I?*

Ex:

1. Find someone who thinks they are shorter than me. Record that person's height.

2. Find someone who thinks they are taller than me. Record that person's height.

3. Estimate my height according to the two known heights.

4. Estimate the bounds you would place on my height. (For instance, if you saw someone about my height commit a crime, what range of heights would you give to investigators so that it would be of use in identifying suspects?)

IQ1: Give the heights of the people who you thought were taller and shorter than me, and explain how you came up with your bounds for my height, and whether the bounds you gave had to be as far apart as the two heights you knew.

The goal when making estimates is to try and make them "safe" but "useful"; i.e. you are *pretty sure* about lower and upper bounds on your estimate, but the bounds are close enough together to make the estimate usable.

IT1: Form a group of 3 or 4, and based on comparisons, fill in Table 8.1 with a reasonable estimate and bounds for each of the following:

- my height (from above Ex:)
- mass of a block of wood (comparing to known masses)
- volume of liquid

8.1.3 Less Familiar Comparison

Often it's not easy to make a clear comparison with something very similar, and so the bounds and thus the estimate have to be a bit more fuzzy.

IT2: Based on your experience, fill in Table 8.2 with suggested bounds and a reasonable estimate for each of the following:

- height of this building (in metres)
- length of this building (in metres)
- mass of lab table (in kilograms)

IQ2: For one of those quantities, explain how you came up with the bounds and the estimate. Was the range of values for this comparison proportionally larger than for the familiar comparisons above? Explain.

8.1.4 Logarithmic scale

If several people make estimates, they will no doubt vary. However, they will probably still be in a common ballpark. This can be more easily observed by plotting the values on a *logarithmic scale*, such as the one in Figure 8.1. On a logarithmic scale, the distance of a number from the left end of the scale is proportional to the logarithm of the number. Figure 8.2 and Figure 8.3 show some other possibilities. (Logarithmic scales are often identified by the number of **cycles** they show.) A *cycle* is the space between two numbers

which differ by a factor of ten. So, between 1 and 10 is one cycle, between 2 and 20 is one cycle, between 5 and 50 is one cycle, etc.
Note that there is *no* zero on a logarithmic scale. All numbers are positive.

Where would zero be, if you wanted to show it?

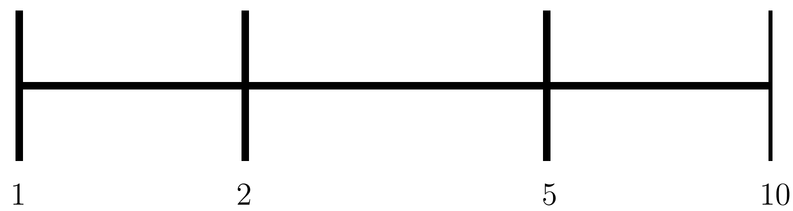


Figure 8.1: Logarithmic scale

There are many things which we *perceive* on a logarithmic scale (such as the volume of music).

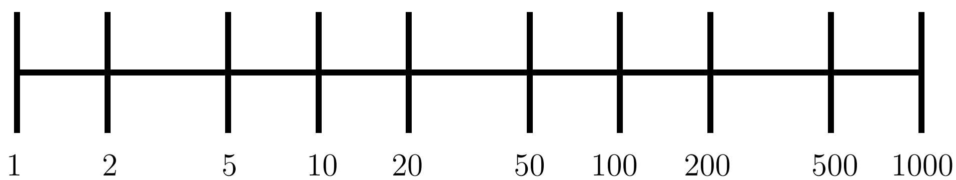


Figure 8.2: Three cycle logarithmic scale

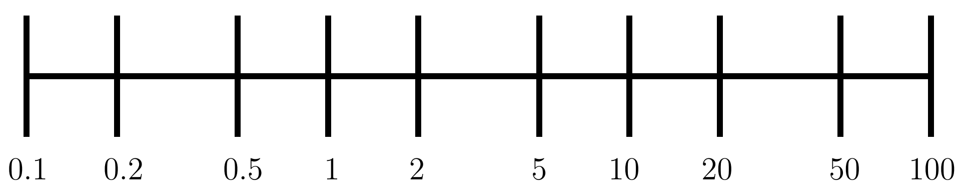


Figure 8.3: Logarithmic scale with numbers less than one

IT3: For one of the quantities estimated in **IT2**, use the logarithmic scale of Figure 8.4 and mark each of the class estimates on it.

IQ3: Do all of the estimates for each quantity fall within a single cycle of the scale? (In other words, between 2 and 20, 5 and 50, 10 and 100, etc.) Explain.

For any question, always include a sentence or two of explanation, even if a single word answer such as 'yes' or 'no' is possible.

8.2 Order of Magnitude Calculations

More complex quantities can be estimated by performing calculations with estimates. For instance, sometimes certain quantities can be measured but others must be estimated. These calculations are called **order of magnitude calculations**[1], since their purpose is to give a result which is within an order of magnitude (i.e. a factor of ten) of the result of the detailed calculation.

How big is an order of magnitude on a logarithmic scale?

Since an order of magnitude calculation is supposed to be within one order of magnitude, there should be some number, call it K, between 1 and 10 so that $(value \times K)$ is an upper bound and $(value/K)$ is a lower bound. The smaller K is the better. A value of 2 for K means you estimate the correct value to be within a factor of 2 of your calculation; a value of 1.5 for K means you estimate your value to be within 50%, (i.e. a factor of 1.5), of your calculated value, etc.

Illustration of Order of Magnitude Calculation

The next question we want to answer is: *What is my body mass index (BMI)?*

What is BMI? The BMI is the

$$BMI = (mass\ of\ a\ person\ in\ kg)/(\ height\ in\ m)^2$$

OR

$$BMI = 703 \times (weight\ of\ a\ person\ in\ lb)/(\ height\ in\ in)^2$$

(A BMI between 18.5 and 25 is considered "normal".)

[1]or, *"back of the envelope calculations"*

1. Find someone who thinks they are lighter than me. Record that person's mass or weight.
2. Find someone who thinks they are heavier than me. Record that person's mass or weight.
3. Estimate my mass or weight according to the two known masses or weights.
4. Estimate the bounds you would place on my mass or weight. Would they have to big as far apart as the two heights you know? Explain.

Now calculate my BMI based on your estimates of my height and mass or weight.

IQ4: For your calculation of my BMI above, suggest a value for K, between 1 and 10 as above, so that

- $(value \times K)$ is an upper bound
- $(value/K)$ is a lower bound

(Remember you want as small a value for K as is reasonable.) Was one bound harder to estimate than the other? Explain your answer.

When is a calculation an Order of Magnitude Calculation?

Any time you have to do a calculation using an estimated quantity, you are performing an **order of magnitude calculation**. The *order of magnitude* of a quantity refers specifically to the power of ten in its measurement. For instance, the height of the building would be in metres, while the length would be in tens of metres. In more general terms, the order of magnitude of a quantity refers to the cycle of a logarithmic scale to which the quantity belongs. Thus we could say that the order of magnitude value for the length of the science building is

- around 100 metres
- between 50 and 200 metres

Both of these are order of magnitude estimates.

When are Order of Magnitude Calculations used?

Order of magnitude calculations are quite commonly done in science *before* an experiment is performed. This is so that the range of expected data can be determined. They are also often done *as* the data are being collected to see if the experimental results appear to be in the correct ballpark.

> An order of magnitude calculation is *any* kind of calculation which will produce an answer which should be close to the *"real"* answer. Any calculation involving at least one estimated quantity is an order of magnitude calculation. Generally, the more estimated quantities involved in an order of magnitude calculation, the wider the distance between the upper and lower bounds produced.

8.2.1 Uncertainties

A quantity that is bounded can be expressed as an estimate with an **uncertainty**. (This is a little less cumbersome than giving the estimate, the lower bound, and the upper bound.) Usually it's easiest to express uncertainties in linear (i.e. non-logarithmic) terms, so that an estimate can be given which is *"plus or minus"* some amount. In order to do this, it may require adjusting one of the bounds so that the uncertainty can be the same in both directions. For instance, the length of the building was estimated to be between 50 and 200 metres. If I think it's probably around 100 metres I could modify my estimate of *"between 50 and 200 metres"* to be *"between 50 and 150 metres"* which I could state as *"*100 ± 50 *metres"*.

> If you have upper and lower bounds for a quantity, then the uncertainty can be *estimated* as one half of the range; i.e.
>
> $$uncertainty \approx 1/2(upper\ bound - lower\ bound)$$
>
> (A better determination of the uncertainty will be given in a later exercise.)

IT4: In the same group of 3 or 4, using the upper and lower bounds for the list of quantities in Task 1 above, and give final estimates with an uncertainty using your upper and lower bounds in Table 8.3.

IQ5: Did any of your estimates *not* fall midway between your upper and lower bounds? If so, how did you choose your uncertainty? If not, how would you choose your uncertainty if that happened?

Mathematically, the uncertainty in a quantity is usually expressed using the symbol Δ. So in other words, if mass has the symbol m, then the symbol Δm should be interpreted as *"the uncertainty in m"*. In that case you would write

$$m \pm \Delta m$$

to mean the mass with its uncertainty. Uncertainty is always given as a positive value, but it can be added or subtracted from the quantity to which it belongs.

8.2.2 Comparing Quantities with Uncertainties

Quantities with uncertainties are said to **agree** if the ranges given by the uncertainties for each overlap. For instance, if I estimated the length of the athletic complex as *"between 60 and 90 metres"* which I could state as *"75 ± 15 metres"*, and I estimated the length of the science building as 100 ± 20 metres, then I would say that the lengths of the two building agree since the ranges overlap. In other words, they *may* be the same; without more careful measurement I couldn't say for sure that they are different.

IQ6: Do the answers from different groups for one item in Task 4 *agree* with each other? Give values of different groups to help explain your answer.

8.3 Recap

By the end of this exercise, you should understand the following terms:

- estimate
- bound
- range of values for a quantity
- logarithmic scale
- order of magnitude calculation

- uncertainty
- whether quantities agree

All of these concepts will be important in later labs and exercises.

Before you leave, get your lab template checked off (but DON'T hand it in!) and hand in the answers to your in-lab questions. This is the way all of the labs and exercises will work.

8.4 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	6	________	50
Post-lab Questions	0	________	0
Pre-lab Tasks	0	________	0
In-lab Tasks	4	________	50
Post-lab Tasks	0	________	0

8.5 Template

My name:
My student number:
My partner's name:
My other partner's name:
My lab section:
My lab demonstrator:
Today's date:

quantity	estimate	units	upper bound	lower bound
my height				
mass of block of wood				
volume of liquid				

Table 8.1: Estimates for Task 1

quantity	estimate	units	upper bound	lower bound
height of building		m		
length of building		m		
mass of lab table		kg		

Table 8.2: Estimates for Task 2

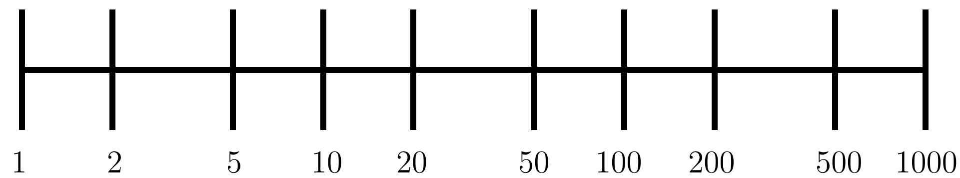


Figure 8.4: For Task 3

quantity	estimate	units	upper bound	lower bound	uncertainty
my height					
my BMI					
mass of block of wood					
volume of liquid					

Table 8.3: Determining Uncertainties for Task 4

Chapter 9

Exercise on Repeated Measurements

9.1 Note

The format of this exercise is not the "standard" one. It's an attempt to see how a different format works and what people prefer.

9.2 Introduction

Lots of experiments involve measurements are repeated. Repeating measurements allows the experimenter to be more accurate and precise in conclusions drawn from the experiment

To illustrate many of the concepts involved, we'll do an experiment. The question we want to answer is:

Has the Canadian mint changed the composition of pennies in the last decade?

This is actually somewhat similar to what Archimedes had to figure out a couple of thousand years ago. One of the most basic ways to try and figure this out is to see if the mass of the pennies has changed.

9.3 Mean and median

1. Weigh 5 pennies and record the values.
2. Identify the largest and the smallest values from any years.
3. Find the value that is in the middle; i.e. two values are as big or bigger and two are as small or smaller.

IT1: Fill in your results in Table 9.1.

Instrument	
reference (or name)	
units	
precision measure	
zero error	
Coin #	mass
1	
2	
3	
4	
5	
middle value	
average	

Table 9.1: Five pennies

The value you have just found is the **median**. It has an equal number above and below. (If you have an even number of measurements, the median is the average of the two in the middle.)

1. Calculate the average of the five values.

IT2: Fill in your results in Table 9.1.

The value you just calculated is also known as the **mean**. ("mean" = "average")

For many sets of data, the mean and the median will be similar. So, since you can find the median with no calculations, it is a simple way to *estimate* the average.

IQ1: Are there any two values in the table which are the same? If so, does that mean those two coins have *exactly* the same mass? (Hint: Would they be likely to have exactly the same mass on any balance that might be used?) Explain.

9.4 Spread of the data

Suppose you pick five coins from the same year. Probably you will find that the five coins do not all have the same mass. Because of this, it makes comparing coins from different years a bit tricky; you have to know what *range* of values are possible *for each year*.

IT3: Fill in your results in Table 9.2.

1. Pick five coins from the same year.

2. Weigh the 5 coins and record the values, along with the year of the coins.

3. Find the median value, and the maximum and minimum values.

4. Find the approximate spread of the values, by taking the difference between the maximum and minimum values and dividing by two.

(Note that Table 9.2 doesn't include information about the measuring instrument, since this has already been recorded in the previous table.)

Look at the median, and the value calculated for the approximate spread. Most of the values should fall between the median *minus* the approximate spread and the median *plus* the approximate spread. Did this occur?

units	
year	
Coin #	mass
1	
2	
3	
4	
5	
median value	
maximum value	
minimum value	
approximate spread	

Table 9.2: Five pennies from the same year

9.4.1 Summarizing the information

The median and the approximate spread give us an easy way to summarize the information in the table. We could write it as follows:

$$\textit{coin weight} \approx \textit{median value} \pm \textit{approximate spread}$$

With only 5 points, this may not seem much shorter than the five values, but if we had 10 (or 100!) measurements, this would be a lot more concise.

9.5 Comparing sets of data

1. Pick five coins from 5 years other than the year you've looked at already.

2. Weigh the 5 coins and record the values, along with the years of the coins.

IT4: Fill in your results in Table 9.3.

Does it look like the composition of the pennies for some other year is different? For which year?

units		
Coin #	mass	year
1		
2		
3		
4		
5		

Table 9.3: Pennies from other years

Now take 5 pennies from a year that looked different. Repeat the analysis above to summarize the results for the second year.

IT5: Fill in your results in Table 9.4.

units	
year	
Coin #	mass
1	
2	
3	
4	
5	
median value	
maximum value	
minimum value	
approximate spread	

Table 9.4: Five pennies from another year

IQ2: Does it look like the composition of the pennies is the same for those two years or not?

IQ3: If the masses of the coins for two years differ, does that mean that the composition (i.e. the material the coins are made of) has changed? Is there anything else that could account for the difference?

9.5.1 Statistics

In question **IQ2** above, all we could ask was whether it *looked* like the composition of the coins was different, since we had no way of determining how much variation would be small enough to ignore. In statistics, there is a quantity which can be calculated for an average to determine whether some other measurement is far enough away that it should be considered "different". That quantity is called the **standard deviation**, and it has the following properties:

- About 2/3 of the measurements should be between that average *minus* one standard deviation and the average *plus* one standard deviation.
- About 95% of the measurements should be between that average minus *two* standard deviations and the average plus *two* standard deviation.
- To turn the previous one around, the standard deviation is about 1/4 of the difference between the biggest and smallest measurements. (It will actually be a little smaller than that.)

Calculating the standard deviation

Table 9.5 is set up to help you calculate the standard deviation for the data for one of the years you chose.

1. Copy the data from Table 9.2 to fill in the first column and the average.
2. Subtract the average from each mass to fill in the second column.
3. Square the second column values to fill in the third column.
4. Add up the third column values and fill in the appropriate cell.
5. Use the formula in the table to calculate σ, the standard deviation. *(Don't worry about the last row yet.)*

IT6: Fill in your results in Table 9.5.

Now you can check the three points listed above to see if they apply in your case.

1. Highlight the rows in the table where the mass is *within average* $\pm\,\sigma$. That should be about 2/3 of the values.

2. Highlight the rows in the table where the mass is *within average* $\pm 2\sigma$. That should be about 95% of the values. Are there any that are outside of this range?

units			
year			
Coin #	mass	mass-average	$(m-a)^2$
1			
2			
3			
4			
5			
average			
sum			
$\sigma = \sqrt{\frac{sum}{n-1}}$			
$\alpha = \frac{\sigma}{\sqrt{n}}$			

Table 9.5: Standard deviation for first year

IT7: Repeat the previous process to fill in Table 9.6. (Again, don't worry about the last row.)

units			
year			
Coin #	mass	mass-average	$(m-a)^2$
1			
2			
3			
4			
5			
average			
sum			
$\sigma = \sqrt{\frac{sum}{n-1}}$			
$\alpha = \frac{\sigma}{\sqrt{n}}$			

Table 9.6: Standard deviation for second year

Comparing averages: step one

The reason for calculating the standard deviation is so that we can compare different averages. In our case, we want to compare the average mass of pennies from one year to the average mass of pennies from another year. We're *almost* ready to do that. If we knew we had a *representative sample* of coins from each year that we used for our average, we'd be in great shape. However, we can't be sure our samples are "representative". (For instance, some coins may be more scratched and worn than others from the same year.) If we have a lot of coins from one year, than the sample will be more representative than if we only have a few from one year. What we need is some quantity that reflects that.

The quantity that we're looking for is called the **standard deviation of the mean**, or the **standard error of the mean**, and is calculated by

$$\textbf{standard deviation of the mean} = \frac{\textit{standard deviation}}{\sqrt{n}}$$

The usual symbol used for the standard deviation of the mean is α, so this is usually written as

$$\alpha = \frac{\sigma}{\sqrt{n}} \tag{9.1}$$

Basically, the standard deviation of the mean is a measure of the range around the mean (i.e. average) from our sample which should contain the average of a "representative" sample.

Since the standard deviation of the mean has $\sqrt{n}$ in the denominator, it will get smaller as the amount of data gets larger, which is what we'd expect. (A larger sample should, by definition, be more representative.)

IT8: Now fill in the last rows of Table 9.5 and Table 9.6.

Comparing averages: step two

This part may sound pretty obvious, but it's important. We only have a hope of comparing two measurements if the instrument we used to measure them is precise enough to show a difference! For example, if we used a balance that only weighed to the nearest gram, *all* of the pennies might look the same. So to determine the uncertainty in the average of several measurements, we need to consider *both* the standard deviation of the mean *and* the precision measure of the instrument. This leads to the following rule:

> The uncertainty in the average of several measurements is the *larger of* the standard deviation of the mean and the precision measure.

The uncertainty in the average is more mathematically valid than the "approximate spread" determined earlier, although the approximate spread probably gave you a useful hint.

units		
precision measure		
year		
average		
std. deviation of the mean (α)		
uncertainty in average		

Table 9.7: Comparing two years

IT9: Complete Table 9.7.

Comparing the coins

Now that you've determined the uncertainty in the average for the two years, you can state whether or not the averages *agree* within their uncertainties, or in this case, whether the average mass of the pennies for two years were the same or not.

IQ4: Based on your calculations, do the average masses for the two years agree or not? (Be sure to state your masses with their uncertainties.) Based on your answer to **IQ3** above, does that mean the composition of the pennies changed between those years? What seems most likely?

How many coins should you measure?

Since all of our statements about whether the coins are the same or not depend on the sample of coins we used, is there any way of measuring enough coins that we don't have to wonder whether we should have measured more?

It turns out that we can measure "enough" coins to be confident. In fact, "enough" may not be that many in some cases. Look again at Equation 9.1. Since α will get smaller as we take more measurements, it seems like there's no limit to the number of useful measurements. However, remember that " *The uncertainty in the average of several measurements is the* larger of *the standard deviation of the mean and the precision measure.* ". Since α will keep getting smaller as more measurements are taken, there will *always* come a point where it will be smaller than the precision measure. After that point, the uncertainty will stay constant, no matter how many more measurements are taken, and so the process becomes mostly pointless.

> The optimum number of measurements has been taken when the standard deviation of the mean and the precision measure are *equal*.

Once you have a few measurements, you can calculate σ and then use it to determine how many measurements would be optimal be rearranging Equation 9.2 to solve for $N_{optimal}$.

$$precision\ measure = \frac{\sigma}{\sqrt{N_{optimal}}} \tag{9.2}$$

IQ5: For one of the sets of coins, determine the optimum number of coins to measure. (Include your calculations.) Would this number of coins be feasible to collect and use in the lab? Explain.

9.6 Recap

By the end of this exercise, you should understand the following terms, and be able to calculate:

- mean
- median
- mode
- standard deviation
- standard deviation of the mean
- uncertainty in the average
- optimum number of measurements

9.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	5	________	40
Post-lab Questions	0	________	20
Pre-lab Tasks	0	________	0
In-lab Tasks	9	________	40
Post-lab Tasks	0	________	0
Bonus		________	5

Chapter 10

Measuring "g"

10.1 Purpose

The purpose of this experiment is to measure the acceleration due to gravity and to see if the effects of air resistance can be observed by dropping various balls and recording fall times.

10.2 Introduction

This experiment will introduce the concept of using uncertainties to compare numbers.

10.3 Theory

10.3.1 Physics Behind This Experiment

For a body falling from rest under gravity, without air resistance, the height fallen at time t will be given by

$$h = \frac{1}{2}gt^2 \tag{10.1}$$

10.3.2 About Experimentation in General

In order to learn anything useful from an experiment, it is critical to collect *meaningful* data.

There are a few things to consider:

1. Data must be correct. (This means values must be recorded accurately, along with units.)
2. Data must be consistent. (Where you have repeated measurements, they should be similar.)
3. Data must be reproducible. (If you or someone else were to come back and do this later, the data should be similar to what you got the first time. This is more determined by your notes about what you do than by the actual data.)

Collaboration

If you are working with a partner, it is important that you both understand ahead of time what has to be done. It is easy to overlook details, but if two people are both thinking then it's much less likely that something important will be missed.

Keep in mind that there may be some individual quantities which must be known when doing an experiment which can change with time. If you do not record them at the time, you may have to redo the experiment completely.

Technique

How you collect the data may have a huge effect on the usefulness of the data. Always consider alternatives which may be better.

Preliminary Calculations

Before you leave the lab you need to do preliminary calculations of important results to see if they are in the right ballpark. This should prevent you from making scale errors (such as using wrong units) and should avoid you forgetting to record time-sensitive values as mentioned above.

Well-Documented Raw Data

If your raw data are too messy or incomplete for you to understand later, you will have to redo the experiment. Always record

- Date
- Experimenters' names and student ID numbers
- Lab section
- Experiment name
- For each type of measurement,
  - Name of device used
  - Precision measure
  - Zero error (if applicable)
  - Other factors in measurement making realistic uncertainty bigger than the precision measure, and bound on uncertainty.
  - Notes about how the measurement was taken or defined.
- For each table of data,
  - Title
  - Number
  - Units for each column
  - Uncertainties for each column; (If uncertainties change for data values in a column, make a column for the uncertainties.)
- For each question asked in the manual,
  - Question number
  - Answer

Most of the questions in this *exercise* are not numbered, but in labs they are. In the exercises, questions are often grouped together to try and develop a "big picture" of what is going on, and so the goal is to write explanations which address a group of questions, rather than handling each one individually. This is the approach which you are to take in writing your *"Discussion of Uncertainties"* in a lab. *Remember that wherever possible, you want to answer questions from experiment, rather than from theory.*

Uncertainties for all Measurements

Make sure that any value you record has an uncertainty. You should record both the precision measure of the instrument used, (if appropriate), and any other factors which may make the actual uncertainty bigger than that and rationale for the size of the realistic uncertainty.

10.4 Procedure

10.4.1 Preparation

Most experiments and exercises will have requirements which must be completed *before* the lab and presented *at the beginning* of the lab period.

Pre-lab Tasks

Wherever you are asked to copy information into the template, you may use the appropriate spreadsheet(s) instead as long as you can show them to get them checked off in the tasks.

PT1: Look up the density of brass, steel, lead, aluminum, wood and cork. (If you find ambiguous information, explain.) Fill these values in Tables 10.1 and and record the reference for them.

PT2: Rearrange Equation 10.1 to solve for g. In other words, complete the following:

$$g =$$

PT3: Print off the “template” sheet of the spreadsheet for this experiment and bring it to the lab (or bring your laptop with the spreadsheet so you can open it).

Pre-lab Questions

PQ1: If a person delays starting the watch after the ball is dropped, but does not delay stopping the watch when the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

PQ2: If a person does not delay starting the watch when the ball is dropped, but delays stopping the watch after the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

PQ3: If a person delays starting the watch after the ball is dropped *and* delays stopping the watch after the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

10.4.2 Experimentation

Apparatus

- stopwatch
- bucket
- dense ball
- less dense ball
- tape measure

Method

Note about groups of 3: There is not really a difference between the roles in a group of 3 and a group of two. For the purpose of the experiment, anyone who is *not* the "dropper" is the "gofer"; the important thing to note is whether the person dropping the ball is the one timing it. Anyone not dropping the ball is functionally equivalent, regardless of which floor he or she is on.

While you are getting the bucket lined up, practice drops with the ping pong ball. Once you have things aligned, you can switch to the dense ball.

1. Measure h.
2. Select a ball which you think should be relatively unaffected by friction and time one drop. Repeat this a few times, to see how consistent your times are.

3. Once you have some consistency in your times,do a calculation to see if this gives you a reasonable result for g or not.

Note: This is not usually *explicit* in a lab, but you should always check when you collect data to see if your results are in the right ballpark, so you can check your data or repeat the experiment if there seems to be a problem. If you wait until you are at home and find your results don't make sense, and your report is due the next day.....

4. Drop the ball several (ie. at least 5) times and record the fall times, as recorded by both the dropper and the gofer in Table 10.3. Calculate the average fall time.

5. Switch position, and repeat the previous steps.

6. Calculate values for g, based on the average times, Determine which of the methods gave the best result and try to figure out why. *Note that "best" has to consider both accuracy and consistency of data.*

7. Use the "preferred method" to collect data for a ball which should be more affected by friction, by dropping the ball several times and recording the fall times, as before.

8. Average the values for t and calculate g for the second type of ball.

In-lab Tasks

IT1: For each instrument you use, copy the pertinent information into Table A.1 or Table A.2 of Appendix A.Continue the list of instruments from previous Table A.1 or Table A.2.

IT2: Record at least 2 experimental factors leading to uncertainty in h *other than the one given* in the table and at least 2 experimental factors *other than reaction time* leading to uncertainty in t in Table 10.2 along with bounds and indication of whether they are random or systematic.

IT3: Identify on the template which person (i.e. you or your partner) is person 'A' and which is person 'B', and get it checked off before you leave the lab. *It would be wise to use the same designation of Person 'A' and Person 'B' as you did when determining your reaction time.* Show this and the completed Table 10.3 to the lab demonstrator before leaving.

1. Calculate the mean, standard deviation and standard deviation of the mean for each of your sets of times for the *"Measuring "g""* experiment.
2. Create tables for your lab report, with proper titles, numbers, etc.
3. For each table, record average time, the standard deviation, the standard deviation of the mean, and the ultimate uncertainty in the average time. (Hint: If you add more rows to Table 10.3 before the calculation of 'g' for σ, α, and the uncertainty in the average, it will be easy to include all of the information you need.)

In-lab Questions

Before leaving the lab, any determination of uncertainties in measurements and other factors affecting uncertainties must be completed. After you leave the lab, you may never see the equipment set up the same way again!

IQ1: What is the *realistic* uncertainty in h, and what experimental factor(s) cause it? This value should be one you feel you can defend as being neither extremely high nor extremely low. (There may be more than one contributing factor.)

IQ2: Was the technique which produced the most *accurate* time also the one which produced the most *precise* (ie. consistent) time? How difficult is it to determine the "best" technique if the most accurate one is not the most precise one?

IQ3: Write up a *Title* and a *Purpose* for this experiment, which are more appropriate than the ones given here. Be sure to include both *quantitative* goals and *qualitative* ones in the "Purpose".

10.4.3 Analysis

Before learning how to analyze uncertainties, there may be few obvious conclusions to be drawn from an experiment.

1. Calculate the standard deviation and standard deviation of the mean for the sets of times for each of the types of ball, and then determine the uncertainty in $\bar{t}$ for each data set.
2. Determine the formula for the uncertainty in g, given the uncertainties in h and $\bar{t}$.

3. Calculate the uncertainty in your values of g using the uncertainties determined above.

Pr-lab Questions

PQ4: For each of the following situations, come up with a physically *plausible* explanation in a sentence or so. (Mistakes in calculations do not count as physically plausible, unless they are errors with units or constants. Systematic errors in measurements are valid, but you must specify in what direction an error must be to have the effect observed.) If you suggest a scale error for one situation, don't simply use the opposite error for the opposite case. (ie. Try to come up with different things.)

- Values for g for both the dense and the non-dense ball are below the expected value of $9.8m/s^2$.
- Values for g for both the dense and the non-dense ball are above the expected value of $9.8m/s^2$.
- The value for g for dense ball is above the expected value of $9.8m/s^2$, but the value for g for the non-dense ball is below the expected value of $9.8m/s^2$.

PQ5: Can you think of any obvious *mistake* which might result in getting the value for g for dense ball below the expected value of $9.8m/s^2$, but the value for g for the non-dense ball above the expected value of $9.8m/s^2$? (Hint: Consider the case where the values for g were not calculated when the experiment was performed, but some days or weeks later.)

Post-lab Discussion Questions

The following questions will be able to be answered after further exercises.

Q1: Were the times given by different methods for the same ball significantly different? (Include your actual calculated values in your explanation.)

Q2: Were the times given by the preferred method for the first ball and the second ball significantly different? (Include your actual calculated values in your explanation.)

Q3: Would it be feasible to take the number of measurements calculated above in the actual experiment?

Q4: Are any of your resulting values for g *higher* than expected? What could explain that? What bounds does that give on factors like reaction time? Explain. (Hint: Look at your answers to the pre-lab questions.)

Q5: Would a more precise stop watch reduce the uncertainty in t or not? Explain.*(Note: Timing* technique *may help, but that's a different matter!!)*

Q6: Would a more precise device to measure h reduce the uncertainty in g or not? Explain.

Q7: Do either of the values for g determined using the preferred technique agree with the accepted value? Explain. (This question can be answered definitively based on your uncertainties.)

> Remember that values agree if the difference between them is less than the sum of their uncertainties; if they do not agree, then you should calculate the percent difference between them. (DON'T calculate the percent difference if they agree!! That's what "agreement" is all about!!)

Q8: Do the two different values for g using the preferred technique suggest friction is significant or not? Explain. (As with the previous question, this can answered definitively based on your uncertainties.)

10.5 Recap

By the time you have finished this lab report, you should know how to :

- collect data and analyze it
- write a lab report which includes:
  - *title* which describes the experiment
  - *purpose* which explains the objective(s) of the experiment
  - *results* obtained, including data analysis
  - *discussion of uncertainties* explaining significant sources of uncertainty and suggesting possible improvements
  - *conclusions* about the experiment, which should address the original objective(s).

10.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	5	________	25
In-lab Questions	3	________	30
Post-lab Questions	8		(in report)
Pre-lab Tasks	3	________	25
In-lab Tasks	3	________	20
Post-lab Tasks	0	________	0
Bonus		________	5

10.7 Template

My name:
My student number:
My partner's name:
My other partner's name:
My lab section:
My lab demonstrator:
Today's date:

Person A is:
Person B is:

The dense ball is made of:
The other ball is :

quantity (material)	density	unit	

Table 10.1: List of quantities

symbol	factor	bound	units
h	bend in tape measure		

Table 10.2: Experimental factors responsible for effective uncertainties

Instrument					
	Times (seconds)				
	Ball one				Ball two
	Technique				
i	g_A	d_B	d_A	g_B	
1					
2					
3					
4					
5					
average					
g					

Table 10.3: Timing data

Chapter 11

Simple Harmonic Motion

11.1 Purpose

The object of this experiment is to study systems undergoing simple harmonic motion.

11.2 Introduction

This experiment will develop your ability to perform calculations with repeated measurements. *Beware that the symbol α is used for angular acceleration, while it is also often used for standard deviation of the mean.*

11.3 Theory

In certain mechanical systems, a particle or body when displaced from its **rest position**, will experience a **restoring force** and undergo an acceleration which is proportional but opposite in direction to the displacement. This can be written as

$$a \propto -s \tag{11.1}$$

where a is the acceleration and s is the displacement. If we call the **constant of proportionality** C, then Equation 11.1 above becomes

$$a = -Cs \tag{11.2}$$

This is the **equation of motion** for the body in question. To determine the motion of the body we must *solve* the equation of motion. The solution follows.

We call the original displacement from the rest position A, the **amplitude** of the motion. If the body is released at position A at $t = 0$, then solutions of the *equation of motion* above are given by

$$s = A\cos(\omega t) \tag{11.3}$$

and for the velocity v

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = -\omega A\sin(\omega t) \tag{11.4}$$

and thus the acceleration is given by

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -\omega^2 A\cos(\omega t)$$

or

$$a = -\omega^2 s \tag{11.5}$$

where ω is called the **angular frequency** of the motion and is given in radians. At any time t the **phase angle** of the system is given by ωt. When this situation arises, (i.e. $a \propto -s$), the resulting situation is known as **S**imple **H**armonic **M**otion. It can be seen from the equations above that a system undergoing SHM will **oscillate** about its rest position. The **frequency** of the oscillation is given by

$$f = \frac{\omega}{2\pi} \tag{11.6}$$

and the **period** of the oscillation will be given by

$$\mathrm{T} = \frac{1}{f} = \frac{2\pi}{\omega} \tag{11.7}$$

For a system exhibiting SHM, the period of oscillation will be independent of the magnitude of the original displacement.

In this experiment, 2 different physical systems will be studied to observe SHM.

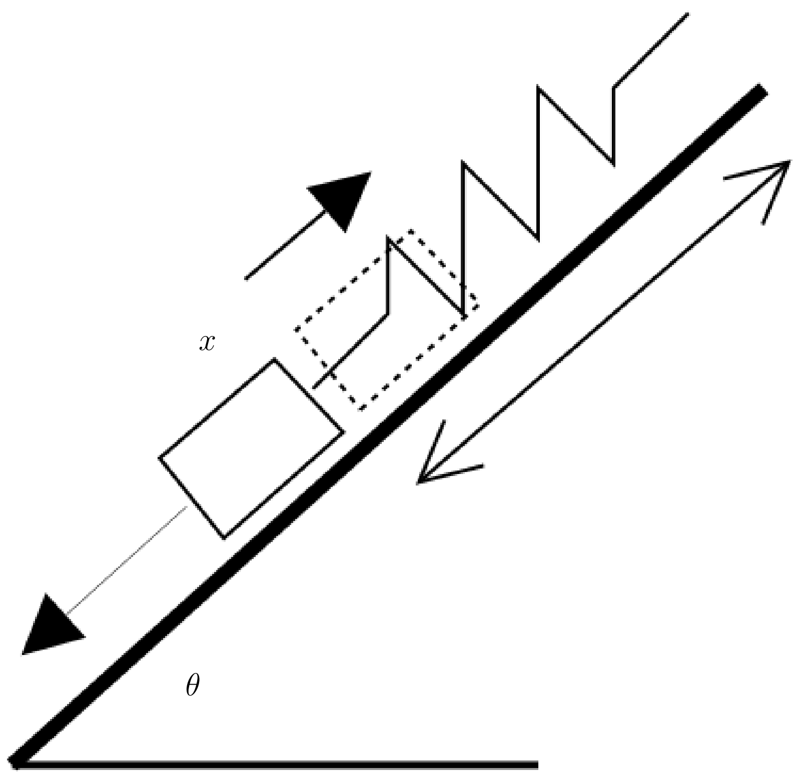


Figure 11.1: Mass on a Spring

11.3.1 Mass on a Spring with Gravity

For a spring, the force exerted is proportional to the distance the spring is compressed or stretched, and the proportionality constant is called k, known as the **spring constant**. This is known as **Hooke's Law**,

$$F = -kx \tag{11.8}$$

For a body attached to the spring, the acceleration of the body will be given by Newton's second law,

$$a = \frac{F}{m} \tag{11.9}$$

and thus combining the two equations gives

$$a = \frac{-kx}{m}$$

or

$$a = -\frac{k}{m}x \tag{11.10}$$

Since $a \propto -x$, then we have satisfied the condition for SHM, and in this case the angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} \tag{11.11}$$

which gives

$$\mathrm{T} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \tag{11.12}$$

11.3.2 Simple Pendulum

The **simple pendulum** consists of an idealized body; *a point mass suspended from a massless inextensible string swinging in a vertical plane solely under the influence of gravity*. Such a pendulum is shown in Figure 11.2. Let us suppose that at some point in the swing of a pendulum the string makes an angle ϕ with the vertical. In that case, the forces on the point mass m are F_T, the tension in the string, acting along the string, and mg, the weight of the pendulum acting straight down. Thus the resultant force acts along the trajectory of the mass and has a magnitude of $mg\sin(\phi)$. The trajectory of the mass is always perpendicular to the tension F_T. As well, the distance s

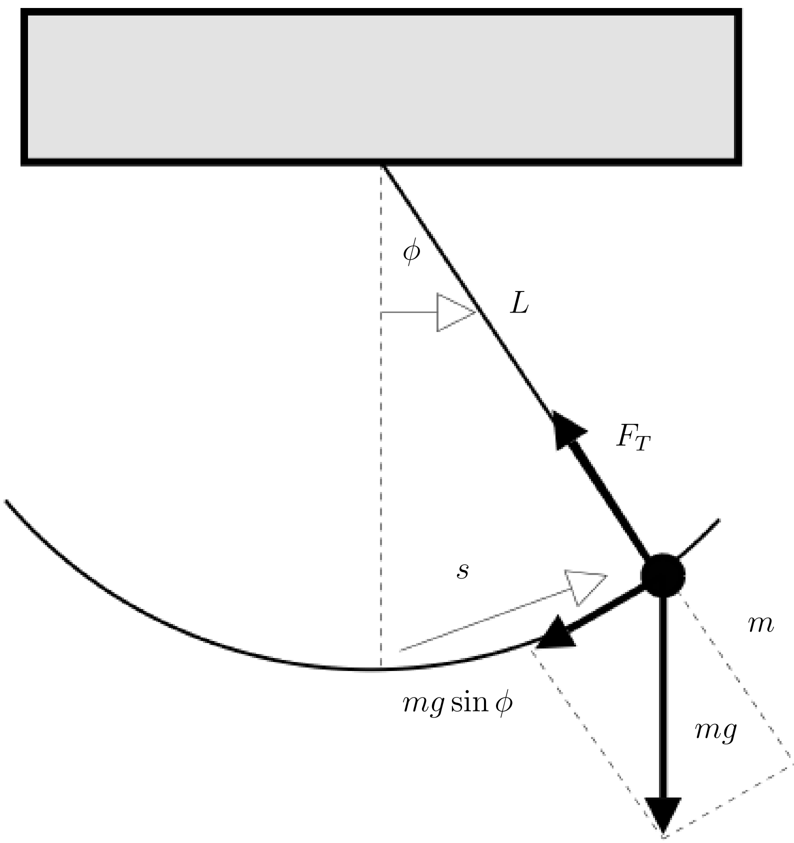


Figure 11.2: Simple Pendulum

along the trajectory from the equilibrium position to the mass is equal to $L\phi$ where L is the length of the string and ϕ is measured in radians.

> There are many times in the study of physics when you will find that *approximations* are made. This is done not because the physicist is lazy, but rather because such approximations make the final result simple and concise. (Usually it also agrees very closely with the "exact" solution and so for most practical purposes is as precise as the "exact" solution and is easier to use as well.) Such approximations, however, place limitations on the system and can only be used if such limitations are acceptable.

In this experiment we are going to make the approximation

$$\phi \approx \sin(\phi) \tag{11.13}$$

This is called the **small angle approximation** and is true for small values of ϕ *for ϕ in radians*. In this case the restoring force is

$$F = -mg\sin(\phi) \approx -mg\phi = -mg\frac{s}{L} = -\frac{mg}{L}s \tag{11.14}$$

Using the fact that $F = ma$ we get

$$a = -\frac{g}{L}s \tag{11.15}$$

Equation 11.15 above is the equation of motion for the pendulum bob. It states that the acceleration a of the bob along the circular path is directly proportional but opposite in direction to its displacement s from the rest position. This is the condition for SHM, and in this case

$$\omega = \sqrt{\frac{g}{L}} \tag{11.16}$$

and so

$$\mathrm{T} = 2\pi\sqrt{\frac{L}{g}} \tag{11.17}$$

11.4 Procedure

11.4.1 Mass on a Spring

Timing oscillations can be done in a couple of different ways; specifically one could measure the time taken between a system reaching the same point at which it is STOPPED, or one could measure the time taken between the same point in the middle of its motion where it is moving FASTEST. While these two measurements should be the same, one is "better".

Q1: Which one of the above timing methods is better and why?

11.4.2 Exercise 1: Mass on a Spring

1. Place a cart on the track with one end of the cart attached to a spring which is attached to one end of the track. Elevate that end of the track *but be sure that the spring does not stretch more than 1/3 the length of the track.* If you now displace the cart a bit and then release it, it should oscillate about its rest position. *Note: always displace the cart* up *the track rather than down, so the cart will not hit the end of the track.* From the description given above for SHM, the period should not depend on the amplitude of the oscillation.

   Q2: How consistent is the rest position? Does this matter? If so, why?

2. Find a displacement which will give at least 4 oscillations, so that you can measure the time for 3.

3. Repeat this 5 times to determine an average time for one oscillation and its uncertainty.

4. Repeat this for 3 different amplitudes.

5. Compare the periods for the different amplitudes.

   Q3: Does the assumption that period is independent of amplitude hold?

11.4.3 Exercise 2: Effect of Initial Displacement of Pendulum

1. With a pendulum length that is suitable and a bob of intermediate mass, compare the period of oscillation given by small amplitude ($\leq 10°$) oscillations with that given by larger amplitude ($\approx 30°$) oscillations. For each of these situations, measure the period of 10 oscillations, calculate the period, and repeat 3 times to determine an average and its uncertainty.

   Q4: Over what range of angles is the approximation of SHM valid based on your data?

   Q5: Over what range of angles is the approximation $\sin(\phi) \approx \phi$ valid?

11.4.4 Exercise 3: Effect of Mass on Pendulum Period

1. Set up another pendulum of the same length as above but with a different mass. Compare the period of this pendulum to the one previously calculated. *Make sure to keep the centre of mass of the pendulum at the same distance from the point of attachment so that L is constant and make sure to keep the angle small.*

   Q6: Is the statement that T is independent of m correct?

   Q7: How consistent can you keep l and other factors between experiments? Does this matter, and if so, why?

11.5 Bonus: Where Did Gravity Go?

In the derivation for the mass on a spring, the angle of the incline was not taken into account. Do the derivation including the force of gravity, and show that the result is the same when the motion is about the equilibrium point (ie. the point where the mass will be at rest with gravity).

11.6 Template

My name:
My student number:
My partner's name:
My partner's student number:
My other partner's name:
My other partner's student number:
My lab section:
My lab demonstrator:
Today's date:

quantity	symbol	measuring instrument	units
linear amplitude			
time			
angular displacement			
mass			
string length			

Table 11.1: Symbol identities

quantity	symbol	value	uncertainty	units

Table 11.2: Given quantities

symbol	value	smallest division	precision measure	units

Table 11.3: Quantities measured only once

symbol	factor	bound	units	s/r/b

Table 11.4: Other factors affecting uncertainties

quantity	symbol	value	units

Table 11.5: Quantities which do not appear in calculations

quantity	symbol	equation	uncertainty

Table 11.6: Calculated quantities

	Times (seconds)			
	Amplitude (cm)			
i				
1				
2				
3				
4				
5				

Table 11.7: Exercise 1 Timing data

	Times (seconds)	
	Displacement (degrees)	
i		
1		
2		
3		
4		
5		

Table 11.8: Exercise 2 Timing data

	Times (seconds)	
	Mass (g)	
i		
1		
2		
3		
4		
5		

Table 11.9: Exercise 3 Timing data

Chapter 12

Moment of Inertia

12.1 Purpose

The purpose of this experiment is to determine empirically the moment of inertia of a body about an axis and to compare this with the theoretical value calculated from the measured mass and dimensions of the body. This experiment will also examine what factors affect an object rolling down an incline.

12.2 Introduction

This experiment tests basic calculations with uncertainties and repeated independent measurements. In this experiment, there are several places involving the choice of certain parameters. How these choices affect the uncertainties involved is important.

12.3 Theory

Understanding rotational motion is a lot easier if you realize that the quantities involved are analogous to the familiar quantities in linear motion, and the resulting equations are analogous. The following two tables should illustrate this.

Some of the equations relating the quantities are shown below.

Linear Motion		Rotational Motion	
name	symbol	name	symbol
force	F	torque	τ
mass	m	moment of inertia	I
distance	x	angle	θ
velocity	v	angular velocity	ω
acceleration	a	angular acceleration	α
momentum	p	angular momentum	L

Table 12.1: Relation of Linear and Rotational Quantities

Linear Motion	Rotational Motion
$F = ma$	$\tau = I\alpha$
$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$

Table 12.2: Relation of Linear and Rotational Definitions

All of the common linear relationships can be turned into their rotational equivalents simply by replacing the linear quantities with the corresponding angular ones.

12.3.1 Theoretical Calculation of Moment of Inertia

The moment of inertia of an object is given by

$$I = \int r^2 dm$$

A table of the moments of inertia for several regular bodies is given in Figure 12.1.

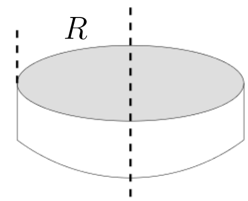
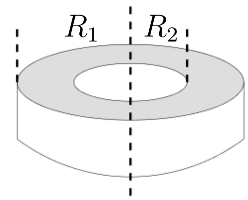
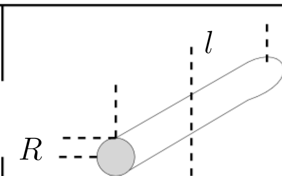
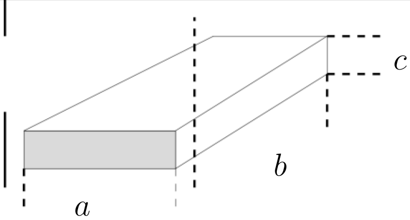
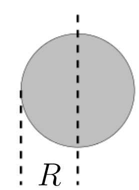
Solid cylinder or disc about axis perpendicular to plane of axis through centre	$\frac{1}{2}MR^2$	
Cylindrical ring about axis perpendicular to plane of ring through centre	$\frac{1}{2}M({R_1}^2 + {R_2}^2)$	
Solid cylinder or disc about transverse axis through centre	$\frac{1}{12}M(3R^2 + l^2)$	
Rectangular bar about axis perpendicular to face at centre	$\frac{1}{12}M(a^2 + b^2)$	
Solid sphere about centre	$\frac{2}{5}MR^2$	

Figure 12.1: Moment of Inertia of Regular Bodies

12.3.2 Experimentally Determining Moment of Inertia by Rotation

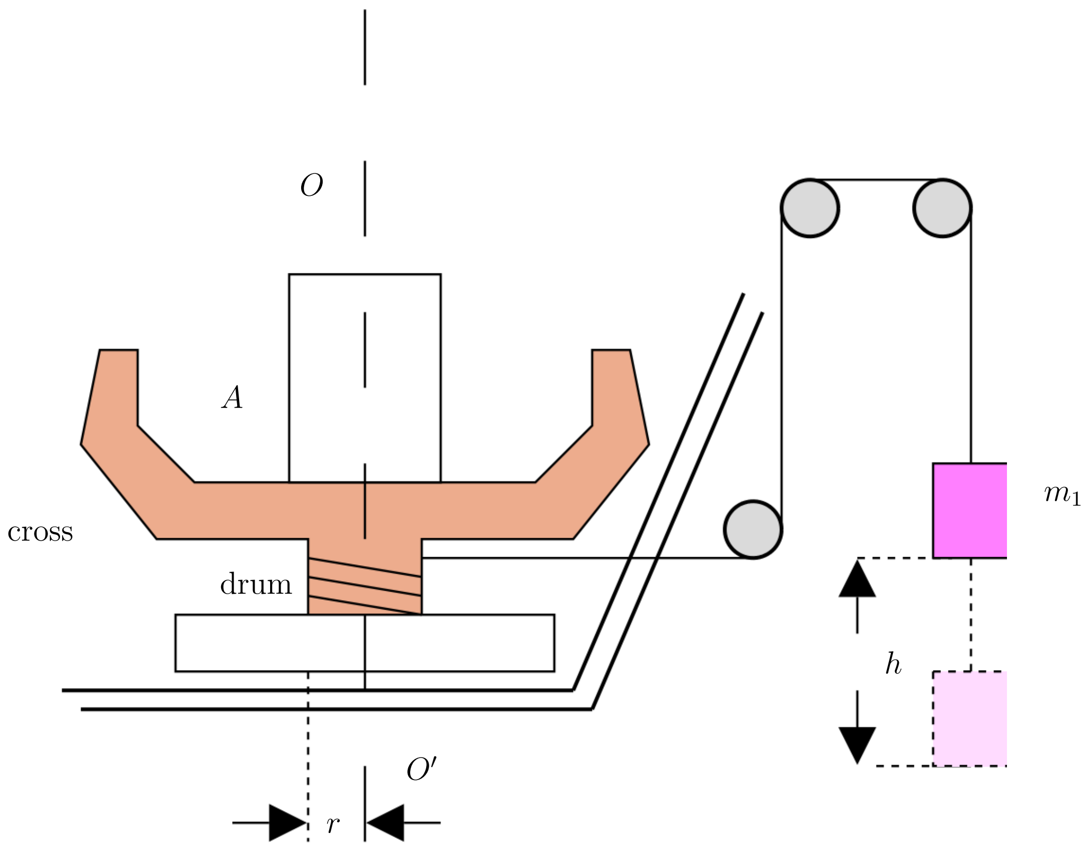


Figure 12.2: System Rotating with Constant Torque

Consider the system as shown in Figure 12.2. The body A, free to rotate about the vertical axis OO', is set in rotation as the result of the mass m_1 falling through a height h. The cord which constrains the motion of the mass m_1 is wound around the drum of the body A which has a radius r. The axis OO' passes through the centre of mass of the body A which has a **moment of inertia** I. When the mass m_1, starting from rest, falls a distance h under gravity, the body A is set in rotation. When moving, the mass m_1 has a velocity v_1 and the body A has an angular velocity ω_1. Clearly,

$$v_1 = r\omega_1 \tag{12.1}$$

If energy is conserved, then

$$m_1gh = \frac{1}{2}m_1v_1^2 + \frac{1}{2}I_1\omega_1^2 \tag{12.2}$$

Since the driving force is constant, the mass m_1 will be uniformly accelerated; consequently its displacement h, time of fall t_1, and its velocity v_1 are related by

$$v_1 = \frac{2h}{t_1} \tag{12.3}$$

Substituting Equations 12.1 and 12.3 into 12.2 we have

$$m_1 g = 2m_1 \frac{h}{t_1^2} + 2I_1 \frac{h}{t_1^2 r^2} \tag{12.4}$$

and

$$I_1 = m_1 r^2 (\frac{g t_1^2}{2h} - 1) \tag{12.5}$$

from which the moment of inertia may be determined in terms of measurable quantities and g, the acceleration due to gravity.

The body A may have such a shape that it would be difficult or practically impossible to compute its moment of inertia from its mass and dimensions. In general, the moment of inertia of a body could only be determined experimentally. However, the moments of inertia for certain regularly shaped objects may be calculated to provide a comparison between experiment and theory.

If the system above is altered to include a regular geometrical mass B having a moment of inertia I_2 about its centre of mass located on the axis OO' then the total moment of inertia of the system about the axis OO' is $I_1 + I_2$ and a larger mass m_2 is required to produce the displacement h in a time t_2 which is comparable to t_1. In this case it should be clear that

$$I_1 + I_2 = m_2 r^2 (\frac{g t_2^2}{2h} - 1) \tag{12.6}$$

From observed values of m_1, t_1, h, and r, I may be computed for the body A. Similarly from observed values of m_2, t_2, h, and r, $I_1 + I_2$ and hence I_2 may be computed for the body B.

12.3.3 Moment of Inertia and Objects Rolling Downhill

Consider the system as shown in Figure 12.3. For an object which is rolling, its energy is given by

$$E = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \tag{12.7}$$

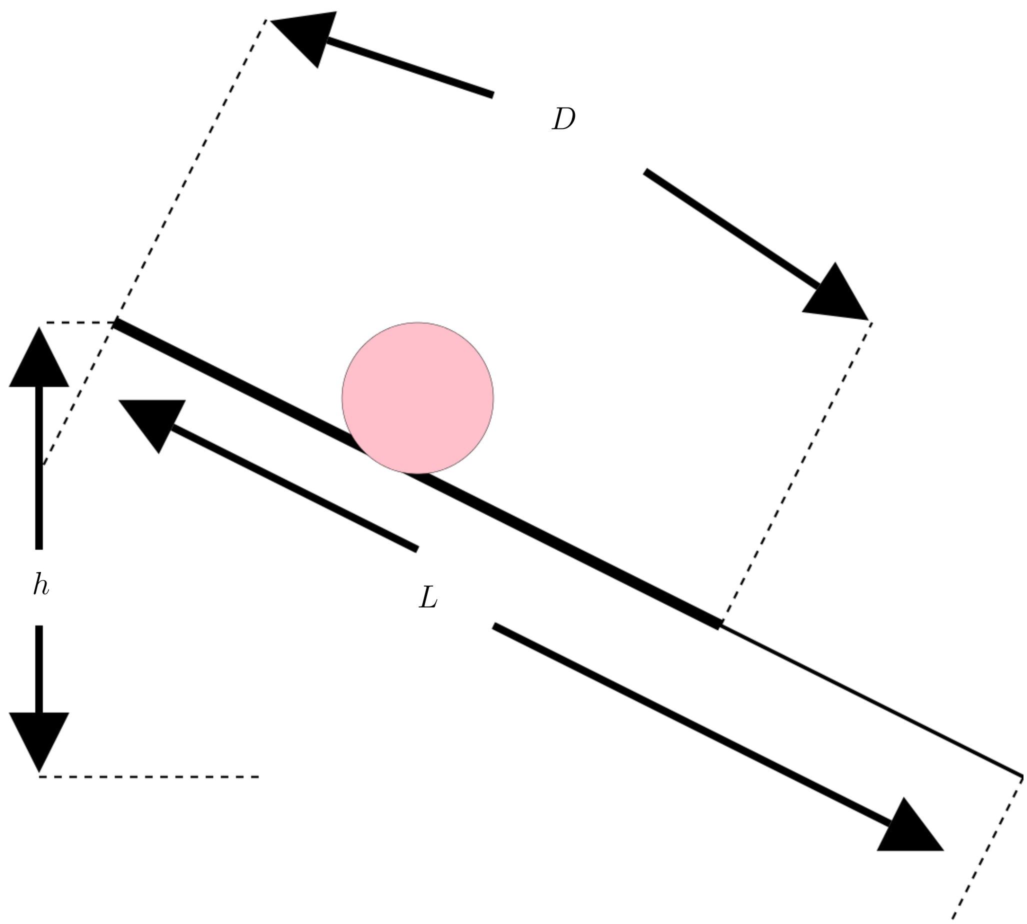


Figure 12.3: Object Rolling Down an Incline

In Equation 12.7 the first term represents the *translational* energy of the object and the second term represents its *rotational* energy. If this energy is provided by gravity, as in the case of an object rolling down an incline, then we can produce the following:

$$E = mgh = \frac{1}{2}m{v_f}^2 + \frac{1}{2}I{\omega_f}^2 \tag{12.8}$$

where h is the height "fallen" by the object, v_f is the final velocity and ω_f is the final **angular velocity** of the object.

If the distance traveled by the object is L, then since the object undergoes constant acceleration, we can show that

$$L = \frac{1}{2}v_f t \tag{12.9}$$

Finally, if the object has a radius R, then its *angular velocity*, ω, is given by

$$\omega = \frac{v}{R} \tag{12.10}$$

and its moment of inertia can be expressed as

$$I = \beta m R^2 \tag{12.11}$$

where β is a constant of proportionality determined by the *shape* of the object. (For instance, $\beta = .4$ for a sphere and $\beta = .5$ for a cylinder.)

Substituting Equation 12.10 and Equation 12.11 into Equation 12.8 gives

$$mgh = \frac{1}{2}m{v_f}^2 + \frac{1}{2}\beta m R^2 \left(\frac{v_f}{R}\right)^2 \tag{12.12}$$

which reduces to

$$mgh = \frac{1}{2}m{v_f}^2 + \frac{1}{2}\beta m{v_f}^2 \tag{12.13}$$

or

$$mgh = \frac{1}{2}m{v_f}^2(1+\beta) \tag{12.14}$$

Now including Equation 12.9 we get

$$mgh = \frac{1}{2}m\left(\frac{2L}{t}\right)^2(1+\beta) \tag{12.15}$$

from which we get

$$1+\beta = \frac{2mgh}{m\left(\frac{2L}{t}\right)^2} \tag{12.16}$$

and finally

$$\beta = \frac{ght^2}{2L^2} - 1 \tag{12.17}$$

> You will notice that in the above equation, only h, t, and L are left; i.e. *only the shape of the object matters, not the size or the mass.* Note the similarity in form to the term in parentheses in Equation 12.5.

If you are only using part of the ramp, then Equation 12.17 becomes

$$\beta = \frac{ght^2}{2DL} - 1 \tag{12.18}$$

where D is the distance used.

12.4 Procedure

12.4.1 Preparation

Pre-lab Questions

PQ1: Does your experience in the *Measuring 'g'* lab give you any insights into how to do the best timing? Explain. (Hint: think about the two roles involved in that experiment, and which role produced better timing.)

PQ2: Based on Equation 12.11, what is β for a sphere? For a cylinder? What would it be for a very thin hoop (where the inner and outer radii are essentially equal)?

PQ3: Based on Equation 12.17, will an object roll faster if β is smaller or larger? Which will roll slower; a sphere or a cylinder?

Pre-lab Tasks

PT1: Copy the equation you determined for ΔI for a ring in the *"Processing Uncertainties" exercise* into Table 12.6.

PT2: From Equation 12.5, determine ΔI_1 given m_1, Δm_1, r, Δr, etc. Copy this into Table 12.6.

PT3: From Figure 12.1, determine ΔI for a cylinder given M, ΔM, R and ΔR. Rewrite the expression for ΔI for the situation where it is the diameter, D, and its uncertainty ΔD which is measured instead of the radius. Copy this into Table 12.6.

PT4: From Equation 12.18, determine $\Delta\beta$ given D, ΔD, t, Δt, etc. Copy this into Table 12.6.

12.4.2 Experimentation

Apparatus

The apparatus consists of a light metal cross mounted on ball bearings so as to rotate in a horizontal plane about a vertical axis, as shown in Figure 12.2. The cross serves as a carrier for the object whose moment of inertia is to be determined. The drum is driven by means of a falling weight connected to the drum by means of a cord wrapped around the drum and a couple of pulleys. The bodies whose moments of inertia are to be determined are a ring, a disc, and a cylinder.

In-lab Tasks

In this experiment, the in-lab tasks are included with each part.

In-lab Questions

In this experiment, the in-lab questions are included with each part.

Method

Part 1: Moment of Inertia of the Cross and Drum

Since the cross and drum combination has a complex shape, it would be difficult to determine its moment of inertia theoretically. Following is the process to determine it experimentally.

1. With the vernier caliper measure the diameter $2r$ of the drum.

2. Take a cord about 4 metres long and after passing it over the top pulley and under the bottom one wind a little more than a metre of it on the drum. Adjust until the possible distance of the fall is about one metre.

3. Make a loop in the free end of the cord and in this loop insert small riders[1], 1/2 to 1 g each, one at a time until the cross rotates with constant speed when started. This mass will serve (it is hoped!) to cancel out the effect of friction on your results. Since these masses are small in comparison with the accelerated mass, their effect on values obtained from Equation 12.5 and Equation 12.6 is negligible. Of course the mass of the riders should not be included as part of m in these equations.

4. Now attach an additional mass m_1 to this cord. This mass is the body whose weight furnishes the driving force. Select and accurately measure the distance of fall h and make several determinations of the time of fall t_1 through this distance. Use these to fill in Table 12.8.

IQ1: To get the best value for I, is the ideal choice for m_1 a large or a small mass? Why? (ie. Consider how the choice of either a very large or a very small mass would affect the experiment.)

IQ2: What is the realistic uncertainty in h and what causes it? How reproduceable is h? (ie. How easy is it to ensure that two different objects are dropped the same distance?)

IQ3: How much uncertainty in m_1 is due to trying to determine constant velocity for the system?

IQ4: How does the mass of the individual riders affect the uncertainty in m_1?

IT1: Do an order of magnitude calculation for I.

Part 2: Moments of Inertia of Regularly Shaped Objects

Experimental Determination

Note that for the *experimental* determination of I, M and R don't matter!

[1]Paper clips work well for this.

1. Place one of the regularly shaped objects on the metal cross so that its centre of mass is on the axis of rotation.

2. Add appropriate riders until the system rotates at a constant speed. As before, the mass of these riders may be neglected in computing the moment of inertia.

3. Now add a mass m_2 to the string in place of m_1 and observe time of fall t_2 over the distance h. The mass m_2 should be much larger since the moment of inertia of the cross, drum, and object is much larger than the moment of inertia of the cross and drum alone.

4. Repeat this several times as in Part 1 and fill in Table 12.9.

5. Repeat the above measurements placing one other regularly shaped object on the cross and fill in Table 12.10.

IQ5: As in Part 1, is the ideal choice for m_2 a large or a small mass? Why? (ie. Consider how the choice of either a very large or a very small mass would affect the experiment.) How should it compare to m_1?

IQ6: Consider the questions in IQ3 and IQ4 above with m_1 replaced by m_2. In other words, in the uncertainty in m_2 similar to the uncertainty in m_1? Explain.

IT2: Do an order of magnitude calculation of I for each object.

Theoretical Determination

Note that for the *theoretical* determination of I, m_1, m_2, r, etc. don't matter!

1. Determine the linear dimensions and the masses of the regularly shaped objects whose moments of inertia are to be determined. (Measure diameter instead of radius to avoid having to determine where the centre is.)

IT3: Do an order of magnitude calculation for I for each object, and see that they are similar to values given previously.

Part 3: Objects Rolling Downhill

In this part, rather than determining the moment of inertia for objects, you will determine the shape constant, β.

> In this part measurement errors will be especially costly; be very careful with your measurements to avoid your values for β being totally meaningless. (For instance, β cannot be negative.)

1. Mark off the positions of the ramp legs[2] on the table top and measure this distance. (This is L.)
2. Elevate the legs at one end of the table by an amount h.
3. Mark starting and ending positions on the ramp for the objects. The distance between these marks is D.
4. For at least 2 objects of different shapes, time the objects rolling down the table, starting at one of the marks and ending at the other. (Do several trials for each to take into account timing errors). Put the data in Table 12.11.
5. Since the size of an object should not enter this problem, try timing two objects of the same shape but different sizes. (Since you already have data for some objects, pick another object of the same shape but a different size from one you've done already.) If there is a significant difference, comment on why you think it might be so.
6. Just as the size of an object should not enter this problem, as long as the shape is constant, similarly the mass of an object should not matter as long as the shape is constant.(Since you already have data for some objects, pick another object of the same shape but a different mass from one you've done already.)

IQ7: By calculating the average times with their uncertainties for the four objects, can you tell whether they will produce significantly different values for β before you actually calculate β? Explain.

[2]Note the important factor is to determine the angle of the ramp, so if the ramp doesn't have legs you want to determine the points of contact with the table underneath.

IQ8: What is the smallest value for h which can be measured reasonably precisely (say, with $\Delta h \lesssim 0.1h$)? What is the smallest value for t which can be measured reasonably precisely (say, with $\Delta t \lesssim 0.1t$)? What do these two suggest about what angles ought to be used to determine β? (Hint: How are height and time related?) Explain.

IQ9: How can you set up the experiment so that the size of the object used does not affect your measurement of the distance D or its uncertainty?

IT4: Do an order of magnitude calculation for β for each object, and see that they are in the correct range.

12.4.3 Analysis

Note that you have two different radii to work with; r, the radius of the drum, and R, the radius of the ring, disc, etc. *Do not* get these confused!

Part 1: Moment of Inertia of the Cross and Drum

Using the data in Table 12.8, calculate the average value of t_1 and its uncertainty. Use this to calculate I_1 and its uncertainty for the cross and drum combination.

Part 2: Moments of Inertia of Regularly Shaped Objects

Experimental Determination

1. Using the data in Table 12.9, compute the moment of inertia $I_1 + I_2$ of the cross, drum, and the regularly shaped object and its uncertainty.

2. Subtract the value of I_1 from the value for $I_1 + I_2$ in order to get the moment of inertia for the regularly shaped object and its uncertainty.

3. Repeat the previous two steps for the data for the other object in Table 12.10.

Theoretical Determination

Compute the theoretical values of the moments of inertia with their uncertainties for the regularly shaped objects used and compare these with the experimental values obtained. (Figure 12.1 gives the theoretical values for the moments of inertia of regularly shaped objects about various axes.)

Part 3: Objects Rolling Downhill

Calculate β and its uncertainty for each object and compare it with what you expect.

Post-lab Discussion Questions

Q1: In Parts 1 and 2, did the use of the small "riders" adequately account for the force of friction? Why or why not?

Q2: Can you come up with a shape which would be *faster* than any of the shapes you've tried? If so, sketch it and explain what makes shapes faster.

Q3: In Part 2, did the theoretical and experimental values for I agree?

Q4: In Part 3, were the values for β independent of size and mass as predicted?

Q5: In Part 3, did the values for β agree with the expected values?

12.5 Bonus

For Part 1: above, try to find the "optimal" value for m_1 experimentally. Does this agree with your predictions in the question asked in that section?

12.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	3	________	10
In-lab Questions	9	________	40
Post-lab Questions	5	________	(in report)
Pre-lab Tasks	4	________	20
In-lab Tasks	4	________	30
Post-lab Tasks	0	________	0
Bonus		________	5

12.7 Template

My name:
My student number:
My partner's name:
My partner's student number:
My other partner's name:
My other partner's student number:
My lab section:
My lab demonstrator:
Today's date:

quantity	symbol	measuring instrument	value	effective uncertainty	units
Part 1 and Part 2					
drum diameter	$2r$				
mass of riders					
mass	m_1				
fall height	h_1				
Part 2 (Theoretical Determination)					
object 1 diameter	$2R_1$				
object 1 mass	M_1				
object 2 diameter	$2R_2$				
object 2 mass	M_2				
Part 3					
length of the ramp	L				
elevation of the end of the ramp	h_3				
distance over which time is measured	D				
Not in equations					

Table 12.3: Quantities measured only once

quantity	symbol	measuring instrument	units
time			
Not in equations			

Table 12.4: Repeated measurement quantities and instruments used

quantity	symbol	value	uncertainty	units

Table 12.5: Given (ie. non-measured) quantities (ie. constants)

quantity	symbol	equation	uncertainty

Table 12.6: Calculated quantities

symbol	factor	bound	units	s/r/b

Table 12.7: Experimental factors responsible for effective uncertainties

Object	Cross and Drum
#	Time (seconds)
1	
2	
3	
4	
5	

Table 12.8: Part 1

Object	
#	Time (seconds)
1	
2	
3	
4	
5	

Table 12.9: Part 2

Object	
#	Time (seconds)
1	
2	
3	
4	
5	

Table 12.10: Part 2

Object	
#	Time (seconds)
1	
2	
3	
4	
5	
Object	
#	Time (seconds)
1	
2	
3	
4	
5	
Object	
#	Time (seconds)
1	
2	
3	
4	
5	
Object	
#	Time (seconds)
1	
2	
3	
4	
5	

Table 12.11: Part 3

Appendix A

Information about Measuring Instruments

Fill this in as you use new measuring instruments so you will have a reliable reference. Put frequently used instruments in Table A.1 and experiment-specific ones in Table A.2.

ref. #	measuring instrument	precision measure	range	units
A1	vernier caliper			
A2	micrometer caliper			
A3	stopwatch			
A4	GL100R balance			
A5				
A6				

Table A.1: Measuring instrument information

ref. #	measuring instrument	precision measure	range	units
B1	spring scale A			
B2	spring scale B			
B3				
B4				
B5				
B6				
B7				
B8				
B9				
B10				
B11				
B12				
B13				
B14				
B15				

Table A.2: Measuring instrument information (continued)

This marking checklist will be used for lab reports this term. You need to print one off and attach it to each lab report you hand in. Lab reports will be marked as follows:

- Start with 90

  For items *not* in italics

- Subtract 1 for each ˜.

- Subtract 2 for each − .

  For items *in italics*

- Subtract 3 for each ˜.

- Subtract 6 for each − .

Note the importance of items in italics. These are very important in a report, and so are weighted accordingly.
The other 10 marks will be based on how well the post-lab discussion questions were answered within the text of the report. *Remember that the answers to these questions should be an integral part of the report, not merely an afterthought.*

Lab Format Checklist (V2.231ng)

A. General

1.	Your own work	________
2.	Complete	________
3.	Clear and appropriate "Purpose"	________
4.	Flows	________
5.	Did not require help on or after due date	________
6.	Correct grammar	________
7.	Correct spelling	________
8.	Complete sentences where required	________
9.	Legible	________
10.	Professionally presented	________
11.	Properly identified (eg. name, partner)	________
12.	On time	________
13.	Checklist included	________
14.	Template included	________

B. Data (for data not in tables)

1.	Your own data	________
2.	Values recorded with uncertainties	________
3.	Sufficient data	________
4.	Reasonable values	________
5.	Reasonable uncertainties	________
6.	Correct number of significant figures	________
7.	Units recorded	________

C. Data in Tables

1.	Neat	________
2.	Column headings informative	________
3.	Units given	________
4.	Uncertainties given	________
5.	Label	________
6.	Number given (eg. "Table #2")	________

D. Calculations and Results

1.	Any required derivations done correctly	________
2.	Analysis explained where needed	________
3.	Correct formulas used	________
4.	Sample calculations shown where needed	________
5.	All required values calculated	________
6.	Uncertainties included	________
7.	Units included	________
8.	Correct number of significant figures	________
9.	Appropriate use of standard form	________
10.	Theoretical or reasonable value	________
11.	Agreement of experiment with theory	________

E. *Error Discussion*

1.	*Sources listed are significant*	________
2.	*Sources are prioritized*	________
3.	*Systematic error consequences*	________
4.	*Evidence: ie test or bound*	________
5.	*Reasonable suggestions for improvement*	________

F. *Conclusions*

1.	*Relate to purpose*	________
2.	*Major results stated*	________
3.	*Comparisons made where appropriate*	________
4.	*Agreement noted when found*	________
5.	*% difference only when no agreement*	________

G. References

1.	Source(s) of constants listed	________

Appendix B

Online Calculators for Statistics and Uncertainties

B.1 Calculators

There are two calculators online to help you get used to uncertainty and statistical calculations. They can be used offline as well if you save the web pages.

B.1.1 Uncertainty Calculator

The calculator, shown in Figure B.1, is divided up into 5 regions:

A. Input

B. Output

C. Functions

D. Operators

E. Mode

Input

Each calculation can have one or two input quantities, designated x and y. The quantities can have uncertainties with them, designated **dX** (for Δx) and **dY** (for Δy). (An uncertainty is assumed to be zero if it is not given.)

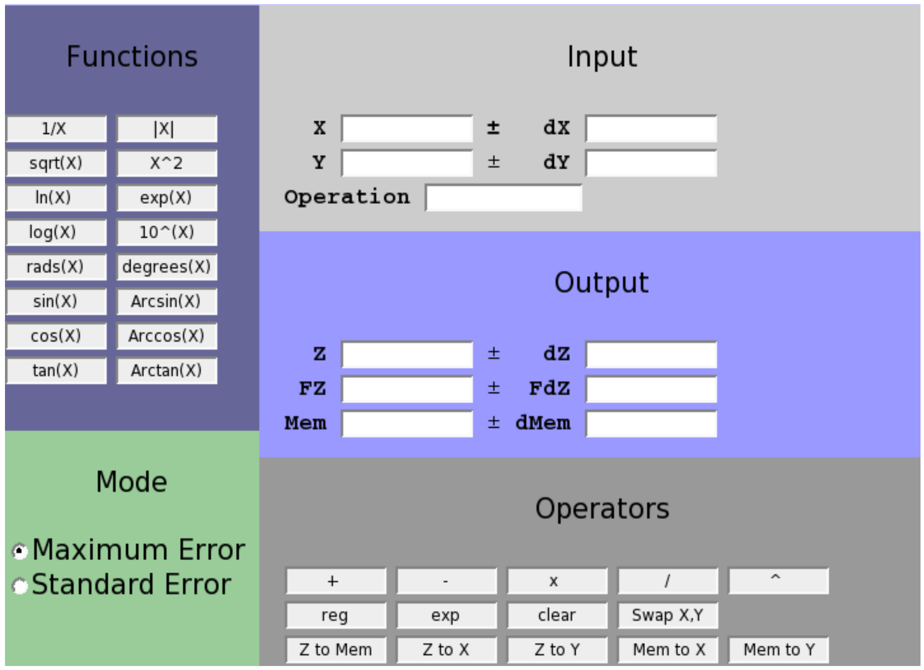


Figure B.1: Uncertainty Calculator

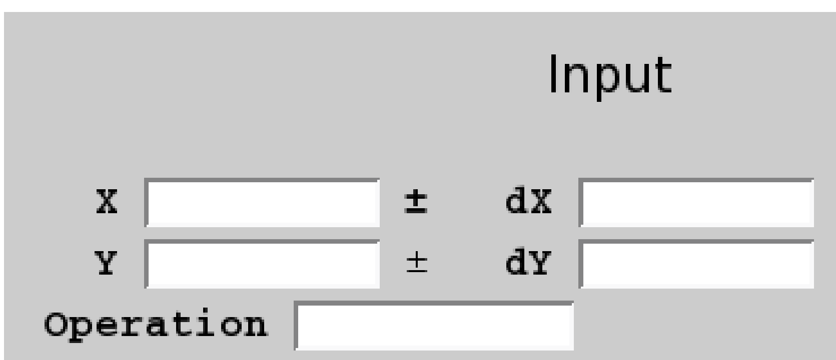


Figure B.2: Input Section

The *"Operation"* field will be filled in by the calculator if you choose an operation to perform on the inputs.

Output

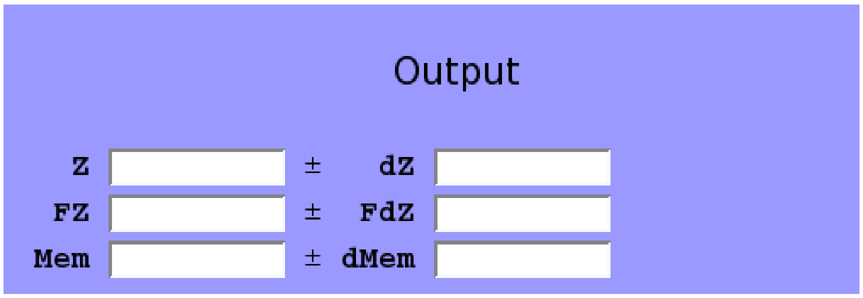


Figure B.3: Output Section

The result of any calculation is shown as z with an uncertainty **dZ** (for Δz). The cells **FZ** (for *formatted* z) and **FdZ** (for *formatted* Δz) show formatted versions of the output and its uncertainty. Most of the time the formatting is correct, although there are a few situations where the formatting is not right. These should be self-apparent, as the uncertainties have several significant figures instead of one.
The contents of the calculator "memory" location is displayed as well.

Functions

Functions are performed on the values in the x register by pressing the corresponding buttons in much the same manner as on a pocket calculator. The result of the function, like any result, is placed in z. If you want to use the result in a further calculation, then you can use the operator buttons to transfer the value in z to x, y, or to be saved in memory. *Note that the trigonometric functions assume angles are in radians, and the calculator has functions to convert between degrees and radians.*

Operators

Notice that there is no "equals" button on the calculator; after typing the inputs to an operation, pressing the desired operator button performs the operation and displays the output. There are three types of operators:

- mathematical operators; $x + y$, $x - y$, $x \times y$, x/y, x^y

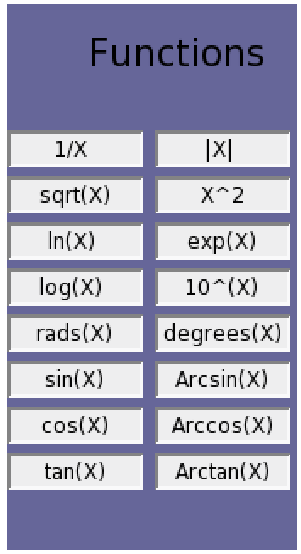


Figure B.4: Function Section

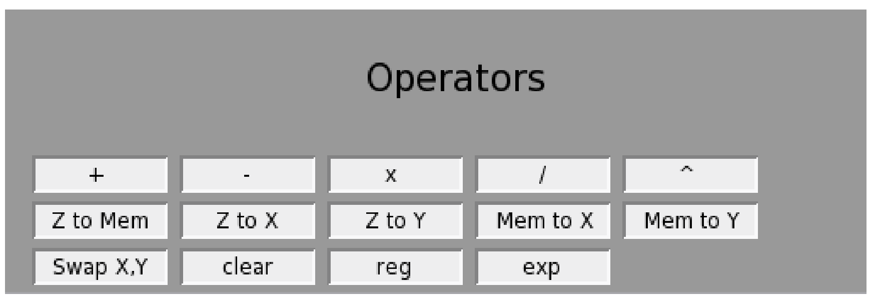


Figure B.5: Operator Section

- movement operators; to move data between the x, y, z registers and the memory location. This includes one to swap the values in the x and y registers and one to clear all of the registers.

- display operators; to change whether the output is displayed in scientific notation or not

Mode

The calculator can perform uncertainty calculations in two ways;

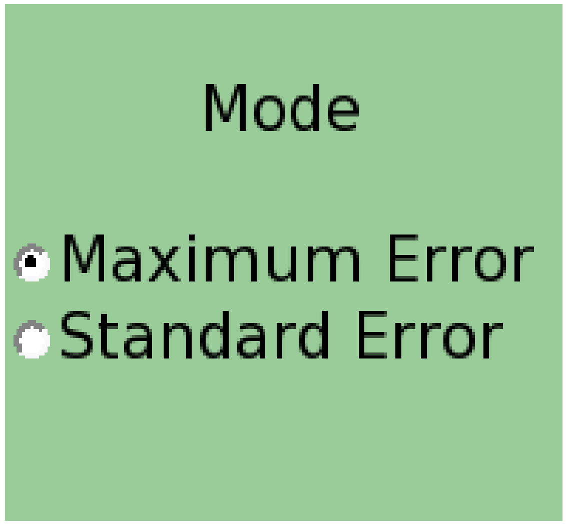


Figure B.6: Mode Section

- determining *maximum uncertainty*, or maximum error, which applies if errors are dependent

- deeming *most probable uncertainty*, or standard error, which applies if errors are independent

The difference between these methods will be small in most cases. In PC131 we will only use maximum uncertainty. Changing this mode will cause subsequent calculations to use the chosen method. Note that the uncertainty for functions will not be affected by the mode switch, since the different modes only apply to combining uncertainties; functions of one variable are unchanged.

B.1.2 What is the Uncertainty Calculator good for?

If you have to repeat a certain calculation several times, it will be convenient to use a spreadsheet. However, if you only have to perform a calculation once, the calculator may save some time. It also makes it easy for you to check that a calculation you have performed is correct, especially if you're not sure of how to do a particular calculation with uncertainties.

T1: Using the calculator, set

- $x = 1$
- $\Delta x = 0.1$

and find Δz if

- $z = \frac{1}{x}$
- $z = \sqrt{x}$
- $z = x^2$

Then use formulas to find the same quantities.

T2: Still using the calculator, with $x = 1$ and $\Delta x = 0.1$, now set

- $y = 2$
- $\Delta y = 0.2$

and find Δz if

- $z = x + y$
- $z = x - y$
- $z = x \times y$
- $z = \frac{x}{y}$

Then use formulas to find the same quantities.

Answers

Check these after you've tried the questions above yourself.

If $x = 1$ and $\Delta x = 0.1$

- $z = \frac{1}{x}$; $\Delta z = 0.1$
- $z = \sqrt{x}$; $\Delta z = 0.05$
- $z = x^2$; $\Delta z = 0.2$

If $y = 2$ and $\Delta y = 0.2$

- $z = x + y$; $\Delta z = 0.3$
- $z = x - y$; $\Delta z = 0.3$
- $z = x \times y$; $\Delta z = 0.4$
- $z = \frac{x}{y}$; $\Delta z = 0.1$

B.1.3 Statistical Calculator

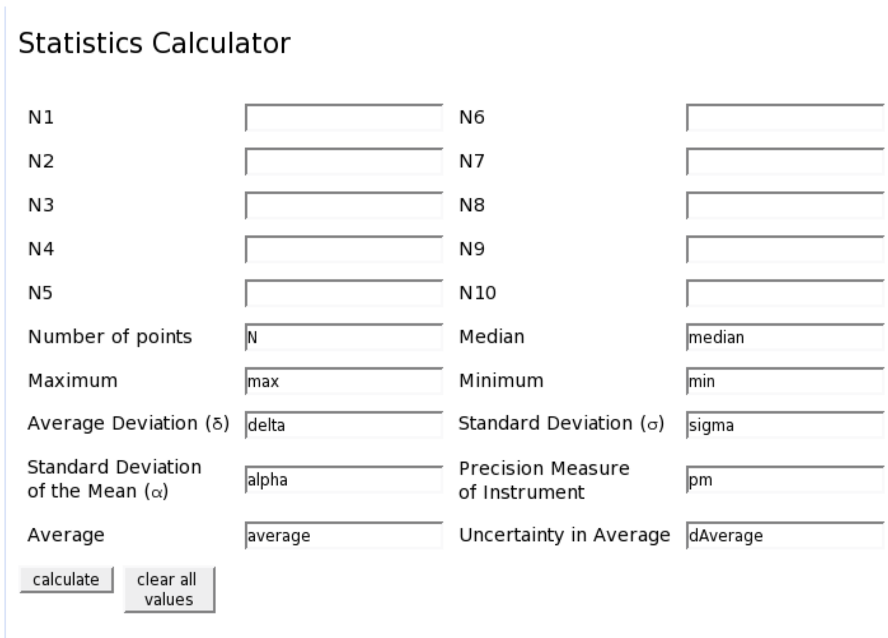


Figure B.7: Statistics Calculator

The calculator, shown in Figure B.7, has several cells:

A. inputs for up to ten numbers, labelled N1 to N10

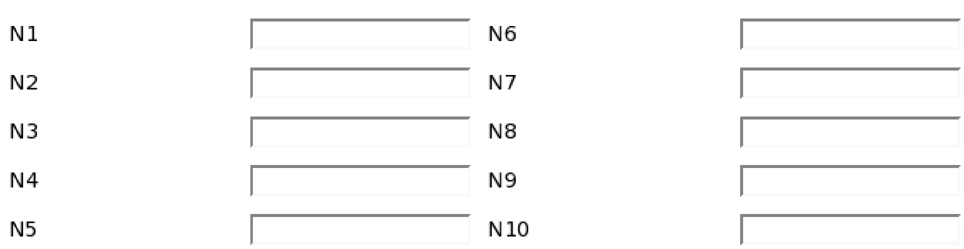


Figure B.8: Inputs

B. an optional input for the precision measure of the instrument used for the measurements

Figure B.9: Precision Measure Input

C. outputs for several statistics about the group of numbers, namely

Figure B.10: Statistical Output

- the number of values (useful to confirm that there were no cells filled with blanks or other non-printing characters)
- the maximum
- the minimum
- the median
- the average deviation of the values
- the (sample) standard deviation
- the standard deviation of the mean

Figure B.11: Average and its Uncertainty

- the average (mean)

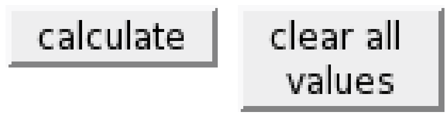


Figure B.12: Buttons

  - the uncertainty in the average; if the precision measure was filled in, this will be the bigger of the precision measure and the standard deviation of the mean

D. a button to perform the calculations

E. a button to clear all values (useful if you want to analyze a small set of values after a larger set, so you don't need to clear each cell individually)

B.1.4 What is the Statistical Calculator good for?

Much like the uncertainty calculator, if you have to repeat a certain calculation several times, it will be convenient to use a spreadsheet, (especially since many statistical functions are built into spreadsheets). However, if you only have to perform a calculation once, the calculator may save some time. It also makes it easy for you to check that a calculation you have performed is correct, especially if you're not sure of how to do a particular calculation .

T3: Using the calculator, type in one data set from your *Measuring 'g'* data. Enter the precision measure of the stopwatch into the appropriate box in the calculator. Press the **calculate** button to determine

- the average (mean), $\bar{x}$
- the uncertainty in the average, $\Delta\bar{x}$
- the average deviation, $\bar{\delta}$
- the sample standard deviation, σ
- the standard deviation of the mean, α

Then use formulas to find the same quantities.

Hint

Just to see that you're doing things correctly, type in the numbers 1, 2, and 3. When you calculate the statistics, you should get:

- the average (mean), $\bar{x} = 2$
- the average deviation, $\bar{\delta} = 0.\bar{6}$
- the sample standard deviation, $\sigma = 1$
- the standard deviation of the mean, $\alpha \approx 0.577$

- If the precision measure is *greater* than 0.577, then the uncertainty in the average, $\Delta\bar{x}$ should be the same as α.
- If the precision measure is *less* than 0.577, then the uncertainty in the average, $\Delta\bar{x}$ should be the same as the precision measure.

Appendix C

Sample Uncertainty Calculations

Here are some sample uncertainty calculations to try with some tips. If you can do these, you should be able to handle just about anything.

> Remember: Either method of calculation,(inspection or algebra), can be used. Which one will be easier to use will depend on the equation you're using. By getting familiar with both methods, you can choose whichever one you prefer.

C.1 Inspection

Here are some examples. In each case, fill in the signs to determine the uncertainty in the results. Remember that you are trying to make the first term as large as possible.

A. (From *"Moment of Inertia"*)

   $I = Kmr^2$

   Assume m and r are positive, and K is a constant.

   $\Delta I = K\,(m \bigcirc \Delta m)\,(r \bigcirc \Delta r)^2 - Kmr^2$

B. (Thin lens equation)

$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$

Assume i and o are positive.

$\Delta\left(\frac{1}{f}\right) = \left(\frac{1}{i \bigcirc \Delta i} + \frac{1}{o \bigcirc \Delta o}\right) - \left(\frac{1}{i} + \frac{1}{o}\right)$

C. (From *"Translational Equilibrium"*)

$\mu = \tan\theta$

Assume θ is in the first quadrant.

$\Delta\mu = \tan\left(\theta \bigcirc \Delta\theta\right) - \tan\theta$

Does it matter whether θ is in degrees or radians?

D. (From *"Moment of Inertia"*)

$\beta = \frac{ght^2}{2L^2} - 1$

Assume h, t, and L are all positive, and g is a constant.

$\Delta\beta = \frac{g(h \bigcirc \Delta h)(t \bigcirc \Delta t)^2}{2(L \bigcirc \Delta L)^2} - \frac{ght^2}{2L^2}$

Where did the -1 go?

C.1.1 Answers

Check these after you've tried the questions above yourself.

A. $+$, $+$ if both are positive.

B. $-$, $-$ if both are positive.

C. $+$ if in the first quadrant, since tan is increasing there. It doesn't matter whether you're in degrees or radians, as long as it's the same for θ and $\Delta\theta$. **Note that if you're using the algebra rules, this doesn't apply; all angles have to be in radians.**

D. $+$, $+$, $-$ if all are positive. Since both terms have the "-1" in them, they will cancel out when subtracted.

C.2 Algebra

Here are some examples. In each case, use the algebra rules to determine the uncertainty in the results. Put in absolute value signs if necessary.

A. (Frequency and period)

   $f = \frac{1}{T}$

   Assume T is positive.

   $\Delta f =$

   Hint: Use either the rule for division or the rule for powers.

B. (Kinematics equation)

   $v = \frac{x}{t}$

   Assume x and t are positive.

   $\Delta v =$

   Hint: Use the rule for division.

C. (Area of a circle)

   $a = \pi r^2$

   Assume r is positive.

   $\Delta a =$

   Hint: Use the rule for powers and the rule for multiplication by a constant.

D. (From *"Moment of Inertia"*)

   $I = Kmr^2$

   Assume m and r are positive, and K is a constant with no uncertainty.

   $\Delta I =$

Hint: Use the rule for multiplication. The rule for powers will also help.

How is the result different if K *has* an uncertainty? (The change should be easy to make if you understand the process.)

E. (Simple pendulum)

$T = 2\pi\sqrt{\frac{\ell}{g}}$

Assume ℓ is positive.

$\Delta T =$

Hint: This will combine some rules.

C.2.1 Hints

Check these after you've tried the questions above yourself.

A. Using the division rule, $A = 1$ and $B = T$. Remember that the uncertainty in "1" is zero.

   Alternative: Using the power rule,

$$\frac{1}{T} = T^{-1}$$

   This result for the uncertainty in the inverse of something is one which is simple and worth memorizing since it comes up so often.

B. Using the division rule, $A = x$ and $B = t$.

C. Since π is a constant, having no uncertainty, then

$$\Delta\pi r^2 = \pi\Delta r^2$$

   (You can use the power rule to figure out Δr^2.)

   This result for the uncertainty in the square of something is one which is simple and worth memorizing since it comes up so often.

D. Break this into three steps (in your head, at least, whether or not you do it on paper).

   Since K is a constant, pull it out like in the previous example.

   Using the multiplication rule, $A = m$ and $B = r^2$. (See the previous example to figure out Δr^2.)

   If K has an uncertainty, then note that using the multiplication rule, you can extend the pattern for A and B to include C, D, etc. for as many terms as you need. So, in this case $A = K$, $B = m$, and $C = r^2$. Then proceed as before.

E. Break this into steps (in your head, at least, whether or not you do it on paper).

You have a constant out front (as in previous examples).

You have a function (a square root) of some things.

Inside the square root, you have a division.

Alternative: You might find it easier to see the square root instead as a *quotient* of two terms. In other words,

$$\sqrt{\frac{\ell}{g}} = \frac{\sqrt{\ell}}{\sqrt{g}}$$

In this case you'll use the division rule with $A = \sqrt{\ell}$ and $B = \sqrt{g}$. Remember that you'll have to use other rules to determine ΔA and ΔB.

Another option is to see the square root instead as a *product* of two terms. In other words,

$$\sqrt{\frac{\ell}{g}} = \ell^{\frac{1}{2}} \quad g^{\frac{-1}{2}}$$

In this case you'll use the multiplication rule with $A = \ell^{\frac{1}{2}}$ and $B = g^{\frac{-1}{2}}$. Remember that you'll have to use other rules to determine ΔA and ΔB.