

PC132 Lab Manual  
Wilfrid Laurier University

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# Chapter 5

## Standing Waves on a String

### 5.1 Purpose

In this experiment we will attempt to use the properties of standing waves on a string to measure the mass density of a string in order to determine which of two sample strings it matches.

### 5.2 Introduction

This experiment will introduce *graphical analysis*, including linearization, graphing and least squares fitting.

This experiment illustrates the situation where you can't directly measure a quantity, but instead must use other physics principles to determine it indirectly. The *process* you go through will take several weeks, as in PC131. The physics involved will be covered in the first couple of labs, so most of your focus will be on how to analyze the results and prepare the report.

This lab will actually be broken into parts, so you will spend several weeks to produce the report. After you know how to do this, you will be able to produce reports much more quickly.

The schedule will be somewhat like this:

- Collect data, including uncertainties in the raw data.
- *Figure out how to produce a linear graph from the data.*

- *Make a linear graph, and from the graph determine the slope and  $y$ -intercept and their uncertainties.*
- Using the graphical quantities, i.e the slope and  $y$ -intercept and their uncertainties, calculate numerical results, including uncertainties.
- Interpret the results and draw quantitative and qualitative conclusions.
- Learn how to write up “Discussion of Uncertainties” and “Conclusions” in your report.

(Notice that two of the steps are new this term; the rest are the same as in PC131.)

After that, you will hand in the lab.

### 5.3 Theory

Consider first a very long string, one end of which is firmly fixed in position (very far away), the other end of which is in your hand. If some tension is kept in the string, and you give the string a flick with your wrist, a pulse will originate from your hand and travel along the string away from your hand. The motion of your hand is perpendicular to the direction in which the pulse travels along the string or **transverse** to it. This transverse motion of the string results in each point on the string moving transverse to the motion of the wave as the wave passes.

Next consider the same long string. If the hand is moved quickly up and down in a regular manner, keeping the string taut, a wave will be generated which will travel along the string away from your hand. This is a **transverse wave** because each particle of the string moves up and down while the wave itself moves horizontally along the string. The wave itself is *traveling* along the string, and thus we have created a **traveling transverse wave** in the string.

This wave will appear, to a person viewing the string from the side, as alternating crests and troughs which move steadily away from the hand along the string. If this viewer watches one particular particle of the string, he will see it move up and down regularly, transverse to the motion of the wave.

Considering this traveling wave, if the viewer took a “fast” snapshot of it, it would look somewhat like Figure 5.1. (The dotted line represents where

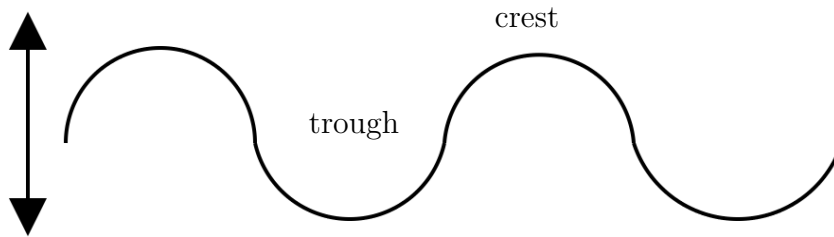


Figure 5.1: Creating a Wave “By Hand”

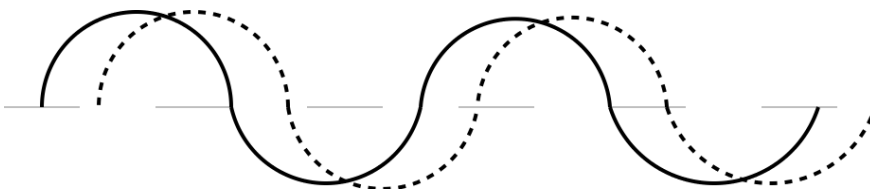


Figure 5.2: Traveling Wave at Different Times

the string would lie if the hand were at rest and the hand and string were both “frozen” for the duration of the snapshot.)

Neglecting the hand, if we were to take another snapshot shortly after the first and superimpose the two snaps, (drawing in the later one as a dashed line), we might see something like Figure 5.2, (i.e. the crests and troughs are moving steadily to the right along the string with a speed  $v$ . If we take a rapid succession of such snaps, we will soon find one in which the string lies exactly over the string in the first picture taken; each crest (or trough) has moved precisely to the position where there was a crest (or a trough) in the first snap.

We say that the distance this wave has traveled is one **wavelength**,  $\lambda$ . It is exactly equal to the distance between any consecutive pair of crests (or troughs) in one of the pictures. The wave thus moves one wavelength  $\lambda$  at speed  $v$ . The time for this to occur is called a period  $P$  of the wave. It is also the time taken for the hand at the end of the string to complete one of its regular (periodic) cycles of oscillation.

The wave’s velocity,  $v$ , is thus equal to the distance  $\lambda$  it traveled in one

period  $P$ , i.e.

$$v = \frac{\lambda}{P} \quad (5.1)$$

In describing the hand's motion, which repeats itself every period  $P$ , it is perhaps more common to think of the number of times per second that the hand makes this cyclical motion. This quantity we call the **frequency**,  $f$ , of oscillation of the hand, and thus of the wave itself. If the length of time per cycle is the period  $P$  (in seconds), then the frequency  $f$  (in cycles/second, or **Hertz**) is its reciprocal.

$$f = \frac{1}{\frac{\text{time}}{\text{cycle}}} = \frac{\text{cycles}}{\text{time}} = \frac{1}{P} \quad (5.2)$$

(Note: 25Hz = 25 cycles per second, or cps.)

We now have two equations relating the wave's period, frequency, wavelength, and velocity; namely

$$v = \frac{\lambda}{P} \quad (5.3)$$

and

$$v = f\lambda \quad (5.4)$$

It can also be shown theoretically that the velocity of the wave traveling along the string is given by

$$v = \sqrt{\frac{T}{\mu}} \quad (5.5)$$

where  $T$  is the **tension** in the string and  $\mu$  is the **mass per unit length** of the string, i.e. its **linear mass density**.

Now, consider the same string, but with much shorter length, so that the fixed end is now close to the free end. Also, consider that the free end is now being "shaken" by an electromechanical device instead of the hand of the now-exhausted student. The electromechanical device can shake the string periodically at a higher frequency than the hand for as long as necessary without fatigue, unlike the student.

The waves generated by the shaker travel along the string as previously, but they now strike the fixed end and are **reflected** there. The actual shape of the string at any instant is a combination of the incident and reflected

waves, and may be quite complex. However, under the proper set of conditions for  $T$ ,  $v$ ,  $\lambda$ , and  $f$ , the incident and reflected waves may combine producing regular shapes on the string. Such waves are called *stationary* or **standing waves**.

These standing waves occur if and only if the distance between the fixed and shaken ends of the string, (which is simply its length,  $L$ ), is some integral number of *half-wavelengths* of the traveling wave. The waves then “fit” nicely into the length of the string. The incident and reflected waves reinforce each other over regions of large amplitude of vibration of the string called **loops**, and cancel each other at points on the string called **nodes** at which the string exhibits no motion at all. Examples of this are shown in Figure 5.3.

Our governing equation is a combination of Equations 5.4 and 5.5

$$f = \frac{1}{\lambda} \sqrt{\frac{T}{\mu}} \quad (5.6)$$

from which the mass density of the string may be determined. In this experiment, the oscillator frequency is fixed and known, and the string tension  $T$  is to be varied with resulting wavelengths  $\lambda$  measured for several different standing waves. The linear mass density  $\mu$  will be determined, and also measured.

Equation 5.6 can be linearized so that a graph can be produced from which  $\mu$  can be determined if  $f$  is known.

## 5.4 Procedure

### 5.4.1 Preparation

Since you may be doing this on your first week of labs, there are no pre-lab requirements.

### 5.4.2 Investigation

The apparatus is set up with a string of length  $L$  between fixed and oscillating ends.

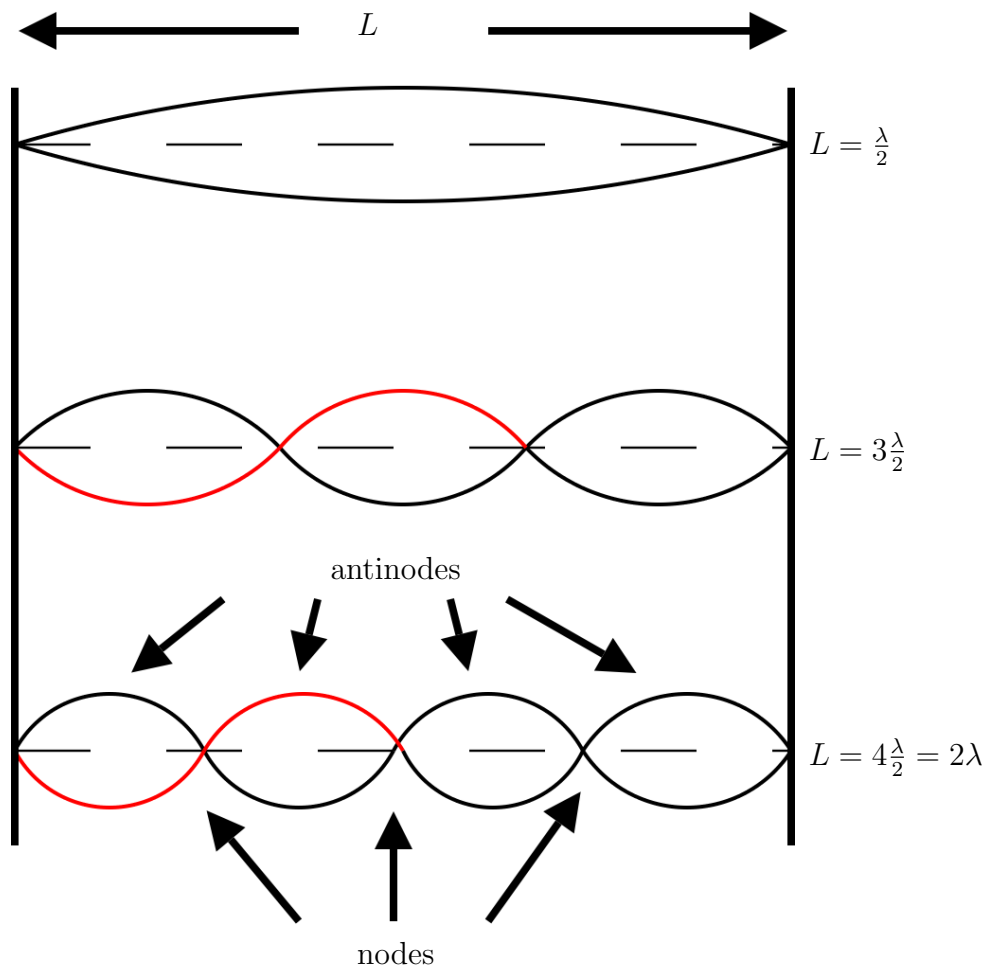


Figure 5.3: Standing Wave Patterns

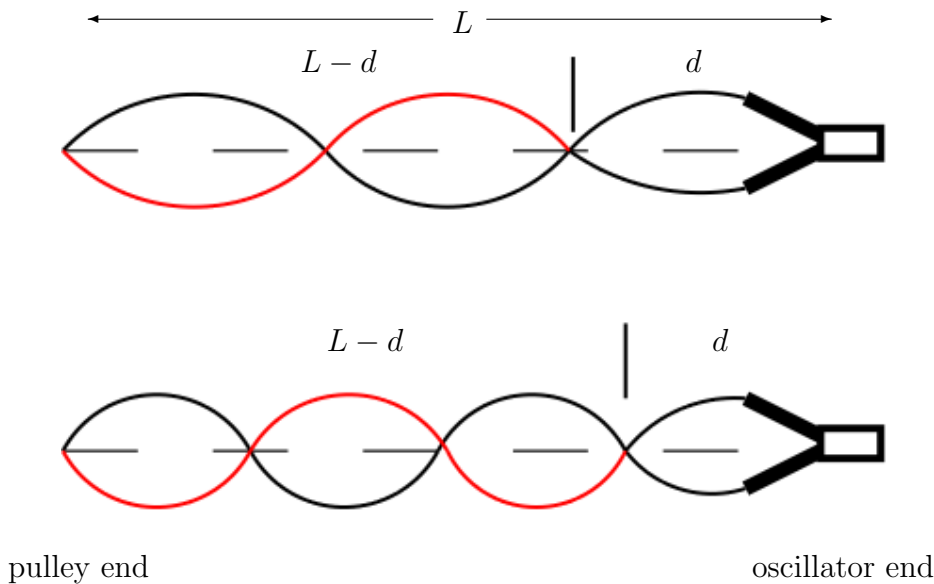


Figure 5.4: Measuring Wavelength

### Method

1. Measure  $L$  and enter the result in Table 5.2. (Be sure to work in MKS units.) Determine the realistic uncertainty in  $L$  and fill in the details in Table 5.4.
2. Plug in the oscillator. The “fixed” end is where the string lies over the pulley. Tension in the string is produced by a weight consisting of a cup filled with a variable amount of copper shot. Adjusting the amount of shot in the cup adjusts the tension in the string.
3. Pour in shot a little at a time until a standing wave pattern with several, (preferably 7 or 8), loops is achieved. (Sometimes it helps to tug on the string by hand to determine whether weight needs to be added or subtracted, as well as to get an idea of how much weight to add or subtract.)
4. Once this is achieved, measure the distance,  $d$ , from the oscillator to the first node. *Note that you should not use the loop closest to the*

*oscillator in your calculations as the “node” at one end is not a part of the string at all.* Enter the data in Table 5.5.

5. Weigh the load pan with the shot in it, and enter the data in Table 5.5.
6. Adjust the mass until you see a noticeable change in the standing wave, (for instance, when it becomes unstable), and weigh the pan again. You can adjust the mass either up or down, whichever is easier. Record this “adjusted mass” in Table 5.5.

Use the difference between the mass and the adjusted mass to determine the uncertainty in the mass,  $\Delta m$ .

7. Add shot to the cup until you get a standing wave pattern with one fewer loop than before and repeat the steps 4 to 6 above for each pattern up to and including one with just two loops.
8. If you can add enough mass to get a standing wave with a single loop, then do so and record the mass. (In this case, your calculation of the wavelength will be less precise due to the inclusion of the oscillator end, but there is no way to avoid it for this case.)
9. Calculate  $\lambda$  using the equations from **IQ7:**, and fill in Table 5.6. If the values are not changing monotonically, you need to make note of the following:

You may find your data actually seems to be giving you two different frequencies of oscillation, where the higher frequency is twice the lower. This is due to the physical operation of the oscillator, and can be explained fairly simply, but that will not be done here. If this is the case for your data, calculate the “effective wavelength” for the odd points by multiplying or dividing the calculated wavelength by 2, (whichever is appropriate), and identify these points in your data and on your graph.<sup>1</sup>

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<sup>1</sup>Normally it’s not a good thing to “adjust” data like this. In this case, however, since the physics of the oscillator makes it oscillate at two frequencies which differ by an *exact* factor of 2, then it makes sense. Also, since the points being adjusted are clearly identified, it is not an attempt to deceive the reader.

10. The linear mass density of the string,  $\mu$ , is measured by selecting a piece of the same kind of string, of length  $\ell$ , and weighing it on a balance. Measure  $\mu$  and  $\ell$  for both sample strings and enter the results in Table 5.2.
11. Determine the realistic uncertainty in  $\ell$  and fill in the details in Table 5.4.
12. It should be obvious that

$$\mu = \frac{\text{mass of string}}{\text{length of string}} = \frac{m_s}{\ell} \quad (5.7)$$

where  $m_s$  is the mass of the sample string from the balance. (Note that  $\mu$  is not *volume* mass density, but *linear* mass density.) Determine the mass densities of the two sample strings with their uncertainties.

### In-lab Questions

**IQ1:** Is there any reason that measuring between nodes should be better than measuring between antinodes? Explain.

**IQ2:** What is the uncertainty in the number of nodes,  $N$ ? Explain.

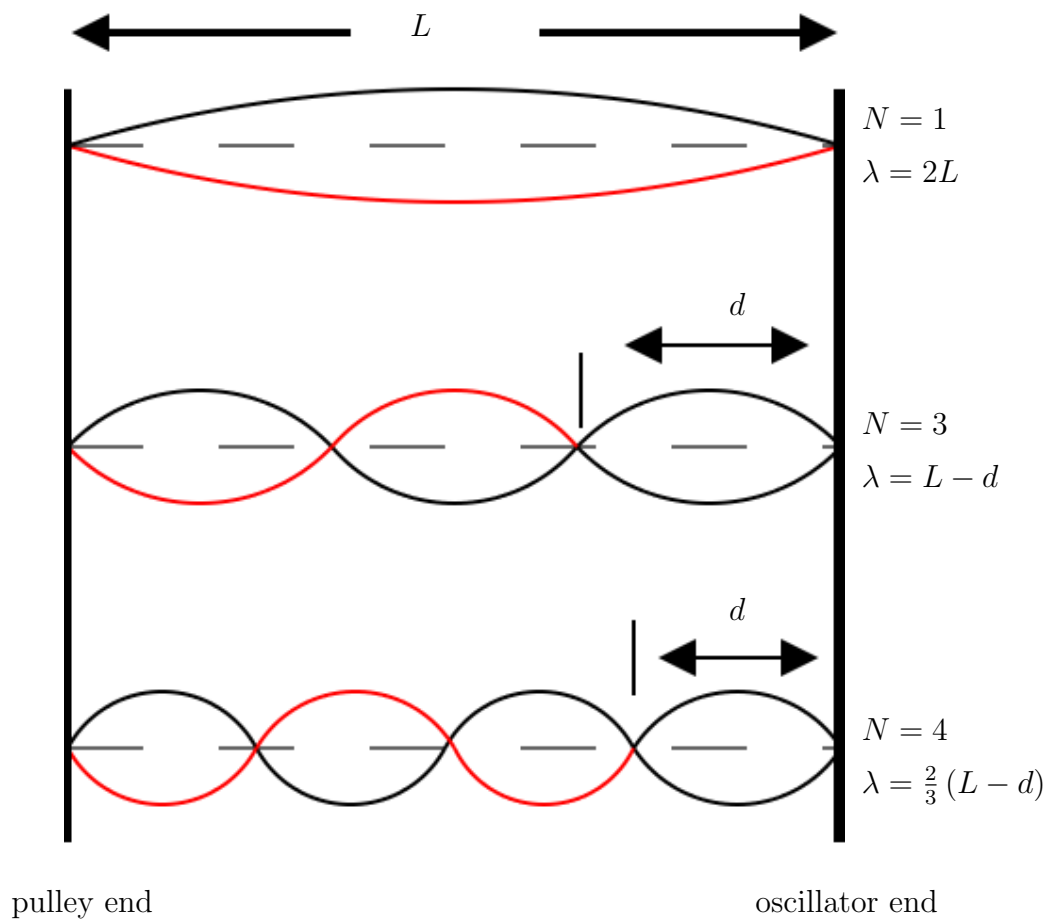
**IQ3:** What is the realistic uncertainty in the length used for each measurement? Is it more due to getting the right mass to make the pattern stable, or is it more due to measuring the position of the node? Does this vary during the experiment? Explain.

**IQ4:** What is the realistic uncertainty in the mass used for each measurement? Does the shot size affect this? Explain.

**IQ5:** What is the realistic uncertainty in the length used for the mass density calculation, and what causes it?

**IQ6:** Why should using a long piece of string be better for calculating the mass density than a short one?

**IQ7:** If the total length of the string is  $L$ , and the last node (near the oscillator) has a length  $d$ , as in Figure 5.5 then determine the equation for the wavelength  $\lambda$  if there are  $N$  nodes visible.

Figure 5.5: Wavelength Relation to  $d$

For instance,

- for  $N = 1$ ,  $\lambda = 2L$ .
- for  $N = 2$ ,  $\lambda = 2(L - d)$ .
- for  $N = 3$ ,  $\lambda = (L - d)$ .
- for  $N = 4$ ,  $\lambda = \frac{2}{3}(L - d)$ .
- for  $N = 5$ ,  $\lambda = \frac{2}{4}(L - d)$ .
- ...

(It might help to note the pattern in the last two, and then re-write the previous two in the same form to determine the general equation. *Note the first one does not fit the equation, since it uses the node ending at the oscillator and the others don't.*)

### In-lab Tasks

**IT1:** Fill in the first and third columns of Table 5.6 to see that both are increasing monotonically.

**IT2:** Rearrange Equation 5.6 to solve for  $\mu$ . Do an order of magnitude calculation<sup>2</sup> of the linear mass density of the string using one data point to show that it is in the right range.

**IT3:** Read over each of the inlab and postlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 5.7.

### 5.4.3 Analysis

This lab can't be completed until after the "*Exercise on Graphing and Least Squares Fitting using a Spreadsheet*" lab exercise.

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<sup>2</sup>If you use a single data point to quickly calculate something, rather than using all of the points for an average, graph, etc., it serves the same purpose as an order of magnitude calculation; it lets you see quickly if the results seem to make sense.

1. For each of the three linearizations of your data, plot the graph, perform the least squares fit, and calculate the linear mass density,  $\mu$ , and its uncertainty from the results.
2. Using the equations for linear mass density,  $\mu$ , and its uncertainty from the linearizing exercise, determine linear mass density and its uncertainty from each linearization.
3. Consider the similarities and differences in results of the three linearizations, and decide which one you will use for your lab report. *If there are big differences between the values of  $R^2$  for the different linearizations, you should see a difference in how linear each graph appears to be. Different types of errors in the original data will affect the different linearizations differently, and so if there's a big difference you will have a hint at what errors may exist in your data.*
4. Depending on the linearization chosen, use the slope or  $y$ -intercept as appropriate to determine a value for  $\mu$ , the linear mass density of the string.
5. Compare the value of  $\mu$  obtained to the values obtained from the string samples.
6. Determine whether the string matches either of the samples, and if so, which one.

### Post-lab Discussion Questions

Answers to the following questions will form the basis of the *Discussion* and *Conclusions* sections of your lab report. Write these sections in paragraph form, with each individual answer underlined or highlighted. At the beginning of each question put the question number in super-script. The goal is to have a flow to the whole section, rather than to have the section appear as a series of statements of unrelated facts. *Be sure to include your numerical results to explain your answers.*

### Comparing linearizations

**Q1:** Did the linear mass densities given by the different linearizations agree within experimental uncertainty? Discuss the similarities and differences in results of the three linearizations, and explain which one you prefer and why. Did your original preference from “*Exercise on Linearizing Equations*” still make sense after you had plotted the data and done the calculations? Why or why not?

### For your preferred linearization

**Q2:** For your linearization, one of the graphical parameters (either the slope or the  $y$ -intercept), was used to determine the linear mass density. The other should have equaled a constant. Did the other parameter give you the expected value for the constant? Explain.

**Q3:** From the shape of your linearized graph, is there any evidence of a systematic error in your data? Explain.

### Experimental insight

**Q4:** Based on whether the scatter in your graph was large or small, would more precise measuring instruments have improved your results? Explain.

**Q5:** Based on your answer to **Q4**, would a more precise metre stick have improved your results? Explain. (Your inlab questions should help you answer this.)

**Q6:** Based on your answer to **Q4**, would a more precise balance have improved your results? Explain. (Your inlab questions should help you answer this.)

**Q7:** Does the magnitude of your calculated mass density agree with either of the sample strings in the lab? Can you identify which type of string you had?

## 5.5 Bonus

What factor determines the maximum number of nodes which can be observed? Are data more reliable for oscillations with many nodes or with few

nodes? Explain your answers.

## 5.6 Recap

By the time you have finished this lab report, you should know how to :

- collect data and analyze it, which may include
  - graphing with error bars
  - least squares fitting
- write a lab report which includes:
  - *title* which describes the experiment
  - *purpose* which explains the objective(s) of the experiment
  - *results* obtained, including data analysis
  - *discussion of uncertainties* explaining significant sources of uncertainty and suggesting possible improvements
  - *conclusions* about the experiment, which should address the original objective(s).

## 5.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	7	_____	70
Post-lab Questions	7	_____	<i>in report</i>
Pre-lab Tasks	0	_____	0
In-lab Tasks	3	_____	30
Post-lab Tasks	0	_____	0

## 5.8 Template

My name:

My partner's name:

My other partner's name:

My lab section:

My lab demonstrator:

Today's date:

Your string identifier is:

quantity	symbol		single/ repeated/
		given/ mine	constant
Not in equations			

Table 5.1: List of quantities

symbol	value	units	instrument			effective uncertainty
			reference ( e.g. A.1)	precision measure	zero error	
$L$						
Not in equations						

Table 5.2: Single value quantities

quantity	symbol	equation	uncertainty
	$T$		
	$\lambda$		
	$\mu$		

Table 5.3: Calculated quantities

symbol	factor	bound	units
Sources of <i>systematic</i> error			
Sources of <i>random</i> error			

Table 5.4: Experimental factors responsible for effective uncertainties

Instrument						
reference (or name)						
units						
precision measure						
zero error						
number of 1/2 wavelengths		distance from end $d$	uncertainty in distance $\Delta d$	hanging mass $m$	adjusted mass $m \pm \Delta m$	uncertainty in mass $\Delta m$
total $N$	used	( )		( )		
1	1*	0				
2	1					
3	2					
4	3					
5	4					
6	5					
7	6					

Table 5.5: Resonance raw data

$\lambda$ ( )	$\Delta\lambda$ ( )	$T$ ( )	$\Delta T$ ( )

Table 5.6: Modified data

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Post-lab		
Hints		
	“think” “suggest” “explain” “how” “why” “what”	“agree” “equal” “do (did, does) ” “significantly different” “support” “verify”

Table 5.7: Lab Report Organization

# Chapter 6

## Exercise on Linearizing Equations

### 6.1 Purpose

The purpose of the exercise is to develop skills in producing linear graphs from various types of data and extracting results.

### 6.2 Introduction

This exercise will develop skills in linearizing data, so that a variety of relationships can be graphed as straight lines.

### 6.3 Theory

Often, the point of a scientific experiment is to try and find empirical values for one or more physical quantities, given measurements of some other quantities and some mathematical relationship between them. For instance, given a marble has a mass of 5 g, and a radius of 0.7 cm, the density of the marble can be calculated given that  $v = 4/3\pi r^3$  and  $\rho = m/v$ . (For the sake of simplicity, uncertainties will be ignored for now, although the calculation of those should be familiar by now.)

Many times, however, rather than having *one* measurement of a quantity, or set of quantities, we may have several measurements which should

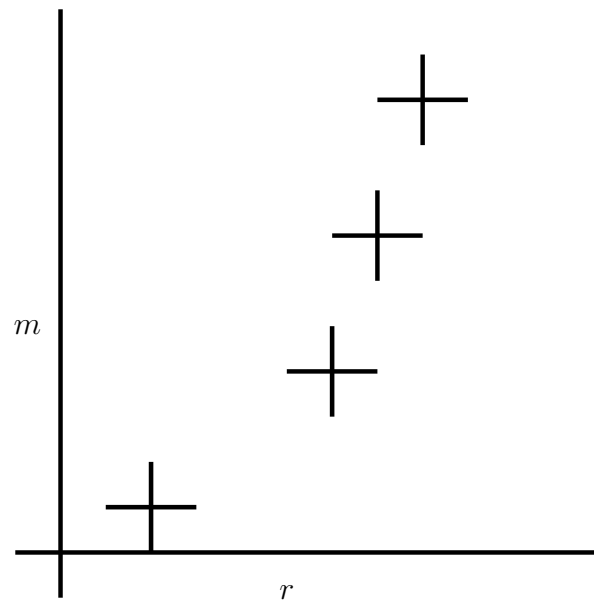


Figure 6.1: Non-linear equation

all follow the same relationships, (such as if we had several marbles made of the same material in the example above), and we wish to combine the results. The usual way of combining results is to create a graph, and extract information (such as the density) from the slope and  $y$ -intercept of the graph.

One may be tempted to ask why a graph should be better than merely averaging all of the data points. The answer is that an average is completely unbiased. The variation of any one point from the norm is no more or less important than the variation of any other point. A graph, however, will show any point which differs significantly from the general trend. Analysis of the graphical data (such as with a least squares fit) will allow such “outliers” to be given either more or less weight than the rest of the data as the researcher deems appropriate. Depending on the situation, the researcher may wish to verify any odd point(s), or perhaps the trend will indicate that a linear model is insufficient. In any case, it is this added interpretive value that a graph has which makes it preferable.

*A plot is better than an average since it may indicate systematic errors in the data.*

The value in fitting the data to an equation is that once the fit has been done, rather than continuing to work with a large amount of data, we can simply work with the parameters of our fit and their uncertainties. In the case of a straight line, all of our data can be replaced by four quantities;  $m, \Delta m, b$  and  $\Delta b$ .

*A fit equation replaces a bunch of data with a few parameters.*

The reason a linear graph is so useful is that it's easier to identify whether a line is straight than it is to identify whether it looks more like  $y = x^2$  or  $y = x^3$ , for instance.

*A straight line is easy to spot with the unaided eye.*

If the data fits an equation of the form  $y = mx + b$ , then it is easy to plot a straight line graph and interpret the slope and  $y$ -intercept, but it is rarely that simple. In most cases, the equation must be modified or **linearized** so that the variables *plotted* are different than the variables *measured* but produce a straight line.

**Linearizing equations** is this process of modifying an equation to produce new variables which can be plotted to produce a straight line graph. In many of your labs, this has been done already.

Look again at  $y = mx + b$ . Note that  $y$  and  $x$  are *variables*, (as each can take on a range of values), while  $m$  and  $b$  are *constants*, (as there is only one value for each for all of the data points). We can linearize an equation if we can get it in the form

$$\text{variable}_1 = \text{constant}_1 \times \text{variable}_2 + \text{constant}_2$$

There are a few things to note:

1. Several constants combined together produces another single constant.
2. Powers or functions of constants are also constants.
3. Constants may have “special” values of 0 or 1 so they appear “invisible”. For example

$$y = mx$$

is still the equation of a straight line, where  $b = 0$ . As well,

$$y = b$$

is the equation of a line where  $m = 0$ .

4. Variables may be combined together to form new variables.
5. Powers or functions of variables are also variables.

*Note that linearizing an equation will produce expressions for the slope and  $y$ -intercept which depend only on the constants in the original equation, not on the original  $x$  and  $y$  variables. This means that the constants can be related to the slope and  $y$ -intercept rather than the original variables.*

### 6.3.1 Techniques for Linearization

If a relationship involves *only* multiplication and division, (including powers), then logarithms can be used to linearize. Sometimes taking roots or powers of both sides of an equation will help.

### 6.3.2 Procedure for Linearization

The steps are as follows:

1. Rearrange the equation to get one variable (or a function of it) on the left side of the equation; this becomes your  $y$  variable.
2. Regroup the right side of the equation to create a term containing the other variable (or some function of it).
3. Use the left-side variable (or the function of it) as your  $x$  variable, and then your slope should be whatever multiplies it; your  $y$  intercept is whatever additive term is left over.

*Note:* It is important to realize that you don't need to understand an equation to linearize it; all you have to know is which parameters are variables (i.e. things you have data for), and which parameters are constants (i.e. things you want to calculate). Of course different experiments involving the same relationship may make different parameters variable, and so how an equation is linearized will depend on the data used. To again consider the above example: The original equations were

$$v = (4/3)\pi r^3 \quad (6.1)$$

and

$$\rho = m/v \quad (6.2)$$

where the quantities  $m$  and  $r$  are measured. (i.e. We have several marbles of the same material, so we can get several measurements of  $m$  and  $r$ , but we expect  $\rho$  to be the same for all of them.) Thus for this situation,  $m$  and  $r$  are variables, and  $\rho$  is a constant. We can combine the two equations to get

$$\rho = \frac{m}{(4/3)\pi r^3} \quad (6.3)$$

or

$$\rho = \frac{3m}{4\pi r^3} \quad (6.4)$$

This equation has a constant on one side, and a mixture of variables and constants on the other. First we should rearrange it to get a variable on the left hand side. Suppose we rearrange the equation, giving

$$m = (4/3)\pi r^3 \rho \quad (6.5)$$

This leaves a variable on the left. From this point on, there are two main possibilities for how to proceed: <sup>1</sup>

### Method I

Now we can create a new variable,  $Y$  such that

$$Y = m$$

By the rule about powers of variables being variables, then we can create a new variable  $X$  given by

$$X = r^3$$

Then equation 6.5 above becomes

$$Y = (4/3)\pi\rho X \tag{6.6}$$

since  $\pi$  is a constant, and  $\rho$  *should* be, and using the rule that combinations of constants produce constants, then we can define  $M$ , a constant, (not the same as  $m$ ), as

$$M = (4/3)\pi\rho$$

so equation 6.6 becomes

$$Y = MX + 0$$

which is the equation of a straight line. (In the case,  $B$ , the  $y$ -intercept is zero.)<sup>2</sup> So if we plot our “modified” variables, we should get a straight line, passing through the origin with a slope  $M$ . How can we get  $\rho$  from the graph? Well, from above

$$M = (4/3)\pi\rho$$

so

$$\rho = \frac{3M}{4\pi}$$

where  $M$  is the slope of the graph.

<sup>1</sup>Usually the process is not as explicit as this. i.e. one doesn't usually create an  $X$  and a  $Y$ , but doing this illustrates the procedure.

<sup>2</sup>Occasionally we can get a situation where the slope is similarly “invisible”, if it is 1 or 0.

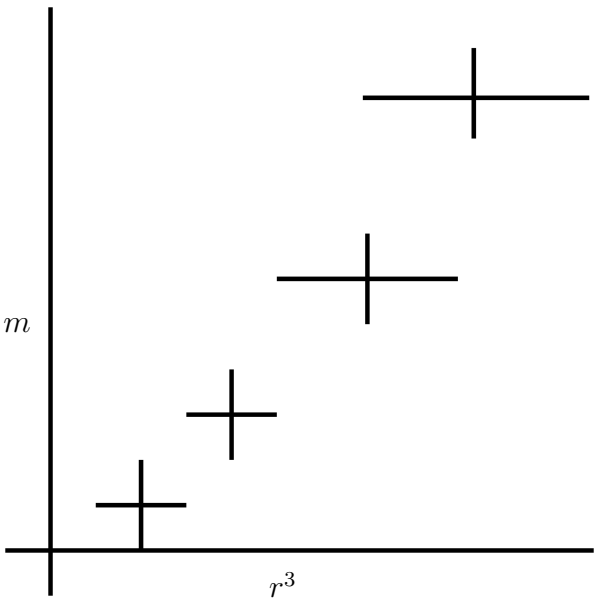


Figure 6.2: One linearization

**Method II**

We can take logarithms of both sides, so that  $Y$  such that equation 6.5 above becomes

$$\ln m = \ln((4/3)\pi\rho) + \ln r^3 \quad (6.7)$$

grouping the terms so one only contains constants (and so the combination should be constant) and one only contains the variable  $r$ . We can bring down the exponent so equation 6.7 becomes

$$\ln m = \ln((4/3)\pi\rho) + 3 \ln r$$

Now we can create new variables,  $Y$  such that

$$Y = \ln m$$

and

$$X = \ln r$$

which is the equation of a straight line. So if we plot our “modified” variables, we should get a straight line. How can we get  $\rho$  from the graph? Well, from above

$$B = \ln((4/3)\pi\rho)$$

so

$$\rho = \frac{3}{4\pi} e^B$$

where  $B$  is the y-intercept of the graph. (In this case, the value you get from the graph for the slope should suggest whether the fit is a good one.)

*Remember that after linearization, our results depend on our graphical quantities of the slope and the y-intercept, rather than on the original measured quantities.*

**6.3.3 Choosing a Particular Linearization**

Often there may be more than one linear form for the equation so there may be more than one “right answer”. In this case, there are a few things which may help you choose.

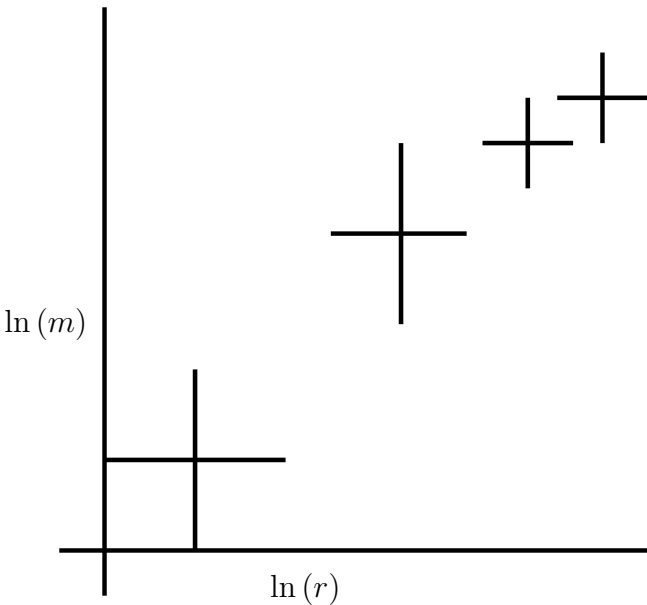


Figure 6.3: Another linearization

### Simple variables

A preferable linearization is one which most simplifies understanding the graph or interpreting the results. For instance, in the above example, it would have been possible to use  $(4/3)\pi r^3$  instead of  $r^3$  as our  $x$  variable, but that would make confusing axis scales and/or units (although it would have made the slope be  $\rho$  with no calculation).

### Spread of data

The spread of data will be different for each linearization. A graph with points which are more equally spaced is generally preferable to one where the points are concentrated in one area.

### Size of error bars

Like the spread of data, the size of the error bars will be different for each linearization. A graph with more equally sized error bars is generally preferable to one where the error bars vary greatly in size for different points.

*Usually it is preferable to separate variables and constants as much as possible in your linearization so that graph variables are easily related to experimental ones.*

## 6.3.4 Uncertainties in Results

After determining how equation parameters relate to graphical quantities as above, uncertainties can be determined as usual. In the above example Method I gives

$$\Delta\rho = \frac{3\Delta M}{4\pi}$$

while for Method II

$$\Delta\rho = \frac{3}{4\pi}e^B\Delta B$$

or

$$\Delta\rho = \rho\Delta B$$

### 6.3.5 Systematic Error in Variables

If there is systematic error in your data, it will show up on the graph.

- If your slope or  $y$ -intercept should have a specific value, such as zero, then a value other than that suggests systematic error.
- Since your graph is supposed to be linear, then if the points actually seem to be on a curve rather than a line there is probably systematic error in the data.

If systematic error is suggested in either of these ways, you should try to determine which variable(s) may be responsible, and whether your data values are above or below their correct value.

## 6.4 Procedure

### 6.4.1 Preparation

You are welcome to go ahead and do as much of the exercise on your own as you wish; you can just bring a sheet with your question answers to the lab. If you get it all done in advance, that's great.

### 6.4.2 Investigation

The fundamental frequency of vibration of a string is given by

$$f = \frac{1}{\lambda} \sqrt{\frac{T}{\mu}}$$

$\lambda$  and  $T$ , are measured variables and  $f$  is a constant, with an uncertainty  $\Delta f$ . Determine  $\mu$  and  $\Delta\mu$ .

There are 3 different ways of doing this. In each case, show what should be plotted, how the error bars for your  $x$  (independent) and  $y$  (dependent) variables are determined, what the slope and  $y$ -intercept will be, and how  $\mu$  and  $\Delta\mu$  come from the slope and  $y$ -intercept and their uncertainties.

For instance, from the example above, using Method I we would say:

- Plot  $m$  vs.  $r^3$ . (In other words, the independent ( $x$ ) variable is  $r^3$  and dependent ( $y$ ) variable is  $m$ .)
- The uncertainty in the dependent variable is  $\Delta m$ .
- The uncertainty in the independent variable is  $3r^2\Delta r$ .
- The slope of the graph will be  $M = (4/3)\pi\rho$ .
- The  $y$ -intercept should be zero<sup>3</sup>.
- The density will be determined from the slope by the equation  $\rho = \frac{3M}{4\pi}$ .
- The uncertainty in the density will be determined from the slope by the equation  $\Delta\rho = \frac{3\Delta M}{4\pi}$ .

### In-lab Questions

For each of the linearizations below, be sure to address all 7 points as in the example above.

**IQ0:** What is the linearization if our dependent variable is  $\lambda$ ? (*See the example above.*)

**Answer:** First we have to rearrange the equation

$$f = \frac{1}{\lambda} \sqrt{\frac{T}{\mu}}$$

to get the *dependent* variable on the left, as follows:

$$\lambda = \frac{1}{f} \sqrt{\frac{T}{\mu}}$$

Then we need to regroup the right side to separate the *independent* variable<sup>4</sup>.

$$\lambda = \frac{1}{f\sqrt{\mu}} \sqrt{T}$$

<sup>3</sup>If the  $y$ -intercept turns out to be something other than zero, then there is some systematic error in our experiment.

<sup>4</sup>There is another possibility here; see if you can figure out what it is.

So the slope is the part that is multiplied by the independent variable,

$$\text{slope} = \frac{1}{f\sqrt{\mu}}$$

which means

$$\sqrt{\mu} = \frac{1}{\text{slope}f}$$

and so

$$\mu = \frac{1}{\text{slope}^2 f^2}$$

The uncertainties in each of the quantities are calculated as usual. Specifically, if  $\mu$  and  $f$  have uncertainties  $\Delta\mu$  and  $\Delta f$ , and after the least squares fit, the slope and  $y$ -intercept will simply be quantities  $m$  and  $b$  with associated uncertainties  $\Delta m$  and  $\Delta b$ , then

$$\begin{aligned} \Delta\mu &= \mu \left( \frac{\Delta\text{slope}^2}{\text{slope}^2} + \frac{\Delta f^2}{f^2} \right) \\ &= \mu \left( \frac{2\text{slope}\Delta\text{slope}}{\text{slope}^2} + \frac{2f\Delta f}{f^2} \right) \\ &= \mu \left( 2\frac{\Delta\text{slope}}{\text{slope}} + 2\frac{\Delta f}{f} \right) \\ &= 2\mu \left( \frac{\Delta\text{slope}}{\text{slope}} + \frac{\Delta f}{f} \right) \end{aligned}$$

using algebra, and *given that the slope should be positive*. (The uncertainty determined by inspection is shown in the template in the example. Remember that it doesn't matter which method you use since they should usually give similar results to one significant figure.)

**IQ1:** What is the linearization if our dependent variable is  $\lambda^2$ ? (See the example above.)

**IQ2:** What is the linearization if our dependent variable is  $\ln(\lambda)$ ? (See the example above.)

**IQ3:** *Without considering the data*, which of the above choices would you prefer and why? (In other words, is there a choice that seems better to work with, regardless of how the data fit?)

### In-lab Tasks

**IT1:** Fill in the headings for the columns in Tables 6.1, 6.2, and 6.3.

**IT2:** Fill in the template with the equations, uncertainties, and units for the slope and  $y$ -intercept for each linearization.

### 6.4.3 Analysis

#### Post-lab Tasks

There are no post-lab tasks for this exercise, although the results of this exercise will be used in later exercises.

#### Post-lab Discussion Questions

There are no post-lab questions for this exercise, although much of the analysis from here will belong in your lab report

## 6.5 Bonus

You can do one of the questions below. In each of the following questions, state the modified variables to be plotted, and state how the unknown(s) may be determined from the graph.

1. The position of a body starting from rest and subject to uniform acceleration is described by

$$s = \frac{1}{2}at^2$$

$s$  and  $t$  are measured variables. Determine  $a$  and  $\Delta a$ .

2. The period of oscillation of a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where  $L$  and  $T$  are measured variables. Determine  $g$  and  $\Delta g$ .

3. The Doppler shift of frequency for a moving source is given by

$$f = f_0 \frac{v}{v - v_0}$$

$f$  and  $v_0$  are measured variables,  $f_0$  is fixed and known. Determine  $v$  and  $\Delta v$ . (Note that even though the notation might suggest otherwise, in this case  $v_0$  is a variable, not a constant.)

4. The impedance of a series  $RC$  circuit is

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$Z$  and  $\omega$  are measured variables. Determine  $R$ ,  $\Delta R$ ,  $C$ , and  $\Delta C$ .

5. The conductivity of an intrinsic semiconductor is given by

$$\sigma = C e^{\frac{-E_g}{2kT}}$$

$\sigma$  and  $T$  are measured variables and  $k$  is a known constant. Determine  $E_g$ ,  $\Delta E_g$ ,  $C$ , and  $\Delta C$ .

6. The relativistic variation of mass with velocity is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m$  and  $v$  are measured variables. Determine  $m_0$ ,  $\Delta m_0$ ,  $c$ , and  $\Delta c$ .

7. The refraction equation is

$$\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$$

$\theta_1$  and  $\theta_2$  are measured variables;  $\mu_1$  is fixed and known. Determine  $\mu_2$  and  $\Delta \mu_2$ .

## 6.6 Recap

By the end of this exercise, you should understand the following terms:

- linear graph
- linearized equation

## 6.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	3	_____	75
Post-lab Questions	0	_____	0
Pre-lab Tasks	0	_____	0
In-lab Tasks	2	_____	25
Post-lab Tasks	0	_____	0
Bonus		_____	5

## 6.8 Template

Trial number	independent $\sqrt{T}$	dependent $\lambda$
1		
2		
3		
4		
5		

Table 6.1:  $\lambda$  Linearization

For this linearization,

The equation for the uncertainty in the independent variable is:

$$\Delta x = \Delta \sqrt{T} \approx \frac{1}{2\sqrt{T}} \Delta T \text{ (using algebra)}$$

The equation for the uncertainty in the dependent variable is:

$$\Delta y = \Delta \lambda$$

The equation for the slope is: **slope** =  $\frac{1}{f\sqrt{\mu}}$

The equation for the  $y$ -intercept is:  **$y$  - intercept** = **0**

The units for the slope are: **mN<sup>-1/2</sup>**

The units for the  $y$ -intercept are: **m**

The equation for the linear mass density,  $\mu$ , is:  $\mu = \frac{1}{\text{slope}^2 f^2}$

The equation for the uncertainty in the mass density,  $\Delta\mu$ , is:

$$\Delta\mu = \Delta \left( \frac{1}{\text{slope}^2 f^2} \right) \approx \frac{1}{\text{slope}^2 f^2} - \frac{1}{(\text{slope} + \Delta\text{slope})^2 (f + \Delta f)^2}$$

(using inspection, given that the slope and  $f$  should be positive)

Trial number	independent	dependent $\lambda^2$
1		
2		
3		
4		
5		

Table 6.2:  $\lambda^2$  Linearization

For this linearization,

The equation for the uncertainty in the independent variable is:

The equation for the uncertainty in the dependent variable is:

The equation for the slope is:

The equation for the  $y$ -intercept is:

The units for the slope are:

The units for the  $y$ -intercept are:

The equation for the linear mass density,  $\mu$ , is:

The equation for the uncertainty in the mass density,  $\Delta\mu$ , is:

Trial number	independent	dependent $\ln(\lambda)$
1		
2		
3		
4		
5		

Table 6.3:  $\ln(\lambda)$  Linearization

For this linearization,

The equation for the uncertainty in the independent variable is:

The equation for the uncertainty in the dependent variable is:

The equation for the slope is:

The equation for the  $y$ -intercept is:

The units for the slope are:

The units for the  $y$ -intercept are:

The equation for the linear mass density,  $\mu$ , is:

The equation for the uncertainty in the mass density,  $\Delta\mu$ , is:



# Chapter 7

## Exercise on Graphing and Least Squares Fitting using a Spreadsheet

### 7.1 Purpose

The purpose of this experiment is to become familiar with using spreadsheet to produce graphs and analyze graphical data.

### 7.2 Introduction

While a spreadsheet has a lot of useful features for data analysis, etc. it was designed primarily for use by people in business rather than for scientists, and so there are some things which scientists wish to do which require a little effort in a spreadsheet. Graphical analysis is one of the areas where this is true, as you shall see. (Even with the extra work, it's a lot more convenient than doing it all by hand.)

## 7.3 Theory

### 7.3.1 Graphing

#### Graph Type

The graph type mainly used by scientists is an **x-y** (or *scatter*) graph<sup>1</sup>. *Do not* choose a line graph!

#### Colour

The default background colour, (grey in many cases), in many spreadsheets just looks bad in graphs; it obscures the data and serves no purpose. *Turn the background colour off!*

#### Gridlines

Grid lines should be either removed or in both dimensions. Gridlines in one direction only look odd on an “xy” graph. *Turn the gridlines off!*

#### Text

The main text of a graph consists of *x*- and *y*- titles, a *main title* and perhaps a *sub-title*. All of these may be set in a spreadsheet.

#### Series

A spreadsheet allows you to plot several different “series” of  $(x, y)$  data. Each series can be *customized*, with choices for many things, including the following:

- Patterns

Each series can be plotted with lines, symbols, or both.

*Do not connect the points like a dot-to-dot drawing!*

<sup>1</sup> As long as there is some *mathematical relationship* between the variables, then an x-y graph illustrates the relationship. However, if the independent variable does not have a numerical value, then this doesn't apply. For instance, if you were graphing reaction time for men and women, then a bar graph would be the logical choice, since there's no *numerical* relationship between “men” and “women”.

*Do not use an arbitrary function just because it goes through all the data points!*

- Markers

There are many possible symbols which can be used for each series.

- Lines

There are several line types available for each series. One important fact about how lines are used to connect points in a series; *all points in a series are joined by lines*, unless the line for that series is turned off.

In science, it is almost always wrong to have a dot-to-dot drawing. It is also wrong to have a curve which has no mathematical significance. For this reason, data points should not be connected by either line segments or a curve like a polynomial which is made to pass through each data point. The only line or curve which should be shown is the result of a fit which is based on some theoretical mathematical relationship.

### Matching up $x$ and $y$ Values

When you create an  $xy$  graph in a spreadsheet, you *don't* input data values as  $(x, y)$  pairs. Instead you select series for each of  $x$  and  $y$ . The way the individual  $x$  and  $y$  values are associated is by where they occur in their respective series.

In Figure 7.1 you can see that the 5<sup>th</sup> point in each series is highlighted. Even though the series all start in different rows and columns, since the number of cells in each is the same, corresponding values can be considered to be related. (If a cell is blank, then the corresponding point or error bar will not be plotted.)

### Error Bars

In a spreadsheet, when you choose *custom* error bars, you can choose series for both  $x$  and  $y$ , and even potentially different series for the + and - directions.

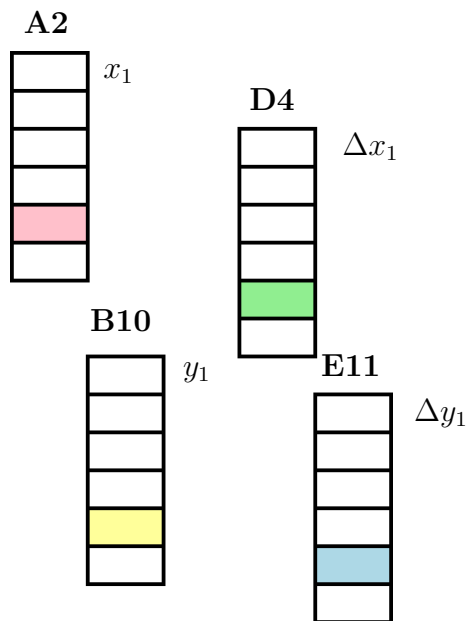


Figure 7.1: Spreadsheet layout with series for  $x$ ,  $y$ , and error bars

You don't necessarily have to put markers on the ends of the error bars; the value in doing so is to make it clear that you're not just using "+" symbols for plotting the data points. Also, if you are including a grid on your graph, error bars without markers on the end may be hard to distinguish. However if the error bars will be clearly identifiable without markers, you don't need to use them.

### 7.3.2 Least Squares Fitting

The point of plotting a graph in an experiment is usually to extract information from the graph; often the data is plotted in such a manner that the model being tested suggests that the data should fit a straight line. If it does, then getting the slope and  $y$  intercept of the line of best fit along with their associated uncertainties is necessary. One of the two usual ways to determine the uncertainty in a graphical quantity is to calculate the **standard error**. (The other involves finding lines of maximum and minimum slope.) The following sections discuss using a spreadsheet to do least squares fitting and to calculate standard errors.

#### Determining the equation of the line by formulas

In Chapter 4, "*Graphs and Graphical Analysis*", the lab manual explains how to calculate a least squares fit to a set of data. This can be done in a spreadsheet by creating additional cells corresponding to each data point which contain, respectively,  $x^2$ ,  $y^2$  and  $xy$ . At the end of the data, these quantities can be totaled to give the sums necessary to do the least squares fit.<sup>2</sup>

---

<sup>2</sup>You may notice that a particular quantity comes up a lot. It is

$$N \left( \sum x_i^2 \right) - \left( \sum x_i \right)^2 \quad (7.1)$$

It only takes a couple of lines of algebra to show that this equals

$$N(N-1)\sigma_x^2 \quad (7.2)$$

where  $\sigma_x^2$  is the sample standard deviation of the  $x$  values.

### Determining the equation of the line using **LINEST()**

Excel and LibreOffice each contain a function to do least squares fitting. Unfortunately it produces a bunch of numbers without indicating what is what. It also has to be configured to do things the way we want. Make sure to configure it to give extra statistics.

*When using **LINEST()** to calculate the least squares fit, always set it to calculate the y-intercept, even when you expect it to be zero! This gives you important information about the data.*

Comparing the result given by the least squares fit using your formulas with your regression output should indicate what several of the quantities are.

(If you use **LINEST()** to do least squares fitting for a lab report, quote the quantities given with the names used in the lab manual. The unidentified block of cells given by many spreadsheets is not very meaningful.)

### Determining uncertainties in the slope and y-intercept

**Case I: Maximum and minimum slopes** If the error bars are large enough, then the line of best fit should go through all of the error bars. In this case, there will be two data points which determine coordinates for a line of maximum slope which crosses all of the error bars. Consider the case for positive slope:

If we label two points  $x_1$  and  $x_2$ , where  $x_1 < x_2$ , then we can see from Figure 7.2 that the steepest line which touches the error bars for both  $x_1$  and  $x_2$  is the line between  $(x_1 + \Delta x_1, y_1 - \Delta y_1)$  and  $(x_2 - \Delta x_2, y_2 + \Delta y_2)$ . The slope of this line will then be

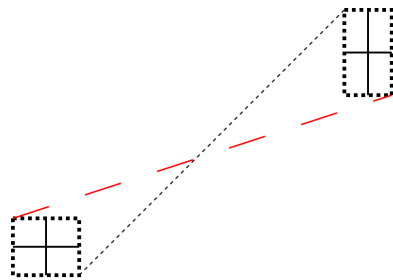
$$m_{max} = \frac{(y_2 + \Delta y_2) - (y_1 - \Delta y_1)}{(x_2 - \Delta x_2) - (x_1 + \Delta x_1)}$$

and then the y-intercept is given by

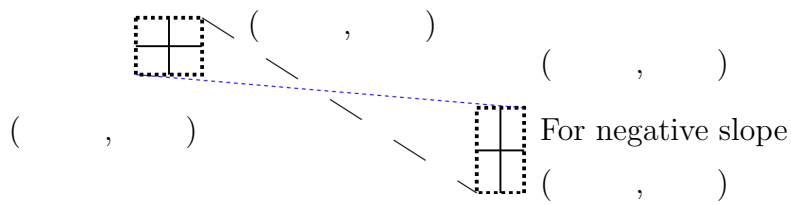
$$b_{min} = (y_1 - \Delta y_1) - m_{max}(x_1 + \Delta x_1) = (y_2 + \Delta y_2) - m_{max}(x_2 - \Delta x_2)$$

Similarly the line with the least slope which touches the error bars for both  $x_1$  and  $x_2$  is the line between  $(x_1 - \Delta x_1, y_1 + \Delta y_1)$  and  $(x_2 + \Delta x_2, y_2 - \Delta y_2)$ . The slope of this line will then be

$$m_{min} = \frac{(y_2 - \Delta y_2) - (y_1 + \Delta y_1)}{(x_2 + \Delta x_2) - (x_1 - \Delta x_1)}$$



For positive slope



For negative slope

Figure 7.2: Maximum and Minimum Slope Coordinates from a Point

and then the  $y$ -intercept is given by

$$b_{max} = (y_1 + \Delta y_1) - m_{min}(x_1 - \Delta x_1) = (y_2 - \Delta y_2) - m_{min}(x_2 + \Delta x_2)$$

The case for a negative slope is shown in Figure 7.2; the analysis is left to the student.

*The points for the maximum and minimum slope will not always be the endpoints on the graph.*

**Index function** To calculate the slope and  $y$ -intercept from a block of data, we can use the *index* function. Its syntax is as follows:

- index(reference, row number, column number)
- reference is the cell range to look in
- row number (starts at one)
- column number (starts at one)

So if we have a data set of 6 values where the  $x$  values start in  $A2$ , and the  $\Delta x$  values start in  $D4$ , then we can get

$$x_2 + \Delta x_2$$

by the formula

$$= \text{INDEX}(A2 : A7, 2, 1) + \text{INDEX}(D4 : D9, 2, 1)$$

(Note that the only difference is in which block of data to use.) You'd probably write the formula as

$$= \text{INDEX}(\$A\$2 : \$A\$7, 2, 1) + \text{INDEX}(\$D\$4 : \$D\$9, 2, 1)$$

so that you could copy it and still refer to the same blocks of data.

**Case II: standard errors** If the error bars are small enough, then the points will be scattered in such a way that no line can be drawn which crosses all of the error bars. In this case, the uncertainties in the slope and  $y$ -intercept reflect the scatter of the points. In this case, the uncertainty in the slope and  $y$ -intercept will be calculated using the **standard errors** in the slope and  $y$ -intercept, in much the same way that the uncertainty for an average value is calculated using the *standard error of the mean*.

### 7.3.3 Displaying Lines

*Unless you are going to give the equation of a line or curve, do not show it on a graph!*

#### Plotting arbitrary lines

To display a line on the graph, such as a best fit line, one can use a series which has not yet been used. When one knows the *equation* of a line, all one needs is two endpoints so that a line can be drawn between them. To allow this, include 2 values at the end of your  $x$  series,  $x_{min}$  and  $x_{max}$  which are the minimum and maximum values from the  $x$  data, respectively. Placing the  $y$  values calculated from the line equation in the corresponding cells of another series will allow a line to be plotted between those points. (Set the format for that series to lines only.)

#### Using “trendline”

There is a built-in feature called *trendline* which allows you to display various fits to data. A *linear* trendline is, in fact, a least squares fit. Unfortunately, this feature does not automatically display the parameters for the fit, so it's not as much use as it could be.

## 7.4 Procedure

### 7.4.1 Preparation

You are welcome to go ahead and do as much of the exercise on your own as you wish; you can just bring your spreadsheet with you to the lab and demonstrate the points indicated. If you get it all done in advance, that's great.

If you do it on your own, then print a copy of the spreadsheet showing *formulas* along with the ones indicated in the post-lab questions. (You only need to show the formulas for rows mentioned in the instructions.) Print the graphs as well.

**Pre-lab Tasks**

**PT1:** Read over Chapter 4, “*Graphs and Graphical Analysis*”, and copy the Equations 4.3 to 4.14 to the appropriate places below. (For instance, copy Equation 4.4 to complete Equation 7.4.) *If you don’t want to print the pages of the manual, copy Equations 4.3 to 4.14 to a piece of paper and bring it with you.*

**PT2:** Open the spreadsheet for this exercise from the web page. On the tabbed pages for “raw data” and the three linearizations, insert rows as needed and type in your linearized data for each of the 3 linearizations. Save the spreadsheet on a USB memory stick or in your WLU account to bring to the lab.

**PT3:** Rewrite the Equations 4.7, 4.8, 4.9, 4.10 for  $m_{max}$ ,  $b_{min}$ ,  $m_{min}$ ,  $b_{max}$  for a line with *negative* slope.

$$m_{min} =$$

$$b_{max} =$$

**PT4:** Fill in the co-ordinates from **PT3** in Figure 7.2.

Raw Data					
i	T	+/-T	lambda	+/-lambda	f
	N	N	m	m	
1	2.00	0.60	2.00	0.20	+/-f
2	0.70	0.50	0.40	0.30	
3	9.34	0.60	8.90	0.40	
4	6.35	0.70	6.70	0.50	

Figure 7.3: Spreadsheet page for raw data

i	$\sqrt{t}$	$\pm\sqrt{t}$	$\lambda$	$\pm\lambda$
	$N^{1/2}$	$N^{1/2}$	m	m
1	1.41	0.21	2.00	0.20
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?

Figure 7.4: Spreadsheet page for first linearization

i	$\pm$ ?	$\lambda^2$	$\pm\lambda^2$
1	?	?	?
2	?	?	?
3	?	?	?
4	?	?	?

Figure 7.5: Spreadsheet page for second linearization

i	$\pm$ ?	$\ln \lambda$	$\pm \ln \lambda$
1	?	?	?
2	?	?	?
3	?	?	?
4	?	?	?

Figure 7.6: Spreadsheet page for third linearization

## 7.4.2 Investigation

### In-lab Tasks

In this exercise the in-lab tasks appear throughout the section.

### Part 1: Plotting a Graph

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 90.

**Setting up the spreadsheet (Excel instructions)** Use the data which is already in the spreadsheet. Only change to your own linearized data after the formulas have all been set up correctly.

- 1 Load the graph from the lab page.
- 2 Insert a chart in the box on the first tabbed page.
  - Make sure it is an  $xy$  (scatter) graph.

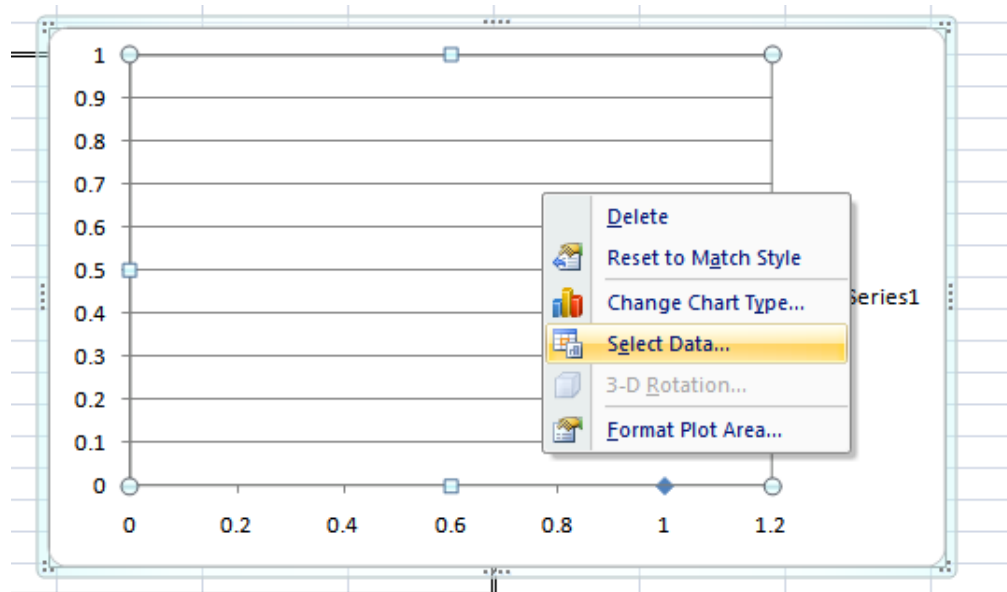


Figure 7.7: Adding a series to the graph: step 1

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 90.

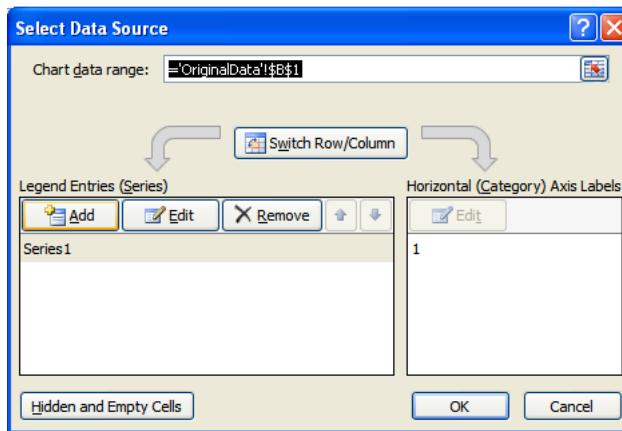


Figure 7.8: Adding a series to the graph: step 2

- The  $x$  series should be **C6** to **C9**.
- The  $y$  series should be **E6** to **E9**.

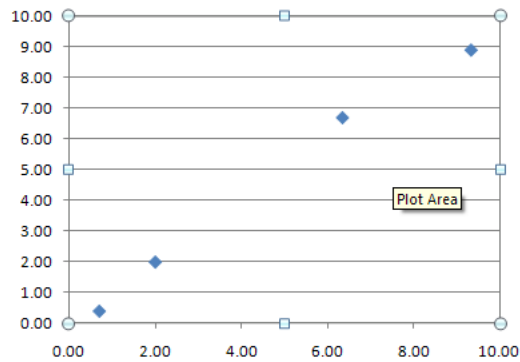


Figure 7.9: Basic graph without error bars

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 90.

- 3 Click on the “*Layout*” tab. You should now see the options for error bars.

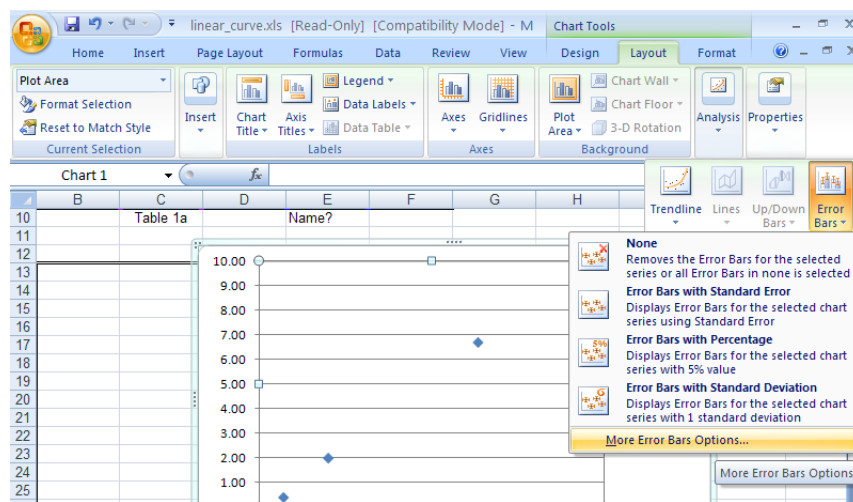


Figure 7.10: Adding vertical error bars

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 90.

- Note options for  $x$  and  $y$  error bars.
- Select  $y$  error bars, and pick *Custom*.

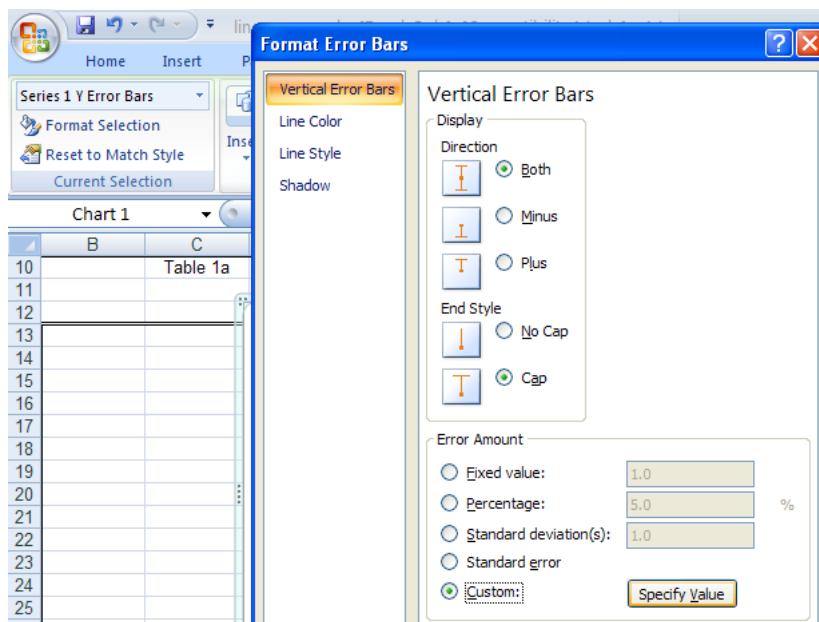


Figure 7.11: Custom vertical error bars

- Select series **F6** to **F9** for both + and -.

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 90.

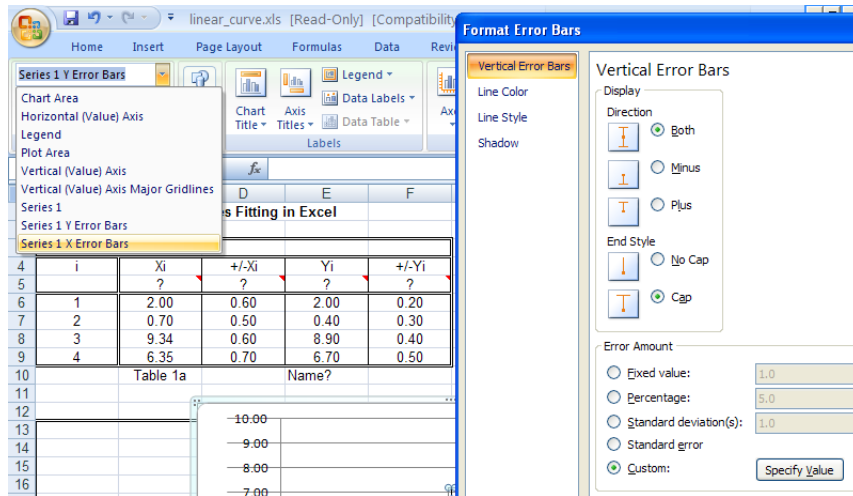


Figure 7.12: Adding horizontal error bars

- Repeat for  $x$  error bars, using D6 to D9.

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 90.

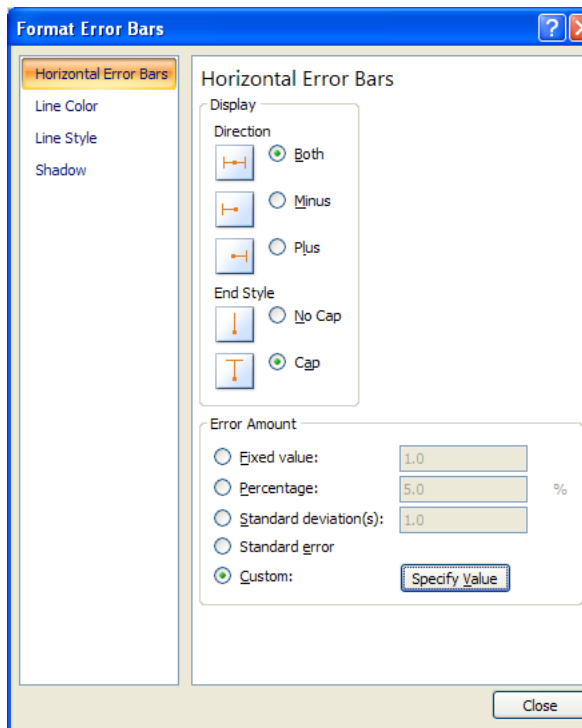


Figure 7.13: Custom horizontal error bars

End of Excel specific instructions for Part 1. Go to page 95.

These instructions are for using LibreOffice. If you are using Excel, go back to page 84.

Setting up the spreadsheet (LibreOffice instructions) *Use the data which is already in the spreadsheet. Only change to your own linearized data after the formulas have all been set up correctly.*

- 1 Load the graph from the lab page.
- 2 Insert a chart in the box on the first tabbed page.
  - Make sure it is an  $xy$  (scatter) graph.

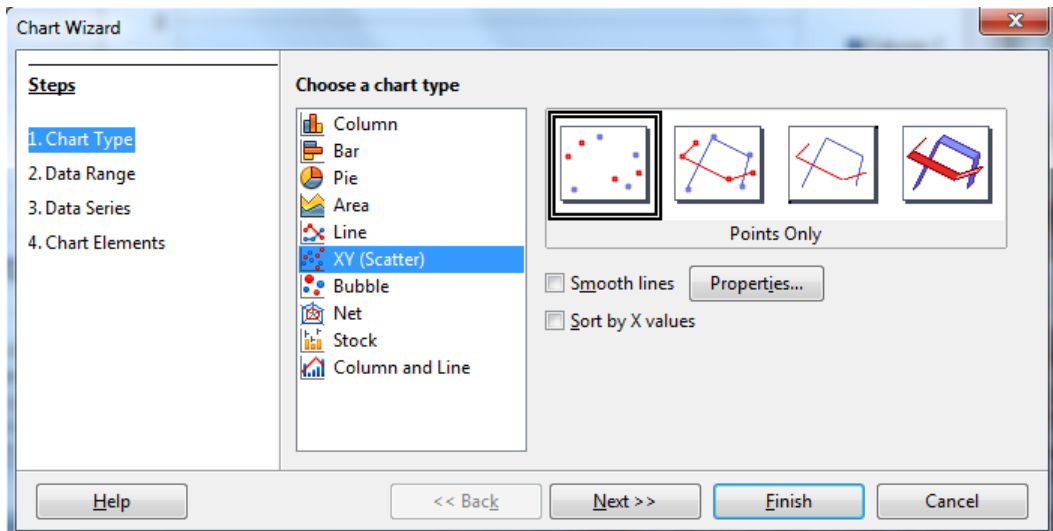


Figure 7.14: Choosing an  $xy$  graph

These instructions are for using LibreOffice. If you are using Excel, go back to page 84.

- Right click on the graph and select **Data Range** to select data.

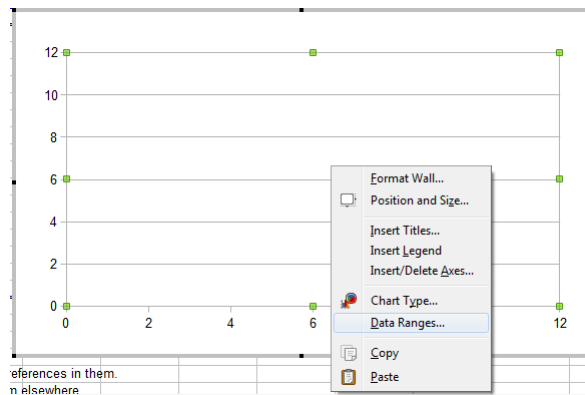


Figure 7.15: Adding a series to the graph: step 1

- Choose options as shown:

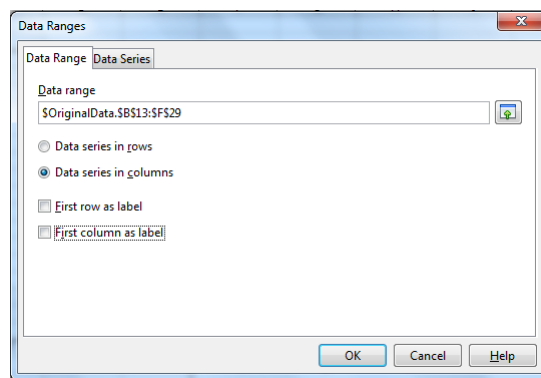


Figure 7.16: Series in columns, without labels

These instructions are for using LibreOffice. If you are using Excel, go back to page 84.

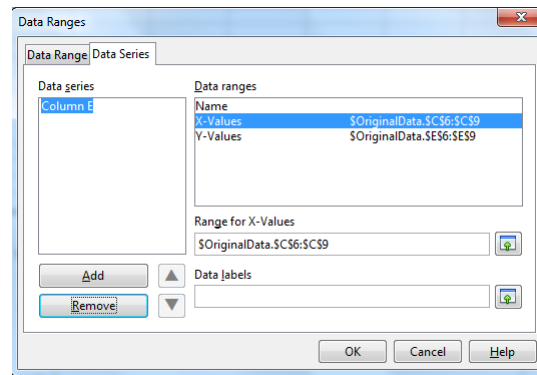


Figure 7.17: Adding a series to the graph: step 2

- The  $x$  series should be **C6** to **C9**.
- The  $y$  series should be **E6** to **E9**.

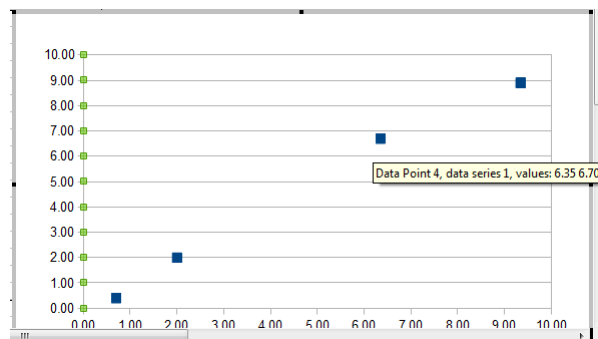


Figure 7.18: Basic graph without error bars

These instructions are for using LibreOffice. If you are using Excel, go back to page 84.

- 3 Right click on a data point to allow you to format the series.

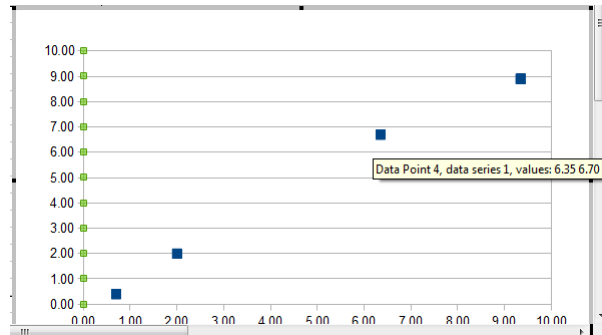


Figure 7.19: Adding vertical error bars

- Note options for  $x$  and  $y$  error bars.
- Select  $y$  error bars, and pick *Cell Range*.

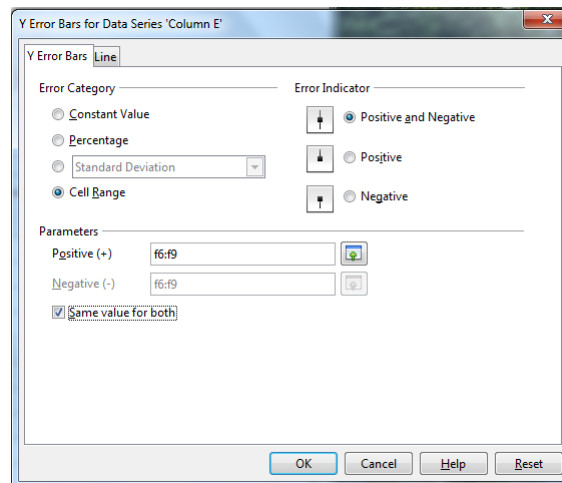


Figure 7.20: Using a cell range for vertical error bars

- Select series **F6** to **F9** and check **Same Value for both** .

These instructions are for using LibreOffice. If you are using Excel, go back to page 84.

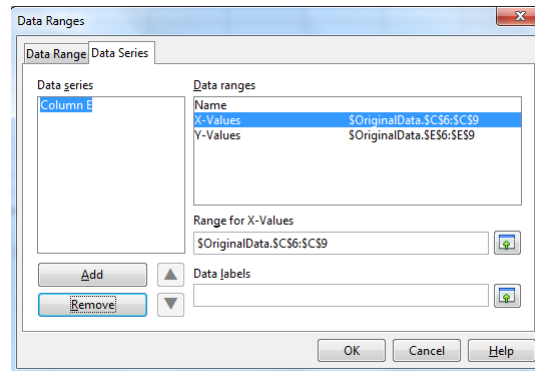


Figure 7.21: Adding horizontal error bars

- Repeat for  $x$  error bars, using **D6** to **D9**.

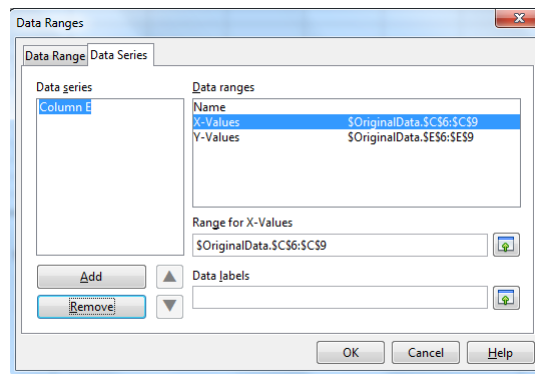


Figure 7.22: Using a cell range for horizontal error bars

End of LibreOffice specific instructions for Part 1

**Common instructions for Part 1.**

At this point there should *not* be a line connecting the data points. If there is, turn it off.

**IT1:** Demonstrate graph as is.

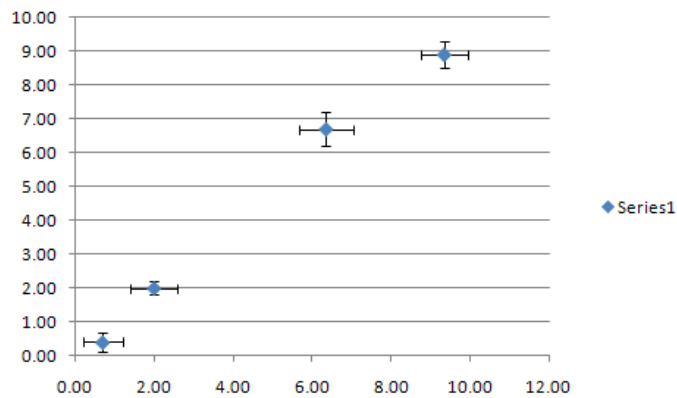


Figure 7.23: Graph with error bars (Excel)

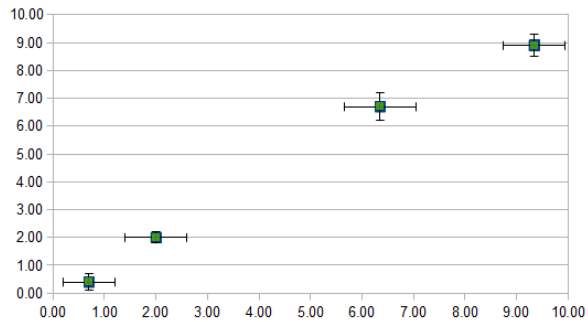


Figure 7.24: Graph with error bars (LibreOffice)

**Part 2: Performing a Least Squares Fit Using Formulas**

In this part, you're going to use formulas to do a least squares fit. Remember that the least squares fit gives values for  $b$ , the  $y$ -intercept, and  $m$ , the slope, as follows:

$$b = \tag{7.3}$$

and

$$m = \tag{7.4}$$

For the linear case,  $S$  can be shown to have a value of

$$S = \tag{7.5}$$

$$\sigma = \tag{7.6}$$

where

$$\nu = \tag{7.7}$$

is the number of **degrees of freedom** mentioned earlier. (Often the symbol  $\nu$  is used for degrees of freedom.)

Also, the **standard error** in the intercept is

$$\sigma_b = \tag{7.8}$$

and the standard error in the slope is

$$\sigma_m = \tag{7.9}$$

$R^2$  is defined as follows:

$$R^2 = \frac{(N \sum x_i y_i - (\sum x_i)(\sum y_i))^2}{(N \sum x_i^2 - (\sum x_i)^2)(N \sum y_i^2 - (\sum y_i)^2)}$$

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 100.

(Excel instructions)

- 1 Go to the second tabbed page, and replace number cells for everything other than data values with formulas. (Make sure to use a function for  $N$ .)

Quantity	Value
Slope	0.9936
Y-intercept	-0.0681
Sum of Squares Error (S)	0.36613
Standard Deviation	0.42786
Standard Error in Slope	0.06208
Standard Error in Y-intercept	0.35669
R <sup>2</sup>	0.99225

Figure 7.25: Least Squares Fit Using Formulas

- 2 Use the values of  $x_{min}$  and  $x_{max}$  in **C31** and **C32** and the equation of the line of best fit to calculate corresponding  $y$  values in **F31** and **F32**. Add this series to your graph to show the line of best fit. Right click on one of the points, and format the data series. You want to get a line with no endpoints.

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 100.

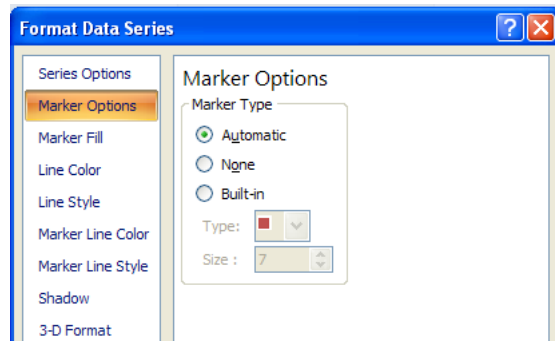


Figure 7.26: Marker options

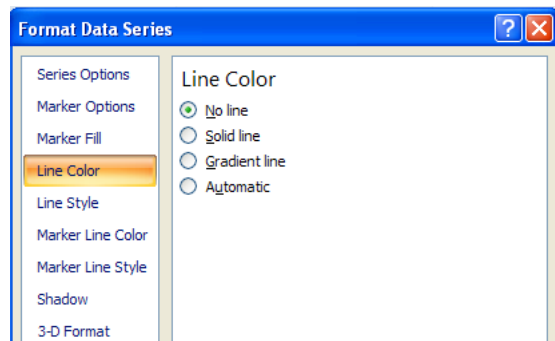


Figure 7.27: Line options

End of Excel specific instructions for Part 2 Go to page 102.

These instructions are for using LibreOffice. If you are using Excel, go back to page 98.

(LibreOffice instructions)

- 1 Go to the second tabbed page, and replace number cells for everything other than data values with formulas. (Make sure to use a function for  $N$ .)

Quantity	Value
Slope	0.9936
Y-intercept	-0.0681
Sum of Squares Error (S)	0.36613
Standard Deviation	0.42786
Standard Error in Slope	0.06208
Standard Error in Y-intercept	0.35669
R <sup>2</sup>	0.99225

Figure 7.28: Least Squares Fit Using Formulas

- 2 Use the values of  $x_{min}$  and  $x_{max}$  in **C31** and **C32** and the equation of the line of best fit to calculate corresponding  $y$  values in **F31** and **F32**. Add this series to your graph to show the line of best fit. Right click on one of the points, and format the data series. You want to get a line with no endpoints.

**These instructions are for using LibreOffice. If you are using Excel, go back to page 98.**

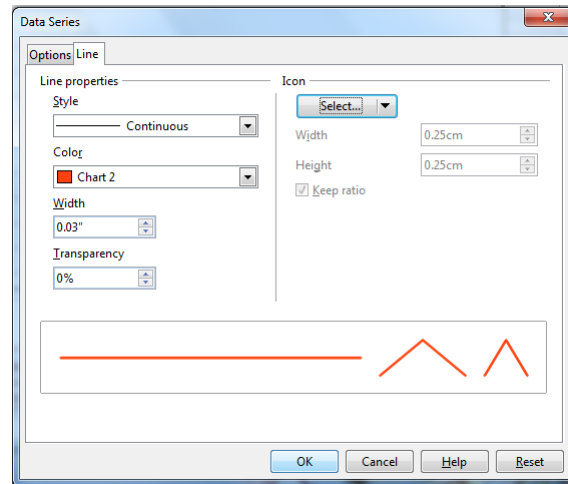


Figure 7.29: Line types

**End of LibreOffice specific instructions for Part 2**

Common instructions for Part 2.

**IT2:** Demonstrate graph as is.

**Part 3: Performing a Least Squares Fit Using Built-in Features**

**These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 105.**

**(Excel instructions)** Here you're going to identify the values produced by Excel's **LINEST()** function by comparing them to the values you got from the formulas.

- 1 Go to the third tabbed page, and use the **LINEST()** function to do a least squares fit.
  - Put the function in **C18**, with *constant*=1 and *stats*=1.
  - Highlight **C18** to **D22**.
  - On a PC, press  $\langle F2 \rangle$  followed by  $\langle \text{CTRL} \rangle \langle \text{SHIFT} \rangle \langle \text{ENTER} \rangle$ .  
(On a MAC, click inside the formula editing box, and press  $\langle \text{COMMAND} \rangle \langle \text{ENTER} \rangle$ .)
  - Fill in **B18** to **B22** and **E18** to **E22** with names from the previous page. (You don't have to fill in the ones that weren't previously calculated.)<sup>3</sup>

---

<sup>3</sup>One of the quantities is the number of **degrees of freedom** mentioned earlier. It should be easy to identify. The two "extra" quantities produced are the Regression Sum of Squares given by

$$SSR = \left( \sum (y_i - \bar{y})^2 \right) - S$$

which can be shown to be given by

$$SSR = \left( \sum y_i^2 \right) - N\bar{y}^2 - S$$

and  $F$  given by

$$F = \frac{\frac{SSR}{\nu_R}}{\frac{S}{\nu}}$$

which you may find out about in a statistics class when discussing *Analysis of Variance*.

These instructions are for using Excel. If you are using LibreOffice, skip ahead to page 105.

	A	B	C	D	E	F
13						
14						
15						
16						
17						
18		?	0.99359843	-0.06806879	?	
19		?	0.06208041	0.35668978	?	
20		?	0.99225289	0.42786005	?	
21		?	256.160768	2	?	
22		?	46.8938716	0.36612844	?	
23						
24						
25			Table 2b	Name?		
26						

Figure 7.30: Least Squares Fit Using LINEST

End of Excel specific instructions for Part 3 Go to page 108.

These instructions are for using LibreOffice. If you are using Excel, go back to page 103.

(**LibreOffice instructions**) Here you're going to identify the values produced by LibreOffice's **LINEST()** function by comparing them to the values you got from the formulas.

- 1 Go to the third tabbed page, and use the **LINEST()** function to do a least squares fit.
  - Put the function in **C18**, with *constant*=1 and *stats*=1.
  - Use the function wizard.

The screenshot shows the LibreOffice Calc interface. The formula bar at the top displays the formula: `=LINEST(OriginalData.E6:E9,OriginalData.C6:C9,1,1)`. Below the formula bar, the 'Function Wizard' dialog box is open, showing the function name 'LINEST' and its arguments. The main spreadsheet area shows the output of the LINEST function, which is a 5x3 array of values. The first row of the output is highlighted in blue. The values in the first row are 0.993598431, -0.06806879, and 2. The other rows of the output are: 0.062080412, 0.356689785, ?; 0.992252889, 0.427860049, ?; 256.1607679, 0.366128443, ?; and 46.89387156, 0.366128443, ?.

Output from LINEST		
?	0.993598431	-0.06806879
?	0.062080412	0.356689785
?	0.992252889	0.427860049
?	256.1607679	2
?	46.89387156	0.366128443

Figure 7.31: LINEST using the function wizard

These instructions are for using LibreOffice. If you are using Excel, go back to page 103.

- Choose the “Array” option to show all of the output values.

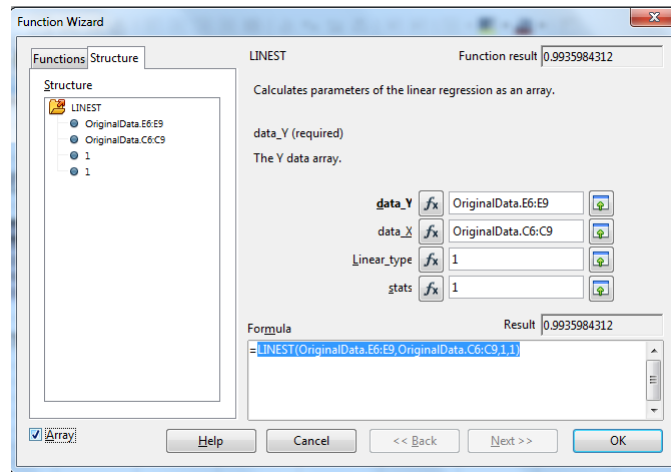


Figure 7.32: Choosing the “Array” option

- Click OK.
- Fill in **B18** to **B22** and **E18** to **E22** with names from the previous page. (You don’t have to fill in the ones that weren’t previously calculated.)<sup>4</sup>

<sup>4</sup>One of the quantities is the number of **degrees of freedom** mentioned earlier. It should be easy to identify. The two “extra” quantities produced are the Regression Sum of Squares given by

$$SSR = \left( \sum (y_i - \bar{y})^2 \right) - S$$

which can be shown to be given by

$$SSR = \left( \sum y_i^2 \right) - N\bar{y}^2 - S$$

and  $F$  given by

$$F = \frac{\frac{SSR}{\nu_R}}{\frac{S}{\nu}}$$

which you may find out about in a statistics class when discussing *Analysis of Variance*.

These instructions are for using LibreOffice. If you are using Excel, go back to page 103.

	A	B	C	D	E	F
13						
14						
15		Output from LINEST				
16						
17						
18		?	0.99359843	-0.06806879	?	
19		?	0.06208041	0.35668978	?	
20		?	0.99225289	0.42786005	?	
21		?	256.160768	2	?	
22		?	46.8938716	0.36612844	?	
23						
24						
25			Table 2b	Name?		
26						

Figure 7.33: Least Squares Fit Using LINEST

End of LibreOffice specific instructions for Part 3

<p><b>Common instructions for Part 3.</b></p>
---

- 3 On the graph, select data series as before, and choose *linear trendline*. See that it fits on top of the best fit line from before, proving it is a least squares fit.

**IT3:** Demonstrate graph as is. Show that the linear trendline is actually a least squares fit.

#### Part 4: Finding Maximum and Minimum Slopes

<p><i>Note that on the “Small Scatter” tab of the spreadsheet, the values in cells <b>D20</b> to <b>D23</b> are <b>point numbers</b>, (i.e. point 1 is the first data point, etc.), and so the values in each of those cells must be integers, and they must be between 1 and <math>N</math>, since each one refers to a <b>data point number</b>.</i></p>
--

- 1 Go to fourth tabbed page, and put in reference to third page (i.e. page using **LINEST()** ) to create meaningful tables.
- 2 Go to fifth tabbed page and add series for lines of maximum and minimum slope using **E20** to **F21** and **E22** to **F23**. Format the series like the line of best fit to have no endpoints.
- 3 Change points referenced in **D20** to **D23** to produce lines of maximum and minimum slope which cross all error bars.
- 4 Look at the cells calculating uncertainty in slope and y intercept and understand how they are calculated using the **index()** function.
- 5 Put in formulas in **C38**, **C39**, **E38**, **E39**, **F38**, **F39** to get endpoints of lines of maximum and minimum slope which go the full width of the graph. Replace these values for the series above. (**E20** to **F21** and **E22** to **F23**).

**IT4:** Demonstrate graph as is. Explain the determination of uncertainties in slope and  $y$ -intercept this way.

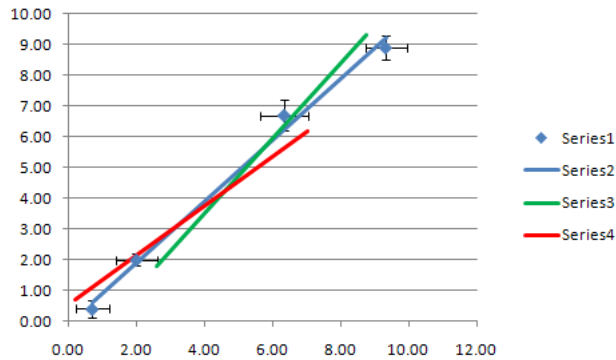


Figure 7.34: Adding lines of maximum and minimum slope

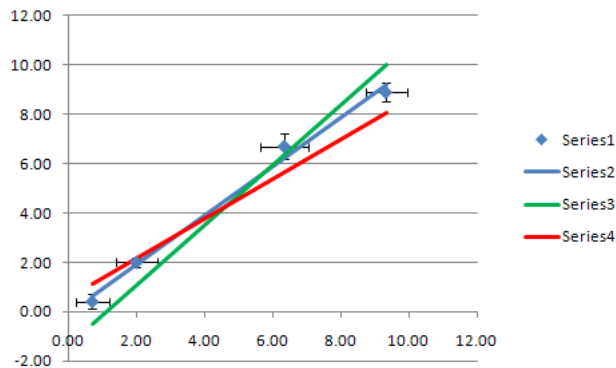


Figure 7.35: Full length lines of maximum and minimum slope

*Once this spreadsheet is set up, you would only usually include the information from either the “small scatter” or the “large scatter” tab in a report; all of the other pages are just in order to make the exercise more organized. (Don’t just print the spreadsheet page, but use it as a guide to figure out what information to include and how it should be organized.)*

### Part 5: Modifying the Spreadsheet for More Data Points

- 1 Insert rows as needed on each tab of the spreadsheet between rows 7 and 8 to allow as many data points as you have in your data from “*Standing Waves on a String*”.
- 2 Copy row 7 and paste it over the inserted rows and the row that was *previously* row 8.
- 3 Modify formulas on the inserted rows on any page where necessary to work correctly with this change. *If you’ve done things correctly, this shouldn’t be necessary.*

**IT5:** Demonstrate spreadsheet as is. Show that the trial numbers and number of data points are correct.

### 7.4.3 Analysis

1. **If not done as a pre-lab task**, on the “*raw data*” tab, fill in your raw data from “*Standing Waves on a String*”.
2. The three tabs following the “*raw data*” tab are for each of the linearizations determined in the “*Exercise on Linearizing Equations*”. **If not done as a pre-lab task**, use the linearization formulas to fill in each of the tables with values calculated from the raw data.
3. For each linearization, replace the first page data with links to your linearization data.
4. For each linearization, adjust the “summary” tab as appropriate, based on whether the scatter is small or large.

### Post-lab Tasks

**T1:** Print off the “summary” tab for each of the three linearizations, making sure you make the appropriate changes for each in rows 4 and 5, and attach them to the lab report, along with the graphs of each of the linearizations.

## 7.5 Recap

By the end of this exercise, you should understand the following terms:

- linear graph
- error bars
- least squares fit
- correlation coefficient
- large scatter of data points
- small scatter of data points

In addition, you should, using a spreadsheet, be able to:

- plot a linear graph
- add error bars
- perform a least squares fit
- show the least squares fit line on the graph with the data

You should also be able to

- determine whether the points on a graph classify as either “small” or “large” scatter, and calculate graphical uncertainties appropriately in either case;
- compare different linearizations of the same function and to explain why one may be preferred over others;
- suggest, based on your graphical uncertainties, whether or not having more precise measuring instruments would improve the results;
- suggest, based on the shape of your graph, whether there is evidence of systematic error in your data.

## 7.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	0	_____	0
Post-lab Questions	0	_____	0
Pre-lab Tasks	4	_____	30
In-lab Tasks	5	_____	50
Post-lab Tasks	1	_____	20

# Chapter 8

## Exercise on Library Research

### 8.1 Purpose

The purpose of this exercise is to introduce you to the library and library research tools.

### 8.2 Introduction

Library research is an essential part of any research project.

### 8.3 Theory

In order for your results to be of any interest or relevance to anyone else, you need to establish things like

- *why* others have done similar research in the past,

**or**

- *why* they haven't;
- *what* others have done in the past,

**and**

- *what* kind of results they have produced.

The library is the most important place to go for all of this information. Even though many journals are now available online, the library has the necessary tools and databases to actually allow you to *find* the kind of articles you're looking for.

*If you find one article that's the kind of thing you're looking for, one way to look for more articles is to use the subject of that article for a further search. Another way is to look at the articles in the bibliography of that one.*

## 8.4 Procedure

### 8.4.1 Preparation

There are no pre-lab questions or tasks for this exercise.

### 8.4.2 Investigation

#### In-lab Questions

There are no specific questions to be completed during the lab time.

#### In-lab Tasks

**IT1:** Show that you have completed the in-lab assignment during the presentation.

The lab time is free for you to use after the presentation is finished. Your task for the week is to follow the instructions given for each of the journal articles listed below.

### Articles

Here are the specific articles you're going to investigate. You're given different information about each one to make the searches a bit different. The articles themselves are about topics which are pertinent to the labs or your university experience.

- A psychology article by Justin Kruger and David Dunning in 1999 about how people perceive their own abilities
- An article titled “Experimental Outcomes of Laboratory Instruction in Elementary College Physics” from the American Journal of Physics in 1952 studying what people learn from physics labs
- An article in “Physics in Canada” by Amy C. Rowat in 2005 about the attractiveness of physics to potential students

### Using Online Databases

1. For each of the journal articles listed, use the online databases to find it.
2. Explain the searches you performed; specifically:
  - indicate which database you used,
  - list search criteria,
  - show list of results,
  - explain how you identified the correct article,
  - state whether the article is available online, or only in print.

### Using Primo and/or databases

3. Search Primo and/or databases to try to find the journals.
4. State which are in the tri-university system (but not online). Explain the Primo or database search you used to find them.
5. Identify any journal which is not in the tri-university system (or online). Explain the Primo or database search you used to determine this.

### Research

6. For all articles identified:

- Print the bibliographic information for the article.(i.e. article name, author(s), journal name, volume and number, year published, page numbers).
- Print the first page of the article.
- In one or two sentences, state one of the major conclusions of the article in your own words. Highlight the location of that information on the first page of the article.

### 8.4.3 Analysis

#### Post-lab Questions

**Q1 to Q3:** Hand in the material required above for each article. Arrange the information by article, in the order of items above. (Each article counts as one question.)

## 8.5 Recap

By the end of this exercise, you should understand the following terms:

- refereed journal
- online database

In addition, you should be able to find

- journal articles
- books

using the library resources.

## 8.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	0	_____	0
Post-lab Questions	3	_____	50
Pre-lab Tasks	0	_____	0
In-lab Tasks	1	_____	50
Post-lab Tasks	0	_____	0



# Chapter 9

## Exercise on Plotting Curves Using a Spreadsheet

### 9.1 Purpose

The purpose of this exercise is to become familiar with using a spreadsheet to produce curves of any function desired.

### 9.2 Introduction

Using a spreadsheet to graph data is convenient once you know what you're doing, and even plotting error bars and straight lines is relatively simple. However, once you need to plot data which is not linear, things become a bit more complicated.

### 9.3 Theory

Consider the data shown in Figure 9.1.

First of all, the graph does not have error bars. However it is also not smooth. *Dot-to-dot drawings do not usually belong in scientific reports; smooth curves are usually more appropriate.* The following discussion should help you to use a spreadsheet to produce non-linear plots.

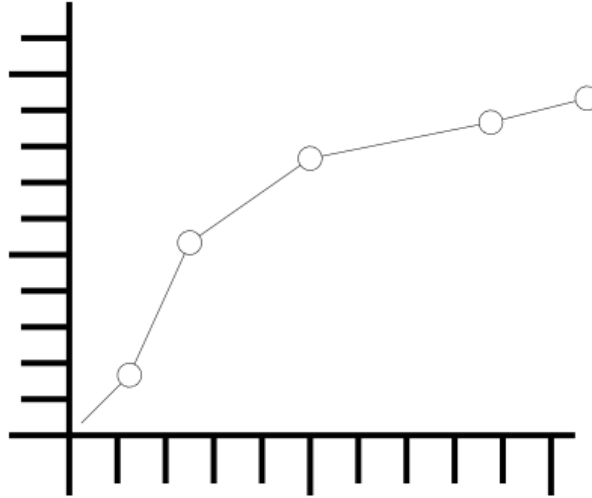


Figure 9.1: Logarithmic Fit the Wrong Way

### 9.3.1 Displaying Curves

You have previously seen how a *line* can be drawn by simply connecting its endpoints. Similarly, a *curve* can be *approximated* by a series of very short line segments between points along the curve. If the points are close enough, the line will look smooth. If we wish to plot a curve in a spreadsheet, proceed as follows:

1. Instead of just using two values,  $x_{min}$  and  $x_{max}$ , for an  $x$ -series, create a series of values  $x_i, i = 0 \dots N$  where

$$x_i = x_{min} + i \left( \frac{x_{max} - x_{min}}{N} \right)$$

so that

$$x_0 = x_{min}$$

and

$$x_N = x_{max}$$

and for each point

$$y_i = f(x_i)$$

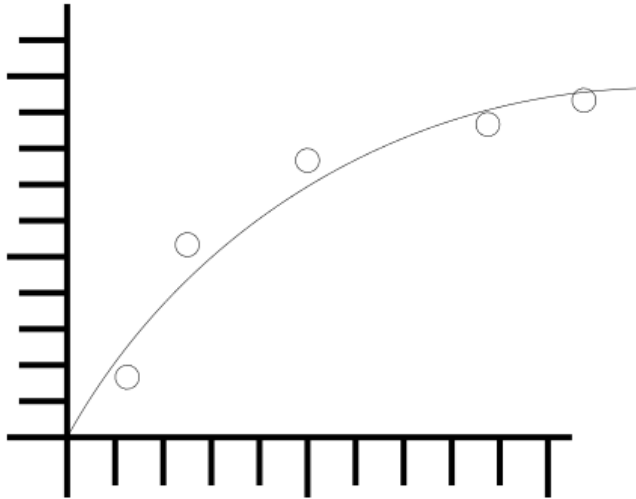


Figure 9.2: Logarithmic Fit the Right Way

Note that even though the  $x$  values for the first and last point are taken from the original data, the  $y$  values are not, since the purpose is to find points on the *curve*.

A value of 100 for  $N$  should be sufficient to make a smooth curve. If not, make it bigger. You can try making it smaller if that many points aren't needed.

2. For each of the  $x_i$ , calculate the corresponding  $y_i$  value from the curve equation and add this series to the graph.
3. Remove the markers and add lines for this series. All points in this series will thereby be joined with line segments. You should be able to produce a graph such as in Figure 9.2. (Error bars have been left off for simplicity. They can be produced in the usual way.)

Note that in the graph of Figure 9.2, none of the data points actually fall on the curve. This is often the case.

### 9.3.2 Piecewise Defined Functions

In some cases, a curve must be made from data which must be fit to different equations in different regions. In this case the trick comes in trying to join the two fit equations smoothly.

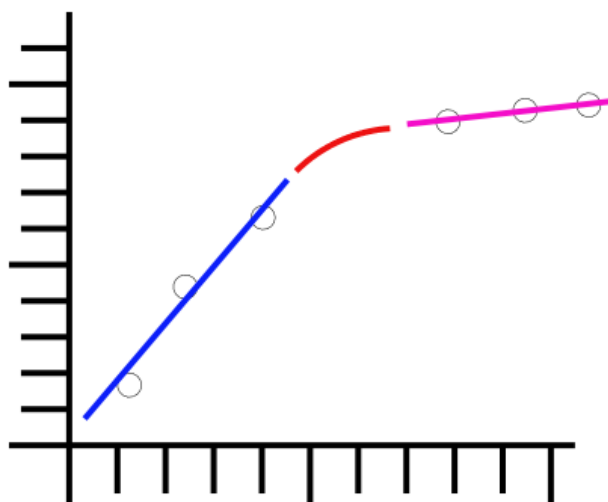


Figure 9.3: Piecewise Defined Function

In this case,

$$f(x) = \begin{cases} F_1(x), & \text{if } x \leq x_1; \\ F_2(x), & \text{if } x \geq x_2. \end{cases}$$

where  $x_2 > x_1$ . To make the function  $f(x)$  *continuous*, we require that

$$f(x_1) = F_1(x_1)$$

and

$$f(x_2) = F_2(x_2)$$

To produce a *smooth* fit, then in addition we require that

$$f'(x_1) = F_1'(x_1)$$

and

$$f'(x_2) = F_2'(x_2)$$

These 4 equations can be solved exactly by a polynomial in 4 unknowns, i.e. an equation of the form

$$f(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$$

and therefore

$$f'(x) = 3\alpha x^2 + 2\beta x + \gamma$$

To solve this, we need to set up a system of equations as follows: If we let

$$\mathcal{A} = \begin{pmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ 3x_1^2 & 2x_1 & 1 & 0 \\ 3x_2^2 & 2x_2 & 1 & 0 \end{pmatrix}$$

and

$$\mathcal{B} = \begin{pmatrix} F_1(x_1) \\ F_2(x_2) \\ F_1'(x_1) \\ F_2'(x_2) \end{pmatrix}$$

Then it should be clear that

$$\mathcal{A}X = \mathcal{B}$$

or

$$X = \mathcal{A}^{-1}\mathcal{B}$$

where

$$X = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

( $F_1'(x_1)$  can be determined by using  $x_1$  and the fit point immediately to its left, and  $F_2'(x_2)$  can be determined by using  $x_2$  and the fit point immediately to its right.) This system can then be solved using the **matrix invert** and **multiply** features of a spreadsheet.

Note: This will produce a *smooth* graph; whether it produces an *accurate* graph remains to be seen, (although smooth transitions are more common than others). This should produce a graph such as in Figure 9.3. Error bars have not been shown so that the smoothness of the curve can be seen easily. As before, error bars should be included unless you are specifically told to omit them.

## 9.4 Procedure

### 9.4.1 Preparation

#### Pre-lab Tasks

**PT1:** Bring the *completed* spreadsheet, (i.e. the one that you have filled in), from the the “*Exercise on Graphing and Least Squares Fitting using a Spreadsheet*” on a flash drive or save it in your WLU account.

**PT2:** Given  $T$  and  $\Delta T$ , give the equation for  $\Delta\sqrt{T}$ . This was done in the “*Exercise on Linearizing Equations*”.

$$\Delta\sqrt{T} =$$

**PT3:** From the linearization, give the equations for  $\mu$  and  $\Delta\mu$  from the graph of  $\lambda$  vs.  $\sqrt{T}$ . This was done in the “*Exercise on Linearizing Equations*”.

$$\mu =$$

$$\Delta\mu =$$

### 9.4.2 Investigation

#### In-lab Tasks

#### Part 1: Plotting the points

The first step is to plot the points on a graph. The method for this should be familiar after the “*Exercise on Graphing and Least Squares Fitting using a Spreadsheet*”.

1. Load the spreadsheet from the lab page, and go to the appropriate tab.
2. Insert a graph in the box marked Graph 1 (non-linearized) on the page.
  - Make sure it is an  $xy$  (scatter) graph.
  - The  $x$  series should be **C5** to **C9**.
  - The  $y$  series should be **E5** to **E9**.
3. Add both  $x$  and  $y$  error bars.
  - Use series **D5** to **D9** for  $x$  error bars.
  - Use series **F5** to **F9** for  $y$  error bars.

**IT1:** Demonstrate the graph as is.

### Part 2: Linearizing the equation

The next step is to linearize the data in order to find a fit equation. The method for this should be familiar after the “*Exercise on Graphing and Least Squares Fitting using a Spreadsheet*”.

1. Replace the numbers with formulas in **C14** to **F14** in Table 2 for linearized data. (The pre-lab task **PT2** should help you with the formulas for column **D**.) When they are correct, *copy* the formulas and *paste special* in **C15** to **C18** to finish the table.
2. In Table 3 use the *LINEST* function to do a least squares fit using the *linearized* data.
3. Insert a graph in the box marked Graph 2 (linearized) on the page.
  - Make sure it is an  $xy$  (scatter) graph.
  - The  $x$  series should be **C14** to **C18**.
  - The  $y$  series should be **E14** to **E18**.

4. Add error bars to the graph.
  - Use series **D14** to **D18** for  $x$  error bars.
  - Use series **F14** to **F18** for  $y$  error bars.
5. Select the data series, and choose a linear trendline.

**IT2:** Demonstrate the graph as is.

### Part 3: Creating points on the curve

1. Put references to the appropriate *LINEST* results in **D41**, **D42**, **F41**, and **F42** according to the linearization equations.
2. Note that formulas are already filled in for **D49** and **D50**, and a number of steps for the curve plot has been chosen. *Hint: In the linear fit, what are  $x$  and  $y$ ?*

From the linearization, we have the equation

$$Y = mX + b$$

So if we replace  $X$  and  $Y$  with the correct quantities, we get

$$\_ = m\_ + b \tag{9.1}$$

(Hint: You can get  $Y$  and  $X$  from **PT3** .)

3. Replace the value in **E56** with the formula using the results of the linearization. Don't forget the  $y$ -intercept.<sup>1</sup> Note that the value for  $\lambda$  is from the curve, not from the original data.

---

<sup>1</sup>Even when the  $y$ -intercept or the slope *should* be zero, use the actual value you get from the linearization for plotting the curve. The curve represents the fit you *got*, not necessarily the one you *expected*.

4. Put a formula in **C57** which can be copied to get a counter, if it hasn't been done already.

Now, by generating several values for  $T$  and using Equation 9.1, you can generate a bunch of values which can be plotted on the original graph. This is the only new step.

5. In **D57**, put the formula for calculating the next value of  $T$ ;

$$T_i = T_{min} + i(T_{max} - T_{min}) / N_{steps}$$

(Be sure to use absolute references as appropriate so that this formula can be copied.)

6. *Copy* the formula in **E57** and *paste special* in **E58**.
7. *Copy* the formulas in **C57** to **E57**, and *paste special* in **C58** to **C70** to finish the table.
8. Add a series to your non-linear graph using the values from **D56** to **E71** to show the curve produced by the fit.
9. Format the data series to get a line with no endpoints.

**IT3:** Demonstrate the graph as is.

#### Part 4: Refining the curve

1. Try changing the number of points for the curve, by changing the number of steps and deleting or adding rows in Table 4 as needed. (When adding rows, you will need to *copy* formulas.)

**IT4:** Demonstrate graph with more points. Show how many are needed to make it smooth.

### 9.4.3 Analysis

#### Post-lab Questions

There are no post-lab requirements since next week final marks will be calculated.

## 9.5 Recap

By the end of this exercise, you should understand how to display any curve on a spreadsheet graph.

## 9.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	0	_____	0
Post-lab Questions	0	_____	0
Pre-lab Tasks	3	_____	30
In-lab Tasks	4	_____	70
Post-lab Tasks	0	_____	0

# Appendix A

## Information about Measuring Instruments

Fill this in as you use new measuring instruments so you will have a reliable reference. Put frequently used instruments in Table A.1 and experiment-specific ones in Table A.2.

ref. #	measuring instrument	precision measure	zero error	units
A1	stopwatch			
A2	GL100R balance			
A3	vernier caliper			
A4	micrometer caliper			
A5				
A6				
A7				

Table A.1: Measuring instrument information

ref. #	measuring instrument	precision measure	zero error	units
B1				
B2				
B3				
B4				
B5				
B6				
B7				
B8				
B9				
B10				
B11				
B12				
B13				
B14				
B15				

Table A.2: Measuring instrument information (continued)

# Appendix C

## Lab Checklist

This marking checklist will be used for lab reports this term. You need to print one off and attach it to each lab report you hand in. Lab reports will be marked as follows:

- Start with 90

For items *not* in italics

- Subtract 1 for each  $\sim$ .
- Subtract 2 for each  $-$ .

For items *in* italics

- Subtract 3 for each  $\sim$ .
- Subtract 6 for each  $-$ .

Note the importance of items in italics. These are very important in a report, and so are weighted accordingly.

The other 10 marks will be based on how well the post-lab discussion questions were answered within the text of the report. *Remember that the answers to these questions should be an integral part of the report, not merely an afterthought.*

## Lab Format Checklist (V3.0g)

## A. General

1. Your own work \_\_\_\_\_
2. Complete \_\_\_\_\_
3. Clear and appropriate "Purpose" \_\_\_\_\_
4. Flows \_\_\_\_\_
5. Did not require help on or after due date \_\_\_\_\_
6. Correct grammar \_\_\_\_\_
7. Correct spelling \_\_\_\_\_
8. Complete sentences where required \_\_\_\_\_
9. Legible \_\_\_\_\_
10. Professionally presented \_\_\_\_\_
11. Properly identified (e.g. name, partner) \_\_\_\_\_
12. On time \_\_\_\_\_
13. Checklist included \_\_\_\_\_
14. Template included \_\_\_\_\_

---

B. *Plagiarism Avoidance*

1. *Data only shared with partner(s)* \_\_\_\_\_
2. *Individual choice of sample data* \_\_\_\_\_
3. *Individual formatting* \_\_\_\_\_
4. *Individual structure of text* \_\_\_\_\_

C. Data

1. Neat \_\_\_\_\_
2. Table column headings informative \_\_\_\_\_
3. Units given \_\_\_\_\_
4. Uncertainties given \_\_\_\_\_
5. Reasonable values \_\_\_\_\_
6. Reasonable uncertainties \_\_\_\_\_
7. Correct number of significant figures \_\_\_\_\_
8. Tables labeled (e.g. "Sample 1 Data") \_\_\_\_\_
9. Tables given numbers (e.g. "Table #2") \_\_\_\_\_

D. Calculations and Results

1. Any required derivations done correctly \_\_\_\_\_
2. Analysis explained where needed \_\_\_\_\_
3. Correct formulas used \_\_\_\_\_
4. Sample calculations shown where needed \_\_\_\_\_
5. All required values calculated \_\_\_\_\_
6. Uncertainties included \_\_\_\_\_
7. Units included \_\_\_\_\_
8. Correct number of significant figures \_\_\_\_\_
9. Appropriate use of standard form \_\_\_\_\_
10. Theoretical or reasonable value \_\_\_\_\_
11. Agreement of experiment with theory \_\_\_\_\_

## E. Linearizations

1. Independent ( $x$ ) variable identified \_\_\_\_\_
2. Dependent ( $y$ ) variable identified \_\_\_\_\_
3. Expected slope identified \_\_\_\_\_
4. Expected  $y$ -intercept identified \_\_\_\_\_

## F. Graphs

1. Title meaningful \_\_\_\_\_
2. Correct graph type and orientation \_\_\_\_\_
3. Background colour and grid appropriate \_\_\_\_\_
4. Plotting data in table \_\_\_\_\_
5. Axis labels meaningful \_\_\_\_\_
6. Correct axis units \_\_\_\_\_
7. Points not connected \_\_\_\_\_
8. Error bars in both dimensions or note if  
too small \_\_\_\_\_
9. Error bars correct size \_\_\_\_\_
10. Line of best fit shown without markers \_\_\_\_\_
11. Number given (e.g. "Graph #3") \_\_\_\_\_

## G. Least Squares Fits

1. Points used for fit clearly identified \_\_\_\_\_
2. Results given meaningful names \_\_\_\_\_
3. Correct units for slope and intercept \_\_\_\_\_
4. Correct indication of "large" or "small"  
scatter \_\_\_\_\_
5. Reasonable points for maximum and min-  
imum slopes if scatter "large" \_\_\_\_\_

---

#### H. *Error Discussion*

1. *Sources listed are significant* \_\_\_\_\_
2. *Sources are prioritized* \_\_\_\_\_
3. *Systematic error consequences* \_\_\_\_\_
4. *Evidence: i.e. test or bound* \_\_\_\_\_
5. *Reasonable suggestions for improvement* \_\_\_\_\_

#### I. *Conclusions*

1. *Relate to purpose* \_\_\_\_\_
2. *Major results stated* \_\_\_\_\_
3. *Comparisons made where appropriate* \_\_\_\_\_
4. *Agreement noted when found* \_\_\_\_\_
5. *% difference calculated only when no agreement* \_\_\_\_\_

#### J. *References*

1. *Source(s) of constants listed* \_\_\_\_\_

