Uncertainty Calculations - Subtraction
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For the following examples, the values of $x = 2 \pm 1$ and $y = 32.0 \pm 0.2$ will be used.

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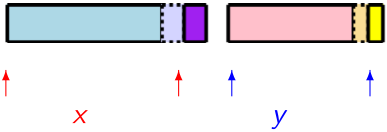
  When subtracting numbers, we add uncertainties.

Graphically,



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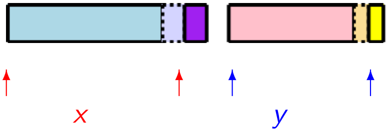

- To subtract, we can reverse the direction of y.

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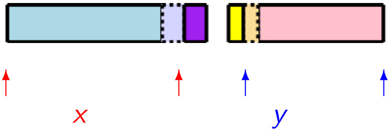

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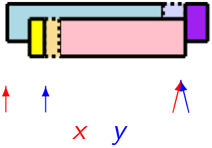

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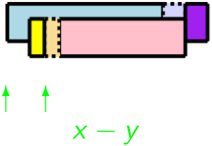

- This is the nominal value of $x - y$.

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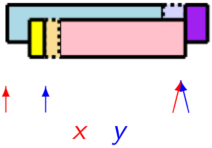

- To find the minimum value of $x - y$, start with the nominal value of $x - y$.

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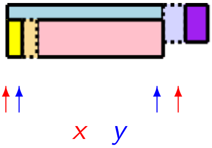
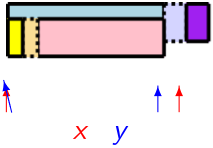


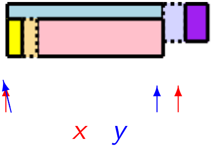
- First we move y by a distance Δx.

Graphically,



- Then we need to move our left pointer by Δy.

Graphically,



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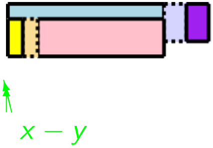

- This is the minimum value of $x - y$.
- It has moved from the nominal value by $\Delta x + \Delta y$.

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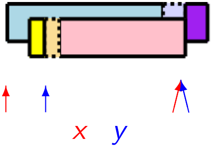
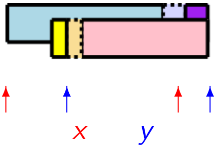


- To find the maximum value of $x - y$, start with the nominal value of $x - y$.

Graphically,



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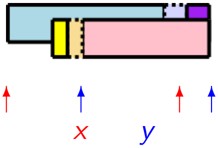

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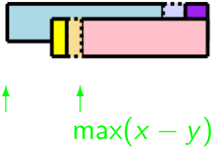

- This is the maximum value of $x - y$.
- It has moved from the nominal value by a distance $\Delta x + \Delta y$.

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$$(2 \pm 1) - (32.0 \pm 0.2) = (2 - 32.0) \pm (1 + 0.2) = -30.0 \pm 1.2$$

2. Uncertainties in final results are usually expressed to one significant figure, so the above result becomes

$$(2 \pm 1) - (32.0 \pm 0.2) = -30.0 \pm 1.2 = -30 \pm 1$$