Uncertainties by Inspection
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It is the difference between the nominal value and the maximum or minimum value.

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(The approximately equals sign is to reflect the fact that these two values may not be quite the same, depending on the function f.)

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Note that this value will be slightly different than the value given by $f_{max} - f$.

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If we have a value of $d = 1.0 \pm 0.1\ \mathrm{cm}$, then $\Delta V = 0.173\ \mathrm{cm}^3$ by this method. Rounded to one significant figure gives $\Delta V \approx 0.2\ \mathrm{cm}^3$ as the value to be quoted.

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