# Uncertainty Calculations - Functions
# Wilfrid Laurier University

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# Calculations with uncertainties

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For the following examples, the values of $x = 2 \pm 1$ and $y = 32.0 \pm 0.2$ will be used.

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Functions with uncertainties

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**Notice that these two different methods may give slightly different results, so that's why there's an "approximately equal" sign.**

# Uncertainty in a function - algebra

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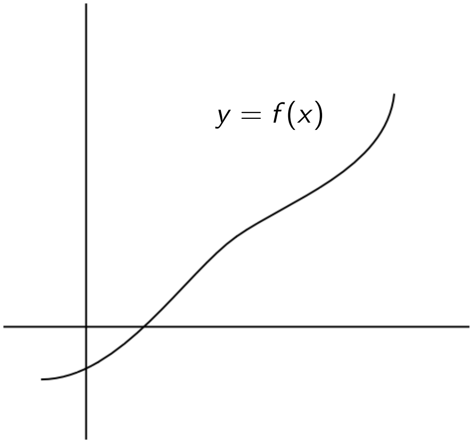
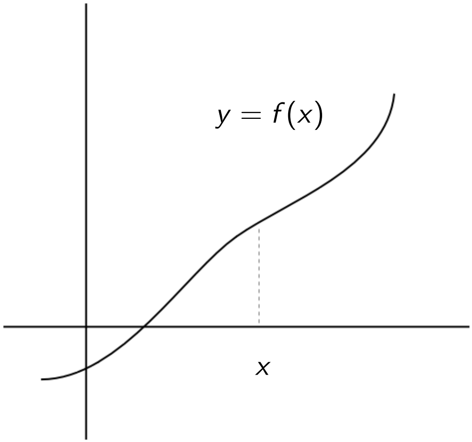
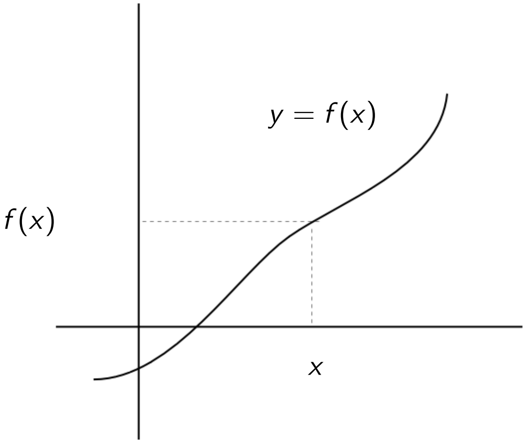
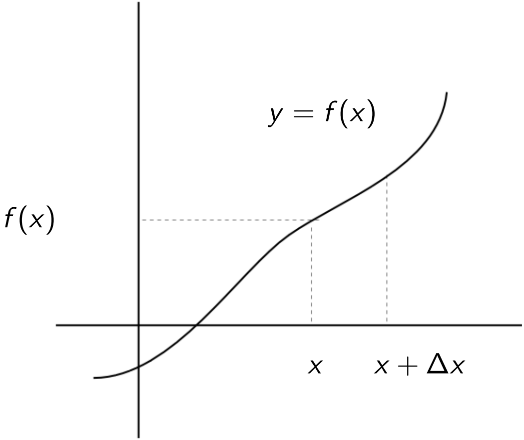
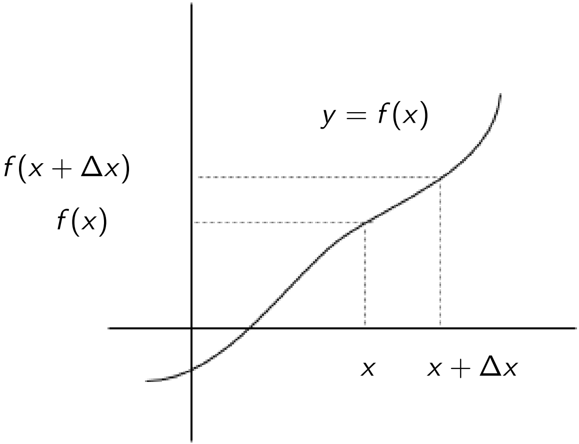
Determining uncertainties algebraically is easiest illustrated graphically.

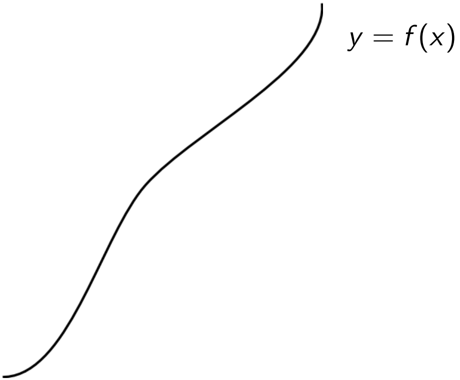

$y = f(x)$


y = f(x)
x


$y = f(x)$
$f(x)$
$x$


$y = f(x)$
$f(x)$
$x$
$x + \Delta x$


$y = f(x)$
$f(x+\Delta x)$
$f(x)$
$x$
$x+\Delta x$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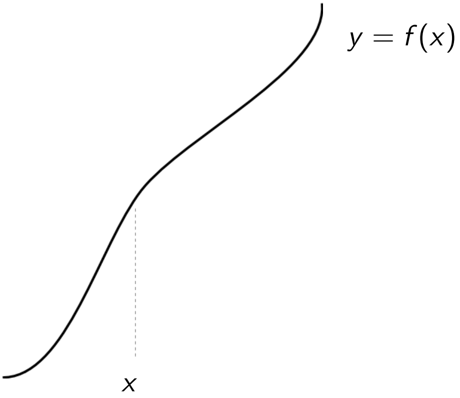
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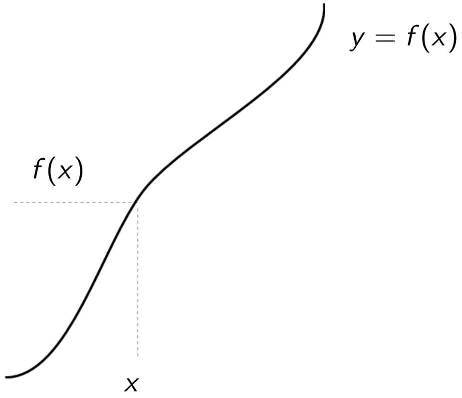
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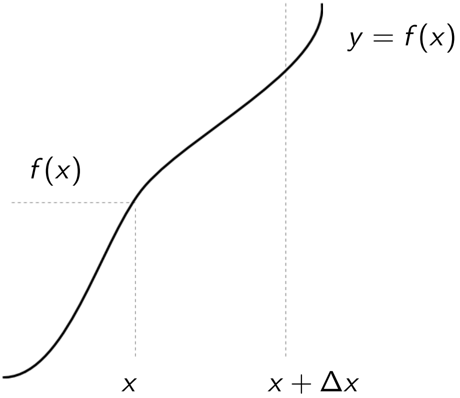
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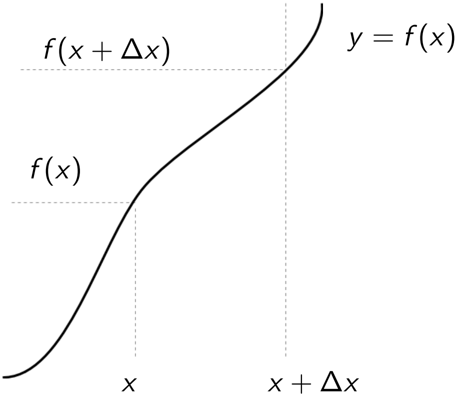
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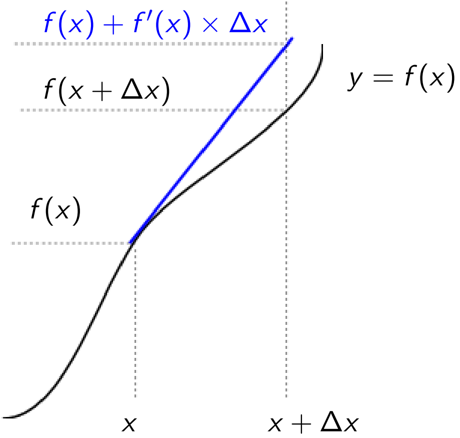
$f(x) + f'(x) \times \Delta x$
$f(x + \Delta x)$
$y = f(x)$
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$x$
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$\rightarrow \Delta z \approx \frac{1}{2\times 1.4} \times 1 \approx 0.4$

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$$\begin{aligned}\sqrt{2 \pm 1} &\approx \sqrt{2} \pm \left(\sqrt{2+1} - \sqrt{2}\right)\\ &\approx 1.4 \pm (1.7 - 1.4)\\ &\approx 1.4 \pm 0.3\end{aligned}$$

# Recap - continued

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2. By algebra, $\Delta z \approx |f'(x)|\, \Delta x$

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   For example,

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   since the derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$