Uncertainty Calculations - Functions Wilfrid Laurier University

Terry Sturtevant

Wilfrid Laurier University

May 13, 2013

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

DQ C

Calculations with uncertainties

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

◆□▶ ◆□▶ ◆三▶ ◆三▶ ・三 ・ への◇

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Following is a discussion of uncertainty in **functions of one** or more variables.

◆□▶ ◆□▶ ◆三▶ ◆三▶ ・三 ・ への◇

Calculations with uncertainties

When quantities with uncertainties are combined, the results have uncertainties as well.

Following is a discussion of uncertainty in **functions of one** or more variables.

For the following examples, the values of $x = 2 \pm 1$ and $y = 32.0 \pm 0.2$ will be used.

◆□▶ ◆□▶ ◆三▶ ◆三▶ ・三 ・ への◇

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainties in Functions

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

▲□▶ ▲圖▶ ▲注▶ ▲注▶ — 注

DQC2

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainties in Functions

Functions with uncertainties

イロト イポト イヨト イヨト

= nar

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Two methods of calculating

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

< ロ > < 同 > < 回 > < 回 > < 回 > <

= nar

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Two methods of calculating

For functions, there are two ways of calculating uncertainties:

イロト イポト イヨト

-

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Two methods of calculating

For functions, there are two ways of calculating uncertainties:

1. by inspection

<ロト < 同ト < ヨト < ヨト -

-

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Two methods of calculating

For functions, there are two ways of calculating uncertainties:

- 1. by inspection
- 2. by algebra

イロト イポト イヨト

DQ C

-

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Two methods of calculating

For functions, there are two ways of calculating uncertainties:

- 1. by inspection
- 2. by algebra

Both ways will be discussed.

イロト イポト イヨト

-

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainty in a function - by inspection

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

< ロ > < 同 > < 回 > < 回 > < 回 > <

DQC2

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainty in a function - by inspection

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

<ロト < 同ト < ヨト < ヨト -

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainty in a function - by inspection

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

 $x = 2 \pm 1$

・ロト ・ 一 ト ・ ヨ ト ・ 日 ト

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainty in a function - by inspection

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

 $x = 2 \pm 1$

 $\rightarrow x$ can be as *big* as 3

・ロト ・ 一 ト ・ ヨ ト ・ 日 ト

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

 $x = 2 \pm 1$

 $\rightarrow x$ can be as *big* as 3

Therefore, if $z = \sqrt{x}$

・ロト ・ 一 ト ・ ヨ ト ・ 日 ト

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

 $x = 2 \pm 1$

 $\rightarrow x$ can be as *big* as 3

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z=\sqrt{2}pprox 1.4$

・ロト ・ 同ト ・ ヨト ・ ヨト

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

$$x = 2 \pm 1$$

 $\rightarrow x$ can be as *big* as 3

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z = \sqrt{2} \approx 1.4$

Then the maximum value of $z=\sqrt{3}pprox 1.7$

・ロト ・ 同ト ・ ヨト ・ ヨト

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

$$x = 2 \pm 1$$

 $\rightarrow x$ can be as *big* as 3

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z = \sqrt{2} \approx 1.4$

Then the maximum value of $z=\sqrt{3}pprox 1.7$

$$ightarrow \Delta z pprox 1.7 - 1.4 = 0.3$$

・ロト ・ 同ト ・ ヨト ・ ヨト

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

$$x = 2 \pm 1$$

 $\rightarrow x$ can be as *big* as 3

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z = \sqrt{2} \approx 1.4$

Then the maximum value of $z=\sqrt{3}pprox 1.7$

$$\rightarrow \Delta z \approx 1.7 - 1.4 = 0.3$$

In general,

・ロト ・ 同ト ・ ヨト ・ ヨト

The uncertainty in a quantity essentially means the difference between its maximum value and its nominal value.

$$x = 2 \pm 1$$

 $\rightarrow x$ can be as *big* as 3

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z = \sqrt{2} \approx 1.4$

Then the maximum value of $z=\sqrt{3}pprox 1.7$

$$\rightarrow \Delta z \approx 1.7 - 1.4 = 0.3$$

In general,

$$\Delta(f(x)) \approx f_{max}(x) - f(x)$$

・ロト ・ 同ト ・ ヨト ・ ヨト

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

・ロト ・ 一 ト ・ ヨ ト ・ 日 ト

DQC2

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

イロト イポト イヨト イヨト

= nar

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

イロト イポト イヨト イヨト

= nar

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

Therefore, if $z = \sqrt{x}$

イロト イポト イヨト イヨト

= ~ ~ ~

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z = \sqrt{2} \approx 1.4$

イロト イポト イヨト イヨト

э.

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z=\sqrt{2}pprox 1.4$

Then the minimum value of $z=\sqrt{1}=1$

イロト イポト イヨト イヨト

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z = \sqrt{2} \approx 1.4$

Then the minimum value of $z = \sqrt{1} = 1$

 $ightarrow \Delta z pprox 1.4 - 1.0 = 0.4$

▲ロ ▶ ▲局 ▶ ▲ 国 ▶ ▲ 国 ▶ ● ● ● ● ● ●

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z=\sqrt{2}pprox 1.4$

Then the minimum value of $z = \sqrt{1} = 1$

$$ightarrow \Delta z pprox 1.4 - 1.0 = 0.4$$

In general,

イロト イポト イヨト イヨト

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as *small* as 1

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z=\sqrt{2}pprox 1.4$

Then the minimum value of $z = \sqrt{1} = 1$

$$ightarrow \Delta z pprox 1.4 - 1.0 = 0.4$$

In general,

$$\Delta(f(x)) \approx f(x) - f_{min}(x)$$

イロト イポト イヨト イヨト

The uncertainty in a quantity essentially means the difference between its nominal value and its minimum value.

 $x = 2 \pm 1$

ightarrow x can be as small as 1

Therefore, if $z = \sqrt{x}$

Then the nominal value of $z=\sqrt{2}pprox 1.4$

Then the minimum value of $z=\sqrt{1}=1$

$$ightarrow \Delta z pprox 1.4 - 1.0 = 0.4$$

In general,

$$\Delta(f(x)) \approx f(x) - f_{min}(x)$$

Notice that these two different methods may give slightly different results, so that's why there's an "approximately equal" sign.

・ロト ・ 同ト ・ ヨト ・ ヨト

Two methods of calculating Uncertainty in a function - by inspection **Uncertainty in a function - algebra**

Uncertainty in a function - algebra

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

イロト イポト イヨト イヨト

 \equiv

DQC2

Two methods of calculating Uncertainty in a function - by inspection Uncertainty in a function - algebra

Uncertainty in a function - algebra

Determining uncertainties algebraically is easiest illustrated graphically.

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

・ロト ・ 一 ト ・ ヨ ト ・ 日 ト

-

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



<ロ> <0</p>

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



<ロ> <0</p>

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



<ロ> <0</p>

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



《曰》《卽》《臣》《臣》

€ 990

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra



Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

€ 990

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

 $x = 2 \pm 1$

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

《日》《圖》《臣》《臣》

€ 990

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

 $x = 2 \pm 1$ Therefore, if $z = \sqrt{x}$

<□ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

 $x = 2 \pm 1$ Therefore, if $z = \sqrt{x}$ $\Delta z \approx |f'(x)| \Delta x$

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

$$x = 2 \pm 1$$

Therefore, if $z = \sqrt{x}$
$$\Delta z \approx |f'(x)| \Delta x$$
$$\approx \left|\frac{1}{2\sqrt{x}}\right| \Delta x$$

€ 990

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

 $x = 2 \pm 1$ Therefore, if $z = \sqrt{x}$ $\Delta z \approx |f'(x)| \Delta x$ $\approx \left|\frac{1}{2\sqrt{x}}\right| \Delta x$

Then the value of $\Delta z pprox \left| rac{1}{2\sqrt{x}} \right| 1$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

nac

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

 $x = 2 \pm 1$ Therefore, if $z = \sqrt{x}$ $\Delta z \approx |f'(x)| \Delta x$ $\approx \left|\frac{1}{2\sqrt{x}}\right| \Delta x$

Then the value of $\Delta z \approx \left| rac{1}{2\sqrt{x}} \right| 1$

 $ightarrow \Delta z pprox rac{1}{2 imes 1.4} imes 1 pprox 0.4$

・ロト ・ 同ト ・ ヨト ・ ヨト

DQ C

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

By inspection,

▲口 → ▲圖 → ▲ 臣 → ▲ 臣 → -

€ 990

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

By inspection,

 $\Delta(f(x)) \approx f_{max}(x) - f(x)$

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

By inspection, $\Delta(f(x)) \approx f_{max}(x) - f(x)$

or

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

By inspection, $\Delta(f(x)) \approx f_{max}(x) - f(x)$ or $\Delta(f(x)) \approx f(x) - f_{min}(x)$

イロト イポト イヨト イヨト ニヨー

Sac

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

By inspection, $\Delta(f(x)) \approx f_{max}(x) - f(x)$ or $\Delta(f(x)) \approx f(x) - f_{min}(x)$ By algebra,

・ロト ・ 同ト ・ ヨト ・ ヨト

DQ C

Calculations with Uncertainties	Two methods of calculating
Uncertainties in Functions	Uncertainty in a function - by inspection
Recap	Uncertainty in a function - algebra

By inspection, $\Delta (f(x)) \approx f_{max}(x) - f(x)$ or $\Delta (f(x)) \approx f(x) - f_{min}(x)$ By algebra, $\Delta f(x) \approx |f'(x)| \Delta x$

Recap

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

<□ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Recap

1. By inspection, $\Delta(f(x)) \approx f_{max}(x) - f(x)$

《曰》 《圖》 《臣》 《臣》

Recap

1. By inspection, $\Delta(f(x)) \approx f_{max}(x) - f(x)$ For example,

《曰》 《圖》 《臣》 《臣》

= nac

Recap

1. By inspection, $\Delta(f(x)) \approx f_{max}(x) - f(x)$ For example,

$$egin{aligned} \sqrt{2\pm1} &\approx \sqrt{2} \pm \left(\sqrt{2+1} - \sqrt{2}
ight) \ &pprox 1.4 \pm (1.7 - 1.4) \ &pprox 1.4 \pm 0.3 \end{aligned}$$

《曰》 《圖》 《臣》 《臣》

= nac

Recap - continued

Terry Sturtevant Uncertainty Calculations - Functions Wilfrid Laurier University

<□ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Recap - continued

2. By algebra, $\Delta z \approx |f'(x)| \Delta x$

《口》 《圖》 《臣》 《臣》

Recap - continued

2. By algebra, $\Delta z \approx |f'(x)| \Delta x$ For example,

<ロト < 部ト < 目ト < 目ト

Recap - continued

2. By algebra, $\Delta z \approx |f'(x)| \Delta x$ For example,

$$\sqrt{2 \pm 1} \approx \sqrt{2} \pm \left(\frac{1}{2 \times \sqrt{2}} \times 1\right)$$
$$\approx 1.4 \pm 0.4$$

<ロト < 部ト < 目ト < 目ト

Recap - continued

2. By algebra, $\Delta z \approx |f'(x)| \Delta x$ For example,

$$\sqrt{2 \pm 1} \approx \sqrt{2} \pm \left(\frac{1}{2 \times \sqrt{2}} \times 1\right)$$
$$\approx 1.4 \pm 0.4$$

since the derivative of \sqrt{x} is $\frac{1}{2\sqrt{x}}$

<ロト < 部ト < 目ト < 目ト

 \equiv

nac