Uncertainty Calculations - Division
Wilfrid Laurier University

Terry Sturtevant

Wilfrid Laurier University

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For the following examples, the values of $x = 2 \pm 1$ and $y = 32.0 \pm 0.2$ will be used.

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The proportional uncertainty in the inverse of a number is the same as the proportional uncertainty in the number.

Division with Multiple Uncertainties

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What if both numbers have uncertainties?

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Remember that if x or y can be negative, we'll need absolute value signs around the appropriate terms, since uncertainty contributions should always be given as positive numbers.

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$$\frac{1}{32.0 \pm 0.2} = \frac{1}{32.0} \pm \left(\frac{0.2}{32.0}\right)\left(\frac{1}{32.0}\right)$$
$$= 0.03125 \pm (.00625)\, 0.03125$$
$$\approx 0.03125 \pm 0.00019$$

Recap - continued

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2. When dividing numbers, we add the *proportional* uncertainties.

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$$\frac{(2 \pm 1)}{(32.0 \pm 0.2)} = \left(\frac{2}{32.0}\right) \pm \left(\frac{2}{32.0}\right)\left(\frac{1}{2} + \frac{0.2}{32.0}\right)$$
$$= 0.0625 \pm 0.0625\,(0.5 + 0.00625)$$
$$= 0.0625 \pm 0.0316$$

3. Uncertainties in final results are usually expressed to one significant figure, so the above result becomes

$$0.0625 \pm 0.0316 = 0.06 \pm 0.03$$