

PC131 Lab Manual
Wilfrid Laurier University

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Chapter 11

Exercise on Repeated Measurements

11.1 Purpose

The purpose of the exercise is to measure your reaction time and to see how it compares to that of your partner, as it is a source of uncertainty in many experiments. You will attempt to measure it two different ways to see if they are equivalent. In doing so, you will be introduced to important concepts in statistics so that you can determine uncertainties in averages. You'll see how to determine the precision of a measuring instrument, and you will also learn to show how two quantities compare by using a graph.

11.2 Introduction

Lots of experiments involve measurements which are repeated. Repeating measurements allows the experimenter to be more accurate and precise in conclusions drawn from the experiment. This exercise will illustrate how to place bounds on a source of uncertainty which can be used in many experiments. Many of these results can be illustrated with bar graphs for comparison purposes. To illustrate many of the concepts involved, we'll do an experiment. The question we want to answer is:

What is a reasonable value for human reaction time?

This will give important insight into analyzing the “*Measuring ‘g’*” experiment.

11.3 Theory

There are actually three different topics in this lab:

1. human reaction time
2. comparing quantities graphically
3. analyzing data with repeated measurements

The first two will be discussed briefly here; the third will be discussed as part of the *Procedure* section.

11.3.1 What is Human Reaction Time?

There are many different situations where *human reaction time* is important, but there are several different *types* of reactions which may be considered, and thus may have different times. Some of the possibilities are:

1. The time it takes to react to a *random* event: this includes situations like hitting the brakes in a vehicle when a child runs onto the street.
2. The time it takes to react to an *anticipated* event: this includes situations like batting a baseball as it crosses home plate.

There are other related things to consider, such as how closely in time people may *synchronize* their actions with other events and/or people, and the possibility of changes to these times with practice.

Random events

Since one cannot anticipate a random event, by definition, there will be a finite time required for a reaction after the event happens.

Anticipated events

When an event is anticipated, “reaction” does not necessarily *follow* the event. In the example of hitting a baseball, it is possible for a batter to swing *before* the ball crosses the plate, (and in fact success requires the swing to begin early), as easily as it is to swing *after* the ball crosses the plate.

Synchronization

In some situations, people must work together at a task, and thus they must try to *synchronize* their actions. In this case, there is a limitation in their ability to act together, and this also reflects a collective reaction time. An example of this is musicians in an orchestra playing in time with the conductor’s baton. A small time difference (either positive or negative) between the motion of the baton and the playing of different instruments turns “music” into “noise”.

Repeatability

No matter which situation occurs involving reaction time, it may be possible for experience to lead to an improvement. Drivers, batters, and musicians can all perform better as they develop focus and experience.

11.3.2 Precision Measure

The **precision** of an instrument refers to *how close two measurements can be and still be distinguished*. Usually instruments with a large range don’t have as much precision, (or, “are not as precise”), as instruments with a small range.

Precision measure of a digital instrument

If you have a digital clock, which shows hours and minutes, how close can two times be and still be different? Obviously, if they are at least 1 minute apart, then they are different. What about a stopwatch that measures to hundredths of seconds? Times that are at least one hundredth of a second apart will be distinct. Since these times are much closer than the times which

the digital clock can distinguish, we say the stopwatch is *more precise* than the clock.

We call this *smallest difference between two measurements which can be distinguished* the **precision measure** of an instrument. A smaller precision measure indicates a more precise instrument.

For a digital instrument, the precision measure is the distance between the value you measure and the next possible value. (If the instrument “auto-ranges”, then the precision measure will change when the range changes. Watch out for that.)

Data with unknown precision measure

Sometimes you may have to work with data from someone else where you are not told the precision measure. In this case you may have to infer the precision measure from the data.

point number	X	Y	Z
1	25	0.86	3.6
2	30	0.92	4.2
3	25	0.68	2.7
4	5	0.74	5.1
\vdots	\vdots	\vdots	\vdots
N	10	0.80	1.9

Table 11.1: Data with unknown precision measure

Each of the columns in Table 11.1 hints at probable values for the precision measure for each of the three variables, X , Y , and Z .

- Note that all of the values in the X column end in either 0 or 5. This suggests that 5 is the smallest increment between the values, and so we can infer a precision measure of 5 for X .
- In the Y column, note that the least significant digit (or last digit) of each value is an even number. This suggests that 0.02 is the smallest increment between the values, and so we can infer a precision measure of 0.02 for Y .

- The values in the Z column don't exhibit any obvious pattern, but they all go to only one place after the decimal, so we can infer a precision measure of 0.1 for Z .

11.3.3 Expressing Quantities with Uncertainties

Mathematically, the uncertainty in a quantity is usually expressed using the symbol Δ . So in other words, if mass has the symbol m , then the symbol Δm should be interpreted as "*the uncertainty in m* ". In that case you would write

$$m \pm \Delta m$$

to mean the mass with its uncertainty. Uncertainty is always given as a positive value, but it can be added or subtracted from the quantity to which it belongs.

Comparing quantities with uncertainties

Quantities with uncertainties are said to **agree** if the ranges given by the uncertainties for each overlap. For instance, if I estimated the length of the athletic complex as "*between 60 and 90 metres*" which I could state as " 75 ± 15 metres", and I estimated the length of the science building as 100 ± 20 metres, then I would say that the lengths of the two building agree since the ranges overlap. In other words, they *may* be the same; without more careful measurement I couldn't say for sure that they are different.

11.3.4 Displaying Comparisons Graphically

Sometimes you need to compare two or more values of one *numerical* variable, (such as mass), where the values are differentiated by some non-numerical parameter,¹ where each of the numerical values has an uncertainty. For instance, in an election poll, you might show percentage of voters favouring each party, (a numerical variable), broken down by party, (a non-numerical parameter). In this case, probably the easiest way to view the comparison is by a **bar graph**, where each bar is for a different value of the non-numeric parameter. ² Such data are shown in Table 11.2. Note that along with the percent support for each party is the uncertainty in the percentage.

Support	Party			
	Conservative	NDP	Liberal	Green
Percent	32	25	10	4
$\Delta\%$	5	4	2	1

Table 11.2: Mythical Poll Results

From Table 11.2, it's clear that

$$\textit{Conservative Support} = 32 \pm 5\%$$

and

$$\textit{NDP Support} = 25 \pm 4\%$$

These two quantities agree within experimental uncertainty, since both include the range from 27% \rightarrow 29%. We can show this comparison graphically by using a **stacked bar graph**.

To turn this into a stacked bar graph³, we need to modify the data slightly, by making a third row, which is a duplicate of the second, and changing the items in the first row by subtracting the values from the second row. This is shown in Figure 11.3. In this way, the minimum value for each row, (i.e. the nominal value minus the uncertainty), is given by the top of the lowest bar,

¹In statistics, these are called “**categorical variables**”, since they divide data into categories, rather than distinguish by a numerical value.

²If the numerical values are percentages, a pie chart may be more useful.

³This may also be called a stacked *column* graph, depending on whether the bars are drawn horizontally or vertically. The principle is the same.

the nominal value is given by the top of the middle bar, and the maximum value, (i.e. the nominal value plus the uncertainty), is given by the top of the top bar. The modified data can then be plotted to show how the different

Support	Party			
	Conservative	NDP	Liberal	Green
Percent - $\Delta\%$	27	21	8	3
$\Delta\%$	5	4	2	1
$\Delta\%$	5	4	2	1

Table 11.3: Modified Data

quantities compare.

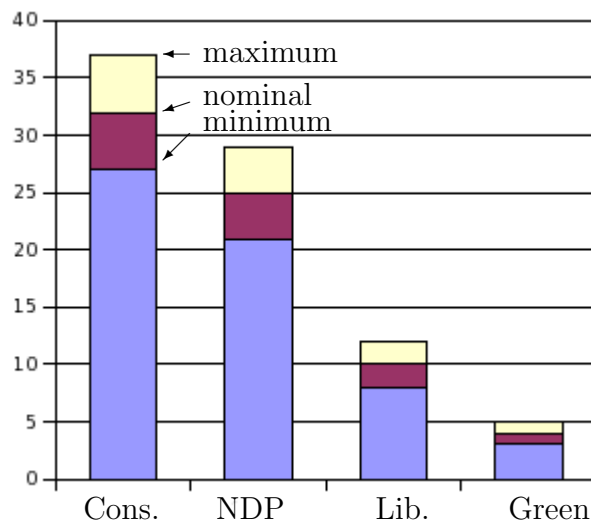


Figure 11.1: Comparing items

From Figure 11.1 we can see that support for Conservatives and NDP is the same, within experimental uncertainties, since a horizontal line can be drawn which passes through the uncertainty range for both. For any other combination, this is not the case.

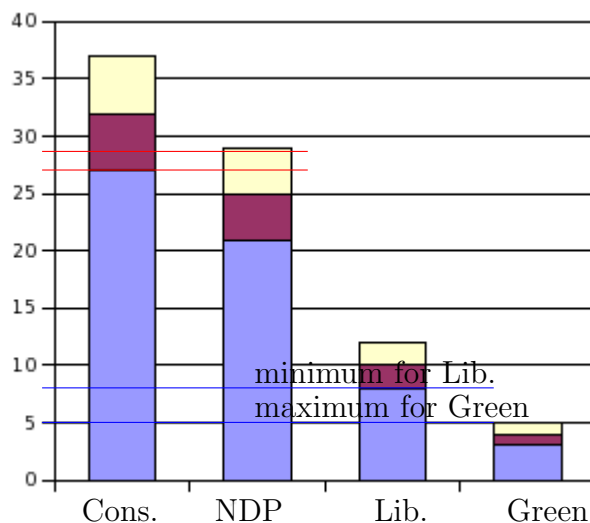


Figure 11.2: Range of overlap

Any line between the two red ones shown in Figure 11.2 crosses the error bars of both the Conservatives and the NDP, so that they are equal within experimental uncertainty. (Note that the top line of the bar graph is at 29, and the bottom one is at 27. This is the same range of values that was calculated earlier.)

If the values agree within experimental uncertainty, then they are the same.

11.4 Procedure

Much of this exercise can be done outside the lab, (except for the parts requiring the “in-lab” measurements of reaction time or your lab partner), using the reaction timer indicated on the website. In exercises like this, you may do as much as you can on your own. Then bring your answers to the in-lab questions and completed in-lab tasks to the lab.

11.4.1 Preparation (before the lab)

Most experiments and exercises will have requirements which must be completed *before* the lab and then presented *at the beginning* of the lab period.

11.4.2 Investigation (in the lab)

Reality check: Without doing any calculations, but using your head: Would you expect your reaction times to be in the ballpark of 1 second, 10 seconds, 0.1 second, or 0.01 second?

In-lab Questions

In this exercise, the in-lab questions are included with each part. Often they will be collected together at the end of the experiment or exercise.

Answer in-lab questions in a sentence or two. Be sure to state any data or results to back up your answer. Don't assume the reader knows the question or has access to your data. Make your responses complete and self-contained.

In-lab Tasks

Whenever you are asked to copy information into the template, you may use the appropriate computer spreadsheet(s) instead so long as you can show them to the IA who can check them off.

Data collection

There are going to be times in the lab when there is more data than there are data collection stations, therefore you will rotate through the stations whenever they become available. The order of collecting data does not matter. Make sure you record the data in the correct table in the template.

In-lab anticipated event measurement

1. Do several (at least 5) trials of the anticipated event test and record the times in the “self” column of Table 11.5. *Negative times are educated guesses and should be included.*
2. Get the reaction times for anticipated events from your partner. Put them in the “partner” column of Table 11.5.
3. Fill in the information about the instrument you used to measure reaction time; include specifically
 - (a) the name
 - (b) the units
 - (c) the precision measure
4. Calculate the average of the values in each column and fill it in.
5. Find the value in each column that is in the middle; (i.e. two values are as big or bigger and two are as small or smaller), and add this information to the row labeled “median value”.
6. Identify the largest and the smallest values in each column and add this information to the rows identified as “maximum value” and “minimum value”.
7. Find the **range** of the values in each column, by taking the difference between the maximum and minimum values. Add this information to the table.
8. In the last row, put in the value of the range divided by 4.

IT1: Fill in Table 11.5 with the information required above.

measure	
Trial #	self
1	46
2	78
3	68
4	126
5	144
average	92.4
median value	78
minimum value	46
maximum value	144
range	98
range/4	24.5

Figure 11.3: Sample of data in Table 11.5

Mean and median

The mean and median are two useful quantities that can be calculated from a set of repeated measurements. The “middle value” you found is the **median**. It has the same number of values above and below. (If you have an even number of measurements, the median is the average of the two in the middle.) The average value you calculated is also known as the **mean**. (“mean” = “average”) For *normally distributed* data, the median should be a reasonable approximation of the mean. In other words, they will probably be the same to one or two significant figures.

For many sets of data, the mean and the median will be similar. So, since you can find the median with no calculations, it is a simple way to *estimate* the average.

IQ1: If several trials produced the exact same measurement for reaction time, such as shown in Table 11.4, is it likely that the next digit would be the same if an instrument with one more decimal place were used? Are there any two values in one of *your* sets of data which are the same? If yes, what are they?

Trial #	time (s)
1	0.3
2	0.3
3	0.3
4	0.3
5	0.3

Table 11.4: Sample reaction times measured with a different instrument

Range of the data

Suppose you have five measurements of reaction time from one person, such as in your sample. Probably you will find that the five reaction times are not all the same. Because of this, it makes comparing times from different people a bit tricky; you have to know what *range* of values are possible *for each person*.

IT2: Copy the appropriate results from Table 11.5 into Table 11.7.

Summarizing the information The median and the $range/4$ give us an easy way to summarize the information in the table. We could write it as follows:

$$\text{range of reaction times} \approx \text{median value} \pm \text{range}/4$$

With only 5 points, this may not seem much shorter than simply stating all five values, but if we had 10 (or 100!) measurements, presenting the results this way would be much more concise than stating all of the values.

Look at the median, and the value calculated for the $range/4$. Most of the values should fall between the median *minus* the $range/4$ and the median *plus* the $range/4$. Did this occur? In other words, did at least 3 values for each of you fall in the range of $median \pm range/4$?

IT3: Fill in the last two rows of Table 11.7. This is to allow comparing you and your partner for *this method*.

median \pm range/4	78 \pm 25
≥ 3 values within median \pm range/4 ?	NO
self and partner overlap? (v/n)	

Figure 11.4: Sample of data in Table 11.7

IQ2: State the *approximate range of* in-lab anticipated event reaction times for you and your partner using the form

$$\text{range of reaction times} \approx \text{median value} \pm \text{range}/4$$

Based on the times, does it seem like *you and your partner* are getting similar results for reaction time? In other words, do the two ranges of times for you and your partner overlap?

Online anticipated event measurement

Now look at the data for anticipated events using the online method.

1. Do several (at least 5) trials of the anticipated event test and record the times in the “self” column of Table 11.6.
2. Get the reaction times for anticipated events from your partner. Put them in the “partner” column of Table 11.6.

Negative times are educated guesses and should be included.

3. Fill in the information about the instrument you used to measure reaction time; include specifically
 - (a) the name
 - (b) the units
 - (c) the precision measure
4. Calculate the average of the values in each column and fill it in.

IT4: Fill in Table 11.6 with the information required above.

Synchronization

This part must be done using the reaction timer on the website.

Usually synchronization involves one person being the *initiator* and the other being the *responder* (like the conductor and a musician).

1. Figure out a method that allows you to be synchronized with your partner.
2. Do several trials of the synchronization test and record the times in Table 11.8.
3. Repeat and reverse the roles of “initiator” and “responder”.

IT5: Fill in Table 11.8. Don’t worry about the greyed out rows now. Did your results fit with your “reality check” at the beginning of the “Investigation” section?

IQ3: How did your synchronization times compare to your anticipated times? Did they overlap, or was one greater than the other? What does this suggest about how easy it is to synchronize compared to responding to anticipated events individually?

IT6: Identify on the template which person (i.e. you or your partner) is person ‘A’ and which is person ‘B’.

Comparing data sets using statistics

In question **IQ2** above, all we could ask was whether it *looked* like the reaction times were different, since we had no way of determining how much variation would be small enough to ignore. In statistics, there is a quantity which can be calculated for an average to determine whether some other measurement is far enough away that it should be considered “different”. That quantity is called the **standard deviation**, and it has the following properties:

- About 2/3 of the measurements should fall between the average of your measurements *minus* one standard deviation and the average *plus* one standard deviation.

- About 95% of the measurements should fall between the average of your measurements minus *two* standard deviations and the average plus *two* standard deviations.
- To turn the previous point around, the standard deviation will be about 1/4 of the difference between the biggest and smallest measurements. (It will actually be a little less than 1/4 of the difference.)

Calculating the standard deviation Table 11.9 is set up to help you calculate the standard deviation for the data for one of the sets of times you chose. The standard deviation is calculated by

$$\text{standard deviation} = \sqrt{\frac{\sum (\text{time} - \text{average})^2}{n - 1}}$$

where n is the number of data points; in this case, 5.

1. Copy the data from Table 11.5 to fill in the first column and calculate the average. **Don't round the average off!**
2. Subtract the average from each time to fill in the second column.
3. Square the second column values to fill in the third column. **Don't round the numbers off! Always keep at least two significant figures.**
4. Add up the third column values and fill in the appropriate cell.
5. Use the formula in the table to calculate σ , the standard deviation.

Trial #	time	time-average $t - \bar{t}$	$(t - \bar{t})^2$
1	46	-46.4	2152.96
2	78	-14.4	207.36
3	68	-24.4	595.36
4	126	33.6	1128.96
5	144	51.6	2662.56
calculations			online calculator
average	\bar{t}	92.4	92.4
sum	$\sum (t - \bar{t})^2$	6747.2	N/A
standard deviation	$\sigma = \sqrt{\frac{\text{sum}}{n-1}}$	41.07	41.07
dev. of the mean	$\alpha = \frac{\sigma}{\sqrt{n}}$	18.37	18.37

Figure 11.5: Sample of data in Table 11.9

IT7: Fill in your results in Table 11.9. Don't worry about the last row for now.

Now you can check the three points listed above to see if they apply in your case.

1. Highlight the rows in the table where the time is *within* ($\text{average} \pm \sigma$). That should be about 2/3 of the values.
2. Highlight the rows in the table where the time is *within* ($\text{average} \pm 2\sigma$). That should be about 95% of the values. Are there any that are outside of this range?

IQ4: State the value you calculated for the standard deviation, as well as the previously calculated value of $\text{range}/4$ from Table 11.5. Was your calculated value for the standard deviation *similar* to the $\text{range}/4$; i.e. was the smaller of the two more than half of the larger of the two? (If they are not similar, check to see if you calculated σ correctly.)

Using the online statistical calculator There is a calculator online to do these statistical calculations. Use it to check your results.

IT8: Use the statistical calculator to verify that your values for the standard deviation and the standard deviation of the mean are correct in Table 11.9. *Write in the calculator answers beside the cells where you calculated the results yourself.*

IT9: Repeat the previous process to fill in Table 11.10 with your partner's data from Table 11.5. (Again, don't worry about the last row.) **You can use the statistical calculator or do the calculations by hand if you prefer. Whichever you do, just leave the extra columns blank.**

Comparing averages: step one The reason for calculating the standard deviation is so that we can compare different averages of similar data sets. In our case, we want to compare the average anticipated event reaction times for the two partners. We're *almost* ready to do that. If we knew we had a *representative sample* of times from each partner that we used for our average, we'd be in great shape. However, we can't be sure our samples are "representative". (For instance, you may have taken a few tries to get familiar with the test.) If we have a lot of times from one person, then the sample will be more representative than if we only have a few. What we need is some quantity that reflects that. The quantity that we're looking for is called the **standard deviation of the mean**, or the **standard error of the mean**, and is calculated by

$$\text{standard deviation of the mean} = \frac{\text{standard deviation}}{\sqrt{n}}$$

The usual symbol used for the standard deviation of the mean is α , so this is usually written as

$$\alpha = \frac{\sigma}{\sqrt{n}} \quad (11.1)$$

Basically, the standard deviation of the mean is a measure of the range around the mean (i.e. average) from our sample which should contain the average of a "representative" sample.

Since the standard deviation of the mean has \sqrt{n} in the denominator, it will get smaller as the amount of data gets larger, which is what we'd expect. (A larger sample should, by definition, be more representative.)

The **standard deviation** indicates how far the *data values* vary from each other. The **standard deviation of the mean** indicates how far the *average* is expected to vary from *the average* from a very large set of measurements.

IT10: Now fill in the last rows of Table 11.9 and Table 11.10.

Comparing averages: step two This part may sound pretty obvious, but it's important. We only have a hope of comparing two measurements if the instrument we used to measure them is precise enough to show a difference! For example, if we used a timer that only weighed to the nearest tenth of a second, *all* of the times might look the same. So to determine the uncertainty in the average of several measurements, we need to consider *both* the standard deviation of the mean *and* the precision measure of the instrument. This leads to the following rule:

The uncertainty in the average of several measurements is the *larger of* the **standard deviation of the mean** and the **precision measure**.

(If you use the statistical calculator, note that it does this step automatically; i.e. if you have typed in the precision measure, it will calculate the uncertainty in the average.)

So, now we can be precise and write

$$\text{reaction time} = \bar{t} \pm \Delta\bar{t}$$

Whenever you calculate the uncertainty in a final result, round the uncertainty to one significant figure. Then round the quantity to the same decimal place, so the last digit in the quantity is in the same decimal place as the uncertainty.

IT11: Complete Table 11.11. Don't worry about the last row for now.

person	self
average (\bar{t})	92.4
standard deviation (σ)	41.1
std. deviation of the mean (α)	18.4
uncertainty in average ($\Delta\bar{t}$)	18.4
$\bar{t} \pm \Delta\bar{t}$	92 ± 18
agree? (y/n)	

Figure 11.6: Sample of data in Table 11.11

Comparing the times

Now that you've determined the uncertainty in the average for the two partners, you can state whether or not the averages *agree* within their uncertainties, or in this case, whether the average anticipated event reaction time for you and your partner were the same or not.

Mathematically, if two quantities a and b , with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if

$$|a - b| \leq \Delta a + \Delta b \quad (11.2)$$

A constant given, *with no uncertainty given*, usually can be assumed to have an uncertainty of zero.

For example, suppose we have two sets of times:

$$T_A = 320 \pm 50ms$$

and

$$T_B = 240 \pm 60ms$$

Do they agree?

Clearly, the two ranges of values overlap; i.e. 270-300 ms is common to both, so we would say they agree. Mathematically,

$$T_A - T_B = 320 - 240 = 70ms$$

$$\Delta T_A + \Delta T_B = 50 + 60 = 110ms$$

Since

$$|T_A - T_B| = 70ms \leq \Delta T_A + \Delta T_B = 110ms$$

they agree. (The absolute value sign covers the case where T_B and T_A are interchanged.)

IT12: Fill in the last row of Table 11.11.

IQ5: If any two reaction times agreed, would that mean they are identical? State both you and your partner's times with their uncertainties. Based on your calculations, do the average times for you and your partner agree or not?

Comparing the two methods (inlab and online)

IT13: Use the statistical calculator to complete Table 11.12. This is to allow comparing both *methods* for one *person*, i.e. you.

IQ6: Did your times agree for the two methods, inlab and online? What would it mean if your times didn't agree for the different methods? Does that mean they are measuring different things, or could there be anything else that could account for the difference?

Checking the validity of the approximations Let's take a quick look back to see how much the "simple" results look like the "statistical" results.

IT14: Complete Table 11.13.

median	78	
\bar{t}	92.4	
$0.5 \leq \text{ratio} \leq 2?$	Yes	
range/4	24.5	
σ	41	
$0.5 \leq \text{ratio} \leq 2?$	yes	

Figure 11.7: Sample of data in Table 11.13

How many trials should you use?

Since all of our statements about whether the times are the same or not depend on the the number of times in the sample we collected, is there any way of knowing whether we have a sufficiently large sample or whether we should obtain more?

It turns out that we can measure “enough” times to be confident. In fact, “enough” may not be that many in some cases. Look again at Equation 11.1. Since α will get smaller as we take more measurements, it seems like there’s no limit to the number of useful measurements. However, remember that “*The uncertainty in the average of several measurements is the larger of the standard deviation of the mean and the precision measure.*” Since α will keep getting smaller as more measurements are taken, there will *always* come a point where it will be smaller than the precision measure. After that point, the uncertainty will stay constant, no matter how many more measurements are taken, and so taking more measurements becomes mostly pointless.

The optimum number of measurements has been taken when the standard deviation of the mean and the precision measure are *equal*.

Once you have a few measurements, you can calculate σ and then use it to determine how many measurements would be optimal by rearranging Equation 11.3 to solve for $N_{optimal}$.

$$precision\ measure = \frac{\sigma}{\sqrt{N_{optimal}}} \quad (11.3)$$

Since the standard deviation of the mean gets smaller as more measurements are taken, if the precision measure is *greater than* the standard deviation of the mean then no more measurements are needed.

IQ7: For one of the sets of times, determine the optimum number of trials to take. (Include your calculations.) Would this number of trials be feasible to collect and use in the lab? Explain. (**Hint:** If $pm > \alpha$, (the *precision measure* is greater than the *standard deviation of the mean*), then $N_{optimal}$ will be *less* than what you already have. If $pm < \alpha$, then $N_{optimal}$ will be *greater*.)

(If you use the statistical calculator, note that it does this step automatically as well; i.e. if you have typed in the precision measure, it will calculate the optimal number of measurements.)

IT15: Read over each of the inlab and postlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 11.14.

Bonus: Random events *If you have time, you can do this in the lab. If not, you can do it using the online reaction time tester.*

1. Do several trials of the random event test and record the times in Table 11.15. *Any negative times are basically “wild guesses” and should be ignored.*
2. Repeat with your partner.

11.4.3 Analysis (after the lab)

Post-lab Discussion Questions

Answer the following questions in paragraph form, with each individual question answer underlined or highlighted, At the beginning of each question put the question number as a super-script. The goal is to have a flow to the whole paragraph, rather than to have it appear as a series of statements of unrelated facts. *Be sure to include your numerical results to explain your answers.*

Repeatability

This applies to all of the previous situations.

1. Look at each of the sets of data and note if the times are changing according to a pattern.

Q1: Was there any indication that something was changing systematically as the trials went on? (For instance, were times getting shorter, longer, or staying the same?) What insight does this give you about doing an experiment such as this?

Q2: Did the in-lab reaction times for *anticipated events* for you and your partner agree within experimental uncertainty?

Q3: Determine the optimum number of trials for *the other method*; (i.e. the *online* measurement for anticipated events). Make sure to include your calculations with your answer. Did the optimal number of times depend on the device you used? Explain.

Q4: As in Figure 11.1, sketch a stacked bar graph to illustrate graphically whether the anticipated event reaction times for you and your partner agree or not. If they do, draw a horizontal line which shows that they agree. If they don't, draw a horizontal line that shows that they don't agree (i.e. one that falls below the higher value and above the lower one.). In this case, the partner name is a categorical variable.

Q5: Was either person's anticipated event reaction time equal to zero within experimental uncertainty by any method?

Q6: Did the two synchronization times for the two situations agree within experimental uncertainty?

Q7: Was either synchronization time equal to zero within experimental uncertainty? How did your synchronization times compare to your anticipated times?

Q8: Did your in-lab and online reaction times for *anticipated events* agree within experimental uncertainty? What does this suggest about how reaction times measured in the lab relate to the speed of your reactions when you are actually performing a task?

11.5 Bonus

Bonuses can be done in or outside of lab time depending on what equipment is needed to do the bonus questions. The bonus questions that are completed outside of lab time are to be handed in with the post-lab questions.

11.5.1 Bonus: Random Event Reaction Time

Using data collected earlier, or using the online reaction time tester, answer the following. (Be sure to include a copy of your data.)

Did the two reaction times for *random events* for the two people agree within experimental uncertainty?

Was either person's reaction time for *random events* equal to zero within experimental uncertainty?

11.5.2 Bonus: Other Factors to Study

Choose any one of the following events to test, depending on the equipment available to you:

1. See if it matters whether a button is pushed or released to stop the time.
2. See if the time taken between repetitions makes a difference.
3. Determine whether the *mode* of stimulus matters (eg. sight versus sound).
4. Determine whether practice does indeed make perfect by doing many repetitions of one test. You need to do at least 30 repetitions to reach a valid conclusion.
5. Figure out whether the length of the sequence for a random event matters.
6. Figure out whether the speed of the sequence for a random event matters. (In other words, if you change the delay time between the steps of the sequence, does it make a difference?)
7. See if you can find a better method of synchronization by testing different approaches.
8. In the lab, check to see whether any person's reaction times are significantly better or worse for the random event.
9. In the lab, check to see whether any group's reaction times are significantly better or worse for a synchronized event.
10. Check to see whether a student's musical ability influences synchronization.

11. Check to see whether a student's athletic ability influences synchronization.
12. (*For musicians*) Given the suggested tempo and time signature of a piece of music, determine the maximum synchronization delay allowable for a performance.

11.5.3 Bonus: Using Calculator Built-in Functions

Note: If you don't have the manufacturer's manual for your calculator you can check the links on the lab web page.

If your calculator has built-in functions to calculate averages and standard deviations, test them out by using your data. Does your calculator calculate the correct value for σ , the standard deviation? If not, how can you correct it?

Write out the instructions needed to do this on your calculator. Include how to clear an old data set before entering a new one. For each step include the calculator's display. (Record the make and model of calculator you have.) *After you have verified the operation of the statistical functions on your calculator, you can use these functions whenever a lab task requires them.*

11.6 Recap

By the end of this exercise, you should understand the following terms, and be able to determine:

- precision measure of a digital instrument
- median of a set of values
- range of a set of values

You should also be able to calculate:

- mean; *the median is a good estimate of the mean (average)*
- standard deviation; *the range/4 is a good estimate of the standard deviation*

- standard deviation of the mean
- uncertainty in the average
- optimal number of measurements

11.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	7	_____	35
Post-lab Questions	8	_____	35
Pre-lab Tasks	0	_____	0
In-lab Tasks	15	_____	30
Post-lab Tasks	0	_____	0
Bonus		_____	5

11.8 Template

My name:

My student number:

My partner's name:

My other partner's name:

My lab section:

My lab demonstrator:

Today's date:

Person A name:

Person B name:

Instrument		
name (or reference)		
units		
precision measure		
Trial #	self	partner
1		
2		
3		
4		
5		
average		
median value		
minimum value		
maximum value		
<i>range</i>		
<i>range/4</i>		

Table 11.5: In-lab anticipated event reaction test

Instrument		
name (or reference)		
units		
precision measure		
Trial #	self	partner
1		
2		
3		
4		
5		
average		

Table 11.6: Other (online) method for measuring anticipated event reaction time

Instrument		
name (or reference)		
units		
precision measure		
person	self	partner
average		
median value		
$range/4$		
$median \pm range/4$		
≥ 3 values within median $\pm range/4$?		
self and partner overlap? (y/n)		

Table 11.7: Summary of results for first (in-lab) method

Instrument		
name (or reference)		
units		
precision measure		
	Responder	
trial i	Person A t_A	Person B t_B
1		
2		
3		
4		
5		
\bar{t}		
σ		
α		
$\Delta(\bar{t})$		
agree?		

Table 11.8: Synchronization data

Instrument			
name (or reference)			
units			
precision measure			
Trial # i	time t_i	time-average $t_i - \bar{t}$	$(t_i - \bar{t})^2$
1			
2			
3			
4			
5			
calculations			online calculator
average	\bar{t}		
sum	$\sum (t - \bar{t})^2$		N/A
standard deviation	$\sigma = \sqrt{\frac{\text{sum}}{n-1}}$		
std. dev. of the mean	$\alpha = \frac{\sigma}{\sqrt{n}}$		

Table 11.9: Standard deviation for self using first (in-lab) method

Instrument			
name (or reference)			
units			
precision measure			
Trial # i	time t_i	time - average $t_i - \bar{t}$	$(t_i - \bar{t})^2$
1			
2			
3			
4			
5			
calculations			online calculator
average	\bar{t}		
sum	$\sum (t - \bar{t})^2$		N/A
standard deviation	$\sigma = \sqrt{\frac{\text{sum}}{n-1}}$		
std. dev. of the mean	$\alpha = \frac{\sigma}{\sqrt{n}}$		

Table 11.10: Standard deviation for partner using first (in-lab) method

Instrument		
name (or reference)		
units		
precision measure		
person	self	partner
average (\bar{t})		
standard deviation (σ)		
std. deviation of the mean (α)		
uncertainty in average ($\Delta\bar{t}$)		
$\bar{t} \pm \Delta\bar{t}$		
agree? (y/n)		

Table 11.11: Comparing two partners (in-lab method)

Anticipated event reaction time		
units		
person	self	
method	in-lab	online
precision measure		
average (\bar{t})		
σ		
α		
$\Delta(\bar{t})$		
agree? (y/n)		

Table 11.12: Summary of results for two methods

Anticipated event reaction time		
units		
method	in-lab	
person	self	partner
median		
\bar{t}		
$0.5 \leq ratio \leq 2 ?$		
$range/4$		
σ		
$0.5 \leq ratio \leq 2 ?$		

Table 11.13: Summary of results for approximate and precise calculations

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Post-lab		
Hints		
	“think” “suggest” “explain” “how” “why” “what”	“agree” “equal” “do (did, does) ” “significantly different” “support” “verify”

Table 11.14: Lab Report Organization

Instrument		
name (or reference)		
units		
precision measure		
trial i	Person A t_A	Person B t_B
1		
2		
3		
4		
5		
\bar{t}		
σ		
α		
$\Delta(\bar{t})$		
agree?		

Table 11.15: Bonus Question data: Random events

Chapter 12

Exercise on Processing Uncertainties

12.1 Purpose

The purpose of the exercise is to develop skills in calculating with uncertainties, which will include calculating uncertainties in calculated values of 'g', recognizing significant sources of error, and writing a "Discussion". Many of these results will be used in future labs, so be sure not to lose this report.

12.2 Introduction

This exercise should help you become familiar with calculations involving uncertainties, and how to address them in lab reports.

The "Discussion of Errors" (or Uncertainties) section of a lab report is where you outline the *reasonable limits* which you place on your results. If you have done a professional job of your research, you should be prepared to defend your results. In other words, you should expect anyone else to get results which agree with yours; if not, you suspect theirs. In this context, you want to discuss sources of error which you have reason to believe are significant.

12.3 Theory

This has been covered previously. (See Chapter 10, “*Uncertain Results*”, in the manual.)

12.3.1 Summary of Rules for algebra (in the form of equations)

Uncertainties in functions of a single variable

$$\Delta f(x) \approx |f'(x)| \Delta x \quad (12.1)$$

Also, remember three important corollaries

$$(e) \quad \Delta(c \times (A \pm \Delta A)) = |c| (\Delta A)$$

$$(f) \quad \Delta\left(\frac{1}{(A \pm \Delta A)}\right) \approx \left|\frac{1}{A}\right| \left|\frac{\Delta A}{A}\right|$$

$$(g) \quad \Delta(A \pm \Delta A)^n \approx |nA^{n-1}| \Delta A$$

Rules for combining multiple variables

The uncertainty in results can *usually* be calculated as in the following examples (if the percentage uncertainties in the data are small):

$$(a) \quad \Delta(A + B) = (\Delta A + \Delta B)$$

$$(b) \quad \Delta(A - B) = (\Delta A + \Delta B)$$

$$(c) \quad \Delta(A \times B) \approx |AB| \left(\left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta B}{B} \right| \right)$$

$$(d) \quad \Delta\left(\frac{A}{B}\right) \approx \left|\frac{A}{B}\right| \left(\left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta B}{B} \right| \right)$$

Note that the first two rules above *always* hold true.

12.3.2 Summary of Rules for inspection (in the form of equations)

There’s really only one rule for inspection.

$$\Delta f(x) \approx f_{max} - f \quad (12.2)$$

or

$$\Delta f(x) \approx f - f_{min} \quad (12.3)$$

12.3.3 Interpretation and Expression of Uncertainties

Sensitivity of Total Uncertainty to Individual Uncertainties

If $f = f(x, y)$, then to find the proportion of Δf due to each of the individual uncertainties, Δx and Δy , proceed as follows:

- To find Δf_x , let $\Delta y = 0$ and calculate Δf .
- To find Δf_y , let $\Delta x = 0$ and calculate Δf .

Simplifying Uncertainties

You do not need to carry uncertainties through if they do not contribute more than 10% of the total uncertainty, since uncertainties are usually only expressed to one decimal place. (However, be sure to give bounds for these uncertainties when you do this.)

Uncertainties and Final Results

Mathematically, if two quantities a and b , with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if

$$|a - b| \leq \Delta a + \Delta b \quad (12.4)$$

A value with no uncertainty given can be assumed to have an uncertainty of zero.

If two numbers do not agree within experimental error, then the *percent difference* between the experimental and theoretical values must be calculated as follows:

$$\text{Percent Difference} = \left| \frac{\text{theoretical} - \text{experimental}}{\text{theoretical}} \right| \times 100\% \quad (12.5)$$

Remember: Only calculate the percent difference if your results do *not* agree within experimental error.

Significant Figures in Final Results

Always quote final answers with one significant digit of uncertainty, and round the answers so that the least significant digit quoted is the uncertain one.

For instance, suppose an experiment determined a value for the speed of light

$$c = 2.953 \times 10^8 m/s$$

and from that experiment the uncertainty was

$$\Delta c = 4.26 \times 10^6 m/s$$

First round the uncertainty to one significant figure; i.e.

$$\Delta c = 4 \times 10^6 m/s$$

Rewrite this value so that it uses the same power of 10 as the value for c . In other words,

$$\Delta c = 0.04 \times 10^8 m/s$$

Since the uncertainty digit is the second one after the decimal, round the value for c to two places after the decimal.

$$c = 2.95 \times 10^8 m/s$$

So putting those together, we end up with

$$c = (2.95 \pm 0.04) \times 10^8 m/s$$

In other words, the uncertainty has only one significant digit, and the last digit we show for c is the uncertain one.

12.3.4 Discussion of Uncertainties

- Spend most of your time discussing the factors which contribute the most to the uncertainties in your results.

- Always give a measured value or a numerical bound on an uncertainty.
- State whether any particular factor leads to a systematic uncertainty or a random one. If it's systematic, indicate whether it would tend to increase or decrease your result.

Types of Errors

- Measurable uncertainties
- Bounded uncertainties
- Blatant filler

Don't use "*human error*" as it's far too vague.

Reducing Errors

1. Avoid mistakes.
2. Repeat for consistency, if possible.
3. Change technique.
4. Observe other factors as well including ones which you may have assumed were not changing or didn't matter.
5. Repeat and do a statistical analysis.
6. The *last resort* would be to change the equipment.

Ridiculous Errors

Anything which amounts to a mistake is not a valid source of error. A serious scientist will attempt to ensure no mistakes were made before considering reporting on results.

12.4 Procedure

12.4.1 Preparation (before the lab)

Pre-lab Tasks

PT1: Copy the data from the results of “*Measuring ‘g’*” into Table 12.2.

PT2: Rearrange Equation 15.1 to solve for g as required in **IQ1** and fill it in the appropriate line on the first page of the template. (*If you’re not using a printed template, write it out on a piece of paper and bring it with you to the lab.*)

PT3: What was the precision measure of the stopwatch from “*Measuring ‘g’*”? Copy this value into the template. (*If you’re not using a printed template, write it out on a piece of paper and bring it with you to the lab.*)

PT4: What were the height and the realistic uncertainty in the height from “*Measuring ‘g’*”, and what factor(s) caused the uncertainty? Copy these values into the template. (*If you’re not using a printed template, write it out on a piece of paper and bring it with you to the lab.*)

PT5: Use the online *statistics calculator*, or do the calculations by hand, to fill in the rows for \bar{t} and $\Delta(\bar{t})$ in Table 12.2.

PT6:

Fill in the following results, based on Chapter 10, “*Uncertain Results*”, in the manual.

$$\begin{array}{r} 24.2 \pm 0.1 \\ + 1.03 \pm 0.02 \\ \hline \pm \end{array}$$

$$\begin{array}{r} 81.2 \pm 0.4 \\ - 29.4 \pm 0.3 \\ \hline \pm \end{array}$$

$$\begin{array}{r} 1.67 \pm 0.07 \\ \times 0.942 \pm 0.002 \\ \hline \pm \end{array}$$

$$\begin{array}{r} 54.3 \pm 0.2 \\ \div 982.1 \pm 0.3 \\ \hline \pm \end{array}$$

$$\begin{array}{r} r = 2.0 \pm 0.1 \text{ cm} \\ A = \pi r^2 = \pm \text{ cm}^2 \end{array}$$

12.4.2 Investigation (in the lab)

This exercise can be done outside the lab if you wish. Alternative instructions are boxed like this. For your lab period bring your answers to the in-lab questions and the completed in-lab tasks to the lab.

In-lab Tasks

Complete the in-lab tasks as you encounter them.

Part 1: Finding the uncertainty by two methods

1. Find the equation for Δg by both methods; (algebra and inspection).
 - To find the equation for Δg by **algebra**:
 - (a) Since the equation for g has two variables, h and t , find which of the rules for multiple variables looks most like the equation for g .
 - (b) Rewrite the equation, replacing A and B with the appropriate quantities from the equation for g .
 - (c) You will have a place where you need to determine the uncertainty in a *function* of h , and a place where you need to determine the uncertainty in a *function* of t . Identify these.
 - (d) For the two places just mentioned, use the function rule to determine the two uncertainties.
 - (e) Replace the results from the function rule in the equation for the uncertainty.
 - (f) Simplify the equation where possible, but don't try to rearrange it in any other way.
 - (g) If you've done things correctly, you should now have an equation for Δg that only involves h , Δh , t , and Δt , so you can plug in values.
 - To find the equation for Δg by **inspection**:
 - (a) The equation for g has two variables, h and t . If you replace h by either $h + \Delta h$ or $h - \Delta h$, which would make g bigger? If you replace t by either $t + \Delta t$ or $t - \Delta t$, which would make g bigger?
 - (b) Make the two substitutions above in the equation for g and call this g_{max} .
 - (c) The uncertainty in g is given by subtracting g from g_{max} . Write this out.
 - (d) Don't try to simplify the equation or rearrange it in any other way.

- (e) If you've done things correctly, you should now have an equation for Δg that only involves h , Δh , t , and Δt , so you can plug in values.
- Using your actual data, and the results from previous exercises, find the numerical values of Δg for one data set by both methods. The two methods should give the same answer when rounded to one significant figure. If they are very different, you have probably made an error somewhere.
 - Using either method, determine the equations for Δg_h and Δg_t .
 - Using your actual data, find the numerical values of the sensitivity of Δg to each of the quantities involved, (i.e. Δg_h and Δg_t), based on your data for the same data set.

IT1: Copy the equations for Δg by inspection and by algebra into the template.

Part 2: Using the online uncertainty calculator

- For one of the values of \bar{t} and $\Delta(\bar{t})$ from Table 12.2, use the uncertainty calculator to calculate $(\bar{t})^2$ and $\Delta((\bar{t})^2)$. (*Hint: This will use a **function** on the calculator.* You may want to do this for the other values of \bar{t} and $\Delta(\bar{t})$ while you're at it.
- Take your values of h and Δh from **PT4** and use the uncertainty calculator to calculate $2h$ and $\Delta(2h)$. (*Hint: This will use an **operator** on the calculator.*

IT2: Fill in these results in Table 12.1. *If you're using the online version of the manual, write the results on a sheet of paper.*

\bar{t}	$\Delta(\bar{t})$	$(\bar{t})^2$	$\Delta((\bar{t})^2)$
h	Δh	$2h$	$\Delta(2h)$

Table 12.1: Using the uncertainty calculator

In-lab Questions

In the last two steps of **IQ1**, you can use the online uncertainty calculator instead of doing the calculations by hand.

IQ1: (From “*Measuring ‘g’*”)

- State the equation for Δg by both methods; (algebra and inspection). State which method you preferred and why.

(Note: you will first have to rearrange the equation.)

- State the numerical values you found for Δg for *one data set* by both methods. Do the two methods give the same answer when rounded to one significant figure? If not, do they give consecutive values?
- State the equations you determined for Δg_h and Δg_t .
- State the numerical values you found for the sensitivity of Δg to each of the quantities involved, (i.e. Δg_h and Δg_t), based on your data *for the same data set*.

For the “*Measuring ‘g’*” experiment, answer the following questions. *Don’t discuss an uncertainty unless you have given a measured value or an estimated bound for it.*

IQ2: From your numerical results for the uncertainty in g , which quantity, (h or t), produced the biggest proportional effect? In other words, which was bigger, Δg_h or Δg_t ?

IQ3: From the answer to question 2, what *experimental factor* from those discussed when you collected the data made the biggest contribution to the uncertainty in the quantity mentioned?

IQ4: From the answer to question 3, what steps might be taken to reduce the uncertainty, keeping in mind the guidelines about the best ways to reduce uncertainties from earlier in this exercise?

IQ5: For the quantity *not* used in question 2, what *factor* from those discussed when you collected the data made the biggest contribution to the uncertainty in the quantity? What steps might be taken to reduce the uncertainty, keeping in mind the guidelines about the best ways to reduce uncertainties?

IQ6: From your results determined for **IT2:** can you figure out whether the uncertainty calculator uses inspection or algebra? How can you figure this out? (*Hint: Figure out the equation for the uncertainty in t^2 by both methods.*)

IT3: Read over each of the inlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 12.3.

12.4.3 Analysis (after the lab)

All of the in-lab questions from this exercise should be incorporated into your *Discussion* for “*Measuring ‘g’*”. This should also be the case for all of the exercises which you used when completing the “*Measuring ‘g’*” lab report.

12.5 Bonus

12.5.1 Proof of earlier result; uncertainty in marble volume

This question is from the “*Uncertain Results*” section of the manual. For a sphere,

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

and so by inspection

$$\Delta V \approx \frac{4}{3}\pi\left(\frac{d + \Delta d}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

Algebraically,

$$V' = 2\pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{2}d^2$$

and so

$$\Delta V \approx \left|\frac{\pi}{2}d^2\right| \Delta d$$

Show that these two methods give the same results if uncertainties are small; i.e. if $\Delta d \ll d$. Remember that uncertainties in final results are usually only expressed to one decimal place, so you can usually discard terms with two or more Δ terms multiplied together; for instance $\Delta A \Delta B \approx 0$

12.6 Recap

By the end of this exercise, you should know how to calculate the uncertainty for :

- quantities which are added
- quantities which are subtracted
- quantities which are multiplied
- quantities which are divided
- functions of quantities, by

- algebraic method
- inspection

In addition, you should understand

- how to use the online uncertainty calculator

12.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	6	_____	75
Post-lab Questions	0	_____	0
Pre-lab Tasks	6	_____	20
In-lab Tasks	3	_____	5
Post-lab Tasks	0	_____	0
Bonus		_____	5

12.8 Template

My name:

My student number:

My partner's name:

My other partner's name:

My lab section:

My lab demonstrator:

Today's date:

g has units of:

The equation for g is:

h has a value (with units) of:

The realistic uncertainty in h is:

The precision measure of the stopwatch (including units) was:

The equation for Δg *by inspection* is:

The equation for Δg *by algebra* is:

i	Times (seconds)				
	Ball one				Ball two
	A dropping		B dropping		
	gofer(B)	dropper(A)	dropper(B)	gofer(A)	
1					
2					
3					
4					
5					
\bar{t}					
$\Delta(\bar{t})$					
g					
Δg					
Δg_h					
Δg_t					
Which is most significant, Δh or Δt ?					

Table 12.2: Uncertainties for g

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Hints		
	“think” “suggest” “explain” “how” “why” “what”	“agree” “equal” “do (did, does) ” “significantly different” “support” “verify”

Table 12.3: Lab Report Organization

Chapter 13

Exercise on Introduction to Spreadsheets

13.1 Purpose

The purpose of this exercise is to introduce you to the use of a spreadsheet as a tool for data analysis by calculating uncertainties using sample experimental data.

13.2 Introduction

A spreadsheet is basically a large programmable calculator with some special features thrown in. Because of this it is very useful for data analysis. It is important that you try to *learn how to use the spreadsheet, rather than simply try to “get the work done”*. A spreadsheet is a handy tool, and these labs give you an opportunity to become familiar with it, if you are not already. It will be well worth the effort to use spreadsheets for it will save you time and careless errors and you will quickly become comfortable using them. Aside from aiding in calculation, a spreadsheet is essentially an organizer which allows you to put all of your data in the proper columns for a quick visual of what you might expect.

13.3 Theory

Some of the information here will be specific to Microsoft Excel, but most should be useful for other spreadsheets as well.

13.3.1 Formulas

The value in using *cell references* instead of numerical values is that if the numbers change, the calculations are performed automatically. In a lab this allows you to set up the spreadsheet and then simply type in new data to see new results. *If you do things properly*, it is even easy to change the number of data points after the fact.

Never use a number where you can use a cell reference. (You can even make references on one page to cells on another page.)

13.3.2 Functions

Functions perform commonly used tasks on a cell or block of cells and return the result in a cell.¹ Some of the most common ones follow.

Common Functions

For blocks of cells

- **average()**
- **count()** counts the non-empty cells

Always use the count function instead of typing in the number of data points so you can change the number later if needed without editing formulas.

- **stdeva()** determines the sample standard deviation
- **max()** finds the maximum value

¹There are functions which return a block of cells, which don't fit with the mathematical notion of a "function", but we won't get into those here.

- **min()** finds the minimum value

For individual values

- **sqrt()** square root
- **abs()** absolute value
- **if()** allows a cell to have a value depending on a condition

Using the Built-in Automation

In you aren't sure what parameters are needed by a function, or in what order, you can simply type in the function until after the left bracket and then you will be prompted for what needs to be filled in.

13.3.3 Copying Formulas and Functions: Absolute and Relative References

Never copy values from one part of the spreadsheet to another where a cell reference could be used instead.

Relative References

The value of using formulas in a spreadsheet is that calculations can be easily repeated. One of the ways this happens is by copying formulas to be applied to different data. When a formula is copied from one location to another, references are usually changed *relative* to the move. In other words, if a formula is copied from one cell to the next one on the right (i.e. where the column address increases by one), then cell references *within* the formula will have their column addresses increased by one as well. If a formula is copied from one cell to the one above (i.e. where the row address decreases by one), then cell references within the formula will have their *row* addresses decreased by one as well.²

Figure 13.1 gives an example of both absolute and relative references and how they affect copying. If the formula given was in the coloured cell, and then copied to the cell shown, the references in the formula would become the ones indicated.

²Note that when you *move* a cell, none of the references inside it are changed.

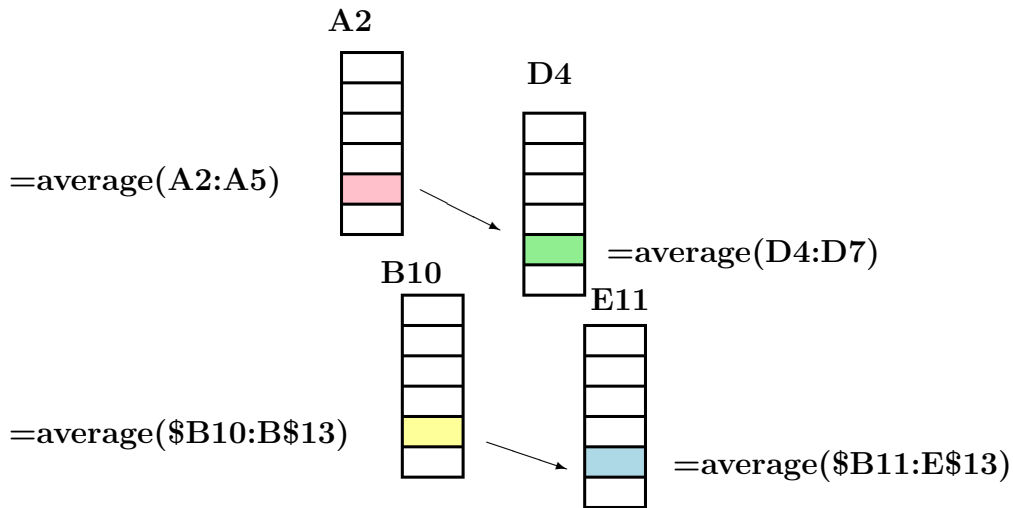


Figure 13.1: Effects of absolute and relative references when copying

Absolute References

Sometimes you don't want cell references in a formula to change when it is copied. In this case what you do is to put a **\$** before the part of a cell reference which you do not want to change. In other words, if you have a formula with a reference to **B13** and you do not want the **B** to change when you copy, then change the reference to **\$B13**. If you don't want the **13** to change when you copy, then change the reference to **B\$13**. If you don't want *either* **B** or **13** to change, then change the reference to **\$B\$13**. (In other words, you make a number a constant by referring to it like **\$B\$13**).

13.3.4 Pasting Options

Sometimes when you copy a formula, you don't want to copy everything about it. For instance, sometimes you just want to get the numerical result. In a spreadsheet, you can't choose what to copy, but you can choose what to *paste*. When you go to paste something, you can **Paste Special** and choose to paste whatever you want. Thus you can paste strings or numbers and not formulas, and you can include formatting (boxes, colours, fonts, etc.) or not.

Always copy formulas from one part of the spreadsheet to another instead of retyping to avoid making mistakes.

13.3.5 Formatting

You can do many things to a cell or block of cells to change its appearance. You can put boxes around it. You can change fonts or how the numbers are displayed by changing the background colour. You can also do many other things.

Before you format make sure that you are done working with your data and copying formulas. This will ensure that you do not have to Paste Special to preserve style information.

13.3.6 Print Preview

When printing a spreadsheet, there are lots of options to make the result more readable. Doing a *print preview* allows this. You can

- turn the grid off.
- include page headers and footers (or not)
- scale to fit the page

Doing a page preview also helps you figure out which page number(s) you need to print, since it may not be obvious.

Preview the printing to avoid wasting reams of paper printing out stuff you don't want.

13.4 Procedure

13.4.1 Preparation (before the lab)

You are welcome to go ahead and do as much of the exercise on your own as you wish; you can just bring your spreadsheet with you to the lab and demonstrate the points indicated. If you get it all done in advance, that's great.

If you do it on your own, then print a copy of the spreadsheet showing *formulas* along with the ones indicated in the post-lab questions. (You only need to show the formulas for rows mentioned in the instructions.)

Pre-lab Tasks

PT1: Verify that you can log in to a computer in one of the public labs on campus with your WLU login before you come to the lab. *Alternatively, bring your laptop to the lab with the lab manual and a spreadsheet program which can open the spreadsheet.*

13.4.2 Investigation (in the lab)

This exercise can be done outside the lab. Alternative instructions, where needed, are boxed like this. You may do that if you wish and bring the answers to the in-lab questions and the completed in-lab tasks to the lab. *In either case, the in-lab tasks are due during the lab period in which this lab is taking place.*

In-lab Tasks

In this exercise, the in-lab tasks are included with each part.

Creating the First Formulas

1. Cell **B14** should contain the average for the steel ball. In that cell, begin by typing a “=” so Excel knows you’re typing a formula. The function you want to use is **average()**. Once you type the left bracket, the function will prompt you for parameters. Select the range from **B7** to **B11** by dragging the mouse, or by typing in **B7:B11**. After this, type the right bracket to finish the formula. After you press **ENTER**, the calculation will take place.
2. Cell **B15** should contain the standard deviation. Follow the same procedure as above, using the function **stdeva()**.
3. Cell **B16** is going to contain the standard deviation of the mean, so you’ll need to use a formula which takes the standard deviation from

B15 and divides it by the square root of the number of measurements. The two functions you'll need are **sqrt()** and **count()**. Do not type in a value for the number of data points, and do not cut and paste the value of the standard deviation. You want to use cell references so that, if the values change, the results will still be correct.

4. Cell **B17** is going to have the uncertainty in the average. This is the bigger of the standard deviation of the mean and the precision measure. Use the **max()** function for this.

IT1: Show the lab demonstrator the table with the results.

Copying the Formulas

1. Modify the formulas so that they can be copied and correctly used for the ping pong ball. In order to correctly use the formulas one of the references must be made absolute. The reference that must be made absolute will be found between **B14** and **B17**.
2. Copy the formulas from **B14** to **B17**, and then use *Paste Special* to paste them in **C14** to **C17**. (Copy formulas only). See that the results are correct. If not, check to see if any references should have been copied differently. If so, make the changes to the steel ball formulas and repeat this step.
3. Notice the column for the brass ball series has one less value. If you have set up the formulas correctly, you should be able to copy them for this ball and the results will be correct. Try this and make changes to the original if necessary so that the copied formulas are correct.

IT2: Show the lab demonstrator the table with the changes you made.

Adapting the Formulas

1. The column for the wood ball series has one more value. If you copy the formulas, you will notice they do not include the last value. If you want to avoid having to correct the formulas, here's what you can do: Right click on the last value for the steel ball (in **B11**). Choose *Insert* so that you can create another cell in this column. This will push the

cells that contain the formulas down and they will point to the range that ends in **B12**. Now you can copy the formulas over to the column containing the wood values. *NOTE, to get the correct answers you have to initially paste in cell E15*. Now drag the cells with the correct formula to the correct position. You can then go back and delete the cell you inserted in **B11**.

This should make the result correct. Try this and see.

IT3: Show the lab demonstrator the formulas and that you have used functions where needed so that the number of data points does not need to be typed in anywhere.

Comparing Values with Uncertainties

1. The times for the steel and ping pong balls can be compared using Equation 10.8. (Hint: You can use the **if()** function for this.) *NOTE, you have to put quotation marks around agree and disagree where it says [value_if_true] and [value_if_false]. Note that if you use “don’t agree” do not put the underscore in the spaces. For conditions which are combinations, such as “greater than OR equal to”, simply put the two appropriate symbols one after the other.*

IT4: Show the lab demonstrator the formulas and that you have used functions where needed so that the whether the two averages agree is correct even if the times change.

Displaying Comparisons Graphically You can set up a graph to show which of the times agree.

1. Set up a table similar to Table 11.3.
2. Plot a stacked column graph (or a stacked bar graph) like Figure 11.1 using the table to show which of the times agree.

IT5: Show the lab demonstrator the graph and demonstrate how it shows which times agree with each other and which don’t.

Save the spreadsheet so you’ll have it for the post-lab tasks. (You can save it in your WLU account or to a flash drive, or simply email it to yourself.)

13.4.3 Follow-up (after the lab)

Post-lab Tasks

T1: Print two versions of the spreadsheet. One should be without gridlines or rows and column labels, and the other should be with them.

13.5 Recap

By the end of this exercise, you should know how to use spreadsheet functions to calculate the :

- mean
- standard deviation
- standard deviation of the mean

In addition, you should understand

- the difference between *absolute* and *relative* addresses in formulas
- how to use a stacked column or bar graph to show whether values with uncertainties agree or not

13.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	0	_____	0
Post-lab Questions	0	_____	0
Pre-lab Tasks	1	_____	10
In-lab Tasks	5	_____	60
Post-lab Tasks	1	_____	30
Bonus		_____	0

Chapter 14

Exercise on Estimation, Bounding, and Order of Magnitude Calculations

14.1 Purpose

The purpose of this exercise is to develop skills in estimating quantities when you can't measure them directly, and to make estimates of some sample quantities.

14.2 Introduction

While we usually think of science as involving *measurement*, there are many times when approximate values for quantities must be produced even when precise measurements are difficult or impossible. This exercise is about how that is done.

14.3 Theory

14.3.1 Estimation

Lots of experiments involve quantities which must be estimated. (For instance, before you *measure* anything, it's good to be able to *estimate* the

result you expect, so you can determine what sort of instrument or method you'll need to perform the measurement.) Some estimates may be better than others, but what really matters is that you have a fair idea about how far off your estimate *could* be.

Bounding

Bounding a quantity is forming an estimate of how far off it could be. An *upper bound* is a numerical limit above the expected value, and a *lower bound* is a numerical limit below the expected value.

Picking Realistic Bounds It's often easy to come up with reasonable bounds for a quantity by using similar known quantities which are pretty clearly above or below. For instance, if you are estimating a person's height, then you can compare with known heights of family members or friends. If you have to estimate the mass of an object, you can compare it to objects with which you are familiar.

Range of Possible Values for a Quantity The **range** of values for a quantity is the difference between its upper and lower bounds.

Familiar Comparison

If you're trying to estimate something, and it's similar to something you know, then you can probably make a pretty good estimate through comparison. In other words, if you can establish an upper and a lower bound, then you can estimate something in between.

14.3.2 Order of Magnitude Calculations

Quantities which are too difficult to be estimated directly can be estimated by performing calculations with estimates. For instance, sometimes certain quantities can be measured but others must be estimated. These calculations are called **order of magnitude calculations**¹, since their purpose is to give a result which is within an order of magnitude (i.e. a factor of ten) of the result of the detailed calculation.

¹or, "*back of the envelope calculations*"

14.3.3 Logarithmic scale

On a logarithmic scale, such as the one in Figure 14.1, the distance of a number from the left end of the scale is proportional to the logarithm of the number. Figure 14.2 and Figure 14.3 show some other possibilities. (Logarithmic scales are often identified by the number of **cycles** they show.) A *cycle* is the space between two numbers which differ by a factor of ten. So, between 1 and 10 is one cycle, between 2 and 20 is one cycle, between 5 and 50 is one cycle, etc. Note that there is *no* zero on a logarithmic scale. All numbers are positive.

14.4 Procedure

14.4.1 Preparation (before the lab)

There is no preparation required for this exercise.

14.4.2 Investigation (in the lab)

Illustration of Comparison

To illustrate, we'll try to make a few simple estimates. The first question we want to answer is: *How tall am I?*

1. Find someone who thinks they are shorter than me. Record that person's height. If that someone is shorter than I am, that person's height is definitely a lower bound for my height.
2. Find someone who thinks they are taller than me. Record that person's height. If that someone is taller than I am, that person's height is definitely an upper bound for my height.
3. Estimate my height according to the two known heights. If, for example, you are closer in height to the person who has the lower bound height than to the person who has the higher bound height, you would estimate your height closer to that of the lower bound person.
4. Estimate the bounds you would place on my height. (For instance, if you saw someone about my height commit a crime, what range of

heights would you give to investigators so that it would be of use in identifying suspects?)

IQ1: Give the heights of the people who you thought were taller and shorter than me. From these heights estimate my height and explain how you came up with my height. What bounds would you place on my height? Are they as far apart as the two heights of the people you knew?

The goal when making estimates is to try and make them “safe” but “useful”; i.e. you are *pretty sure* about lower and upper bounds on your estimate, but the bounds are close enough together to make the estimate usable.

IT1: Form a group of 3 or 4, and use items in the class to make comparisons. Fill in Table 14.1 with a reasonable estimate and bounds for each of the following:

- my height (from above)
- mass of a block of wood (comparing to known masses)
- volume of liquid

Less Familiar Comparison

Often it’s not easy to make a clear comparison with something very similar, and so the bounds and thus the estimate have to be a bit more fuzzy.

IT2: Based on your experience, fill in Table 14.2 with suggested bounds and a reasonable estimate for each of the following:

- height of this building (in metres)
- length of this building (in metres)
- mass of lab table (in kilograms)

IQ2: For one of those quantities, explain how you came up with the bounds and the estimate. Was the range of values for this comparison proportionally larger than for the familiar comparisons above? Explain.

Order of Magnitude Calculations

In labs, order of magnitude calculations are often done with preliminary data to see if the results seem to be as expected. Using the wrong units, a common mistake in labs, shows up easily in order of magnitude calculations. In these calculations, the formula may be the one that will ultimately be used with the complete data, but with incomplete or estimated values it is an order of magnitude calculation. (In these cases, if you're using a calculator, there's no need to type in the known values to more than one or two significant figures since the result will be approximate anyway.)

Since an order of magnitude calculation is supposed to be within one order of magnitude, you should be fairly confident that 10 times the value is *clearly* too big, and 1/10 of the value is *clearly* too small.

Illustration of Order of Magnitude Calculation

The next question we want to answer is: *How many coffees are sold in the Tim Horton's in the Science building every day?*

How can we figure this out?

There are several things we know about this problem. We know:

- They have lineups at certain times of the day.
- They have to sell enough coffee to pay their staff.
- It takes a certain amount of time to serve a coffee.
- There are a fixed number of coffeemakers and pots that process all of the coffee each day.

Each of these facts can be used with a bit of reasoning to produce a calculation of the number of coffees sold in a day, provided we can estimate the quantities involved. An example with a related problem can be used to illustrate.

How many gold parking spaces are there at Laurier?

1. Gold parking lots are for faculty and staff. So, there are probably less gold spaces than the total number of faculty and staff at Laurier, since some will walk, bus, carpool, etc.
2. There are several lots of various sizes, so an estimate of the number of lots and the average size will allow estimating the total number of spots.

In the first case, the lot in front of the science building has maybe 50 spots; (about 25 on each side). I think there are around 10 or 15 gold lots around campus, so let's say ten. If we call the average number of spots in a lot S , and the number of lots L , then the number of spaces, N , is given by

$$N = S \times L$$

or

$$N \approx 50 \times 10 = 500$$

So I'd estimate around 500 gold spots on campus. Does this make sense?

- *Are there more than 50 spots?* Since there are probably about 50 in front of the science building alone, then yes, there are more than 50.
- *Are there less than 5000 spots?* Since Laurier has around 10000 students, there would have to be one faculty or staff member for every two students to be able to use 5000 spots, which doesn't fit with what I observe. (I'd guess that there are more like one faculty or staff person for every 10 students or so.) So, yes, there are less than 5000.

Putting these together, my order-of-magnitude estimate of 500 spots is reasonable.

One of the important things to understand about order of magnitude calculations is that they are somewhat self-correcting; i.e. you're likely to overestimate some quantities but underestimate others, so the effects will cancel somewhat.

IQ3: Using the facts you know about Tim's that I listed earlier, produce two different formulas for the number of coffees in a day, with definitions of each of the quantities in the equation. Using estimates for each of the quantities involved, calculate results for each method. Discuss whether the two results are within an order of magnitude of each other. (If they aren't, suggest which particular estimates are the most suspect.)

When is a calculation an Order of Magnitude Calculation?

Any time you have to do a calculation using an estimated quantity, you are performing an **order of magnitude calculation**. The *order of magnitude* of a quantity refers specifically to the power of ten in its measurement. For instance, the height of the building would be in metres, while the length would be in tens of metres. In more general terms, the order of magnitude of a quantity refers to the cycle of a logarithmic scale to which the quantity belongs. Thus we could say that the order of magnitude value for the length of the science building is

- around 100 metres
- between 50 and 200 metres

Both of these are order of magnitude estimates.

When are Order of Magnitude Calculations used?

Order of magnitude calculations are quite commonly done in science *before* an experiment is performed. This is so that the range of expected data can be determined. They are also often done *as* the data are being collected to see if the experimental results appear to be in the correct ballpark.

An order of magnitude calculation is *any* kind of calculation which will produce an answer which should be close to the “*real*” answer. Any calculation involving at least one estimated quantity is an order of magnitude calculation. Generally, the more estimated quantities involved in an order of magnitude calculation, the wider the distance between the upper and lower bounds produced.

Logarithmic scale

If several people make estimates, they will no doubt vary. However, they will probably still be in a common ballpark. This can be more easily observed by plotting the values on a *logarithmic scale*, such as the one in Figure 14.1.

Where would zero be, if you wanted to show it?

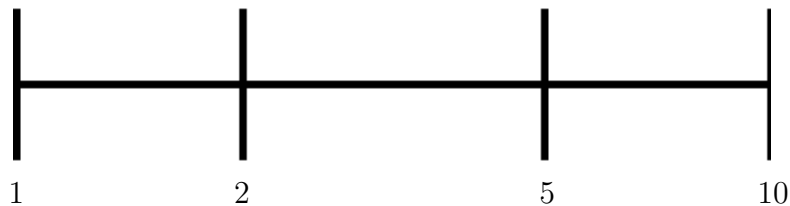


Figure 14.1: Logarithmic scale

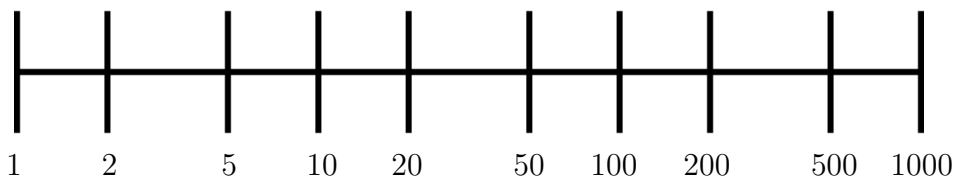


Figure 14.2: Three cycle logarithmic scale

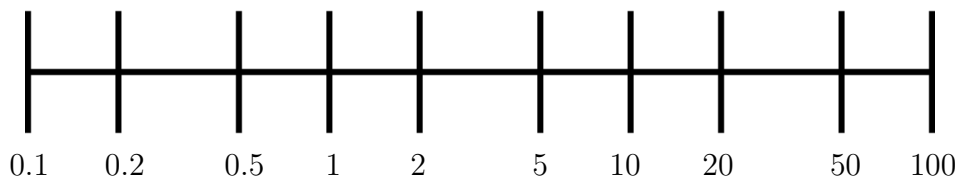


Figure 14.3: Logarithmic scale with numbers less than one

There are many things which we *perceive* on a logarithmic scale (such as the volume of music).

How big is an order of magnitude on a logarithmic scale?

If two numbers are within the same cycle of a logarithmic scale, they are within an order of magnitude of each other.

IT3: For the quantity estimated in **IQ3**, use the logarithmic scale of Figure 14.4 and mark 3 or 4 of the class estimates on it.

IQ4: Do all of the estimates for that quantity fall within a single cycle of the scale? (In other words, between 2 and 20, 5 and 50, 10 and 100, etc.) Explain.

Uncertainties

A quantity that is bounded can be expressed as an estimate with an uncertainty. (This is a little less cumbersome than giving the estimate, the lower bound, and the upper bound.) Usually it's easiest to express uncertainties in linear (i.e. non-logarithmic) terms, so that an estimate can be given which is “*plus or minus*” some amount. In order to do this, it may require adjusting one of the bounds so that the uncertainty can be the same in both directions. For instance, the length of the building was estimated to be between 50 and 200 metres. If I think it's probably around 100 metres I could modify my estimate of “*between 50 and 200 metres*” to be “*between 50 and 150 metres*” which I could state as “ 100 ± 50 metres”.

If you have upper and lower bounds for a quantity, then the uncertainty can be *estimated* as one half of the range; i.e.

$$\text{uncertainty} \approx 1/2(\text{upper bound} - \text{lower bound})$$

IT4: In the same group of 3 or 4, using the upper and lower bounds for the list of quantities in Task 1 above, and give final estimates with an uncertainty using your upper and lower bounds in Table 14.3.

IQ5: Did all of your estimates fall midway between your upper and lower bounds? If not, how did you choose your uncertainty? If so, how would you choose your uncertainty if that happened?

Comparing Quantities with Uncertainties

IQ6: Compare your answers to the answers from one other group. Do the answers from the two groups for one item in Task 4 *agree* with each other? Give values of both groups to help explain your answer.

14.4.3 Analysis (after the lab)

There are no post-lab requirements since next week final marks will be calculated.

14.5 Recap

By the end of this exercise, you should understand the following terms:

- estimate
- bound
- logarithmic scale
- order of magnitude calculation

All of these concepts are important in research.

14.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	6	_____	50
Post-lab Questions	0	_____	0
Pre-lab Tasks	0	_____	0
In-lab Tasks	4	_____	50
Post-lab Tasks	0	_____	0

14.7 Template

My name:

My student number:

My partner's name:

My other partner's name:

My lab section:

My lab demonstrator:

Today's date:

quantity	estimate	units	upper bound	lower bound
my height				
mass of block of wood				
volume of liquid				

Table 14.1: Estimates for Task 1

quantity	estimate	units	upper bound	lower bound
height of building		m		
length of building		m		
mass of lab table		kg		

Table 14.2: Estimates for Task 2

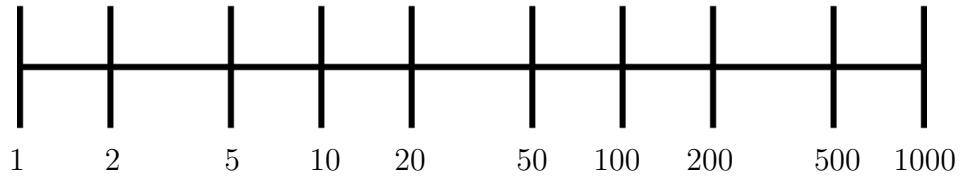


Figure 14.4: For Task 3

quantity	estimate	units	upper bound	lower bound	uncertainty
my height					
mass of block of wood					
volume of liquid					

Table 14.3: Determining Uncertainties for Task 4

Chapter 15

Measuring ‘g’

15.1 Purpose

The purpose of this experiment is to measure the acceleration due to gravity and to see if the effects of air resistance can be observed by dropping various balls and recording fall times.

15.2 Introduction

This experiment will introduce the concept of using uncertainties to compare numbers.

Actually this experiment is really intended to illustrate the *process* you will go through in each lab. The physics involved is extremely basic, so you should be able to focus on how to analyze the results and prepare the report.

This lab will actually be broken into parts, so you will spend several weeks to produce the report. After you know how to do this, you will be able to produce reports much more quickly.

The schedule will be somewhat like this:

- Collect data, including uncertainties in the raw data.
- Calculate numerical results, including uncertainties.
- Interpret the results and draw quantitative and qualitative conclusions.

- Learn how to write up “Discussion of Uncertainties” and “Conclusions” in your report.

After that, you will hand in the lab.

15.3 Theory

15.3.1 Physics Behind This Experiment

For a body falling from rest under the influence of gravity, assuming no air resistance, the height fallen in time t will be given by

$$h = \frac{1}{2}gt^2 \quad (15.1)$$

15.3.2 About Experimentation in General

In order to learn anything useful from an experiment, it is critical to collect *meaningful* data.

There are a few things to consider:

1. Data must be correct. (This means values must be recorded accurately, along with their corresponding units.)
2. Data must be consistent. (Where you have repeated measurements, they should be similar.)
3. Data must be reproducible. (If you or someone else were to come back and do this later, the data should be similar to what you got the first time. This is more determined by the clarity of your note-taking than the actual data recorded.)

Collaboration

If you are working with a partner, it is important that you both understand ahead of time what has to be done. It is easy to overlook details, but if two people are both looking at the material then it’s much less likely that something important will be missed.

Keep in mind that there may be some individual quantities which must be known when doing an experiment which can change with time. If you do not record them at the time, you may have to redo the experiment completely. Make sure you record all the variables that could influence the data. If you don't do this you may end up having to redo the entire experiment.

Technique

How you collect the data may have a huge effect on the usefulness of the data. Always try to use the best method of collecting data.

Preliminary Calculations

Before you leave the lab you need to do preliminary calculations of important results to see if they are in the right ballpark. This should prevent you from making scale errors (such as using wrong units) and forgetting to record time-sensitive values as mentioned above.

Well-Documented Raw Data

If your raw data are too messy or incomplete for you to understand later, you will have to redo the experiment. Always record

- Date
- Experimenters' names and student ID numbers
- Lab section
- Experiment name
- For each type of measurement,
 - Name of device used
 - Precision measure
 - Zero error (if applicable)
 - Other factors in measurement making realistic uncertainty bigger than the precision measure, and bound on uncertainty.

- Notes about how the measurement was taken or defined.
- For each table of data,
 - Title
 - Number
 - Units for each column
 - Uncertainties for each column; (If uncertainties change for data values in a column, make a column for the uncertainties.)
- For each question asked in the manual,
 - Question number
 - Answer

In the labs and exercises, questions are often grouped together to try and develop a “big picture” of what is going on, and so the goal is to write explanations which address a group of questions, rather than handling each one individually. This is the approach which you are to take in writing your “*Discussion of Uncertainties*” in a lab. *Remember that wherever possible, you want to answer questions from your experimental observations rather than from theories.*

Uncertainties for all Measurements

Make sure that any value you record has an uncertainty. You should record the precision measure of the instrument used. Also, list any other factors which may increase the actual uncertainty. Give your rationale for the size of the realistic uncertainty.

15.3.3 Precision Measure of an analog instrument

Many measuring instruments are not digital. Some are **analog**. This means that rather than giving a distinct value, analog instruments show a continuous range of possible values. Consider the scale reading in Figure 15.1.

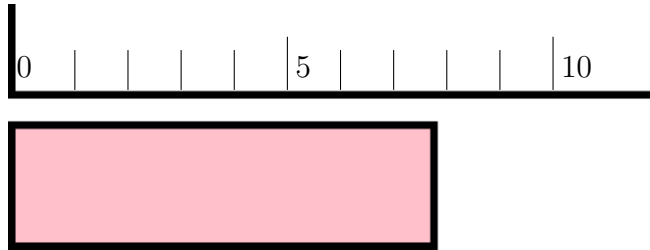


Figure 15.1: An analog scale

First of all, the left edge of the object is lined up with the zero of the scale, so we should be able to read the length of the object from the scale at the right edge of the object. It's pretty clear that that the object ends between the '7' and the '8' of the scale. Now take a look at another object measured with the same scale in Figure 15.2.

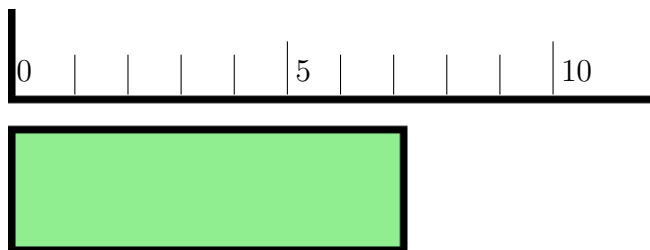


Figure 15.2: Different measurement?

This object also ends between the ‘7’ and the ‘8’ of the scale. The question is: *Can these two objects be distinguished?*

The first object is closer to the ‘8’ than the ‘7’, while the second object is closer to the ‘7’ than the ‘8’, so we might say they can *possibly* be distinguished. What scientists usually do in this situation is to **estimate** one more digit than they know for sure. So, for the first one, I might estimate it to be 7.7 units. (Someone else might estimate it to be 7.8 units, but that’s fine.) The second one I estimate to be 7.2 units. (Someone else might estimate it to be 7.3 units, but that’s also fine.) The *precision measure* of a scale like this would be said to be *one half of the smallest spacing on the scale*. Since this scale has spacings 1 unit apart, the precision measure would be 0.5 units.

15.3.4 Effective Uncertainties

Reality is not always precise

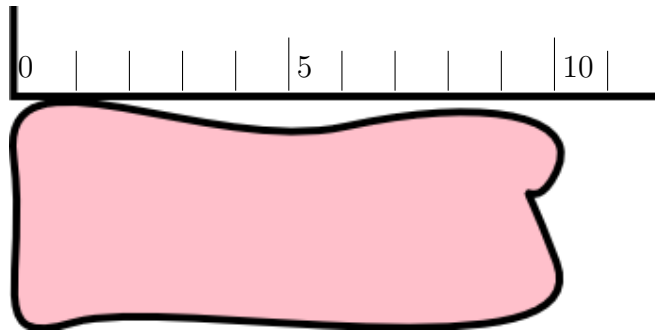


Figure 15.3: Measuring a realistic object

Many times when we take measurements the precision measure of the instrument doesn’t really matter, since we’re trying to measure something “fuzzy”. Look at the object in Figure 15.3. Since it doesn’t have straight edges, measuring the length is a bit problematic. In fact, what we want to measure may depend on why we want to measure it. Consider these questions:

1. If we used this to prop open a window, how big a window opening could we have?
2. If we wanted to determine the area of the object, what length would we want?

3. If we wanted to use the edge for drawing straight lines on paper, what length of line could we draw?

In a situation where the precision measure isn't really the limitation on the precision of a measurement, we *estimate* a **realistic** or **effective** uncertainty based on whatever sort of limits make sense.

For instance, when measuring the height for this experiment with a tape measure, you notice that it's hard to ensure that there is no bend in the tape. You can look at the difference between a measurement where the tape is as straight as you can make it and a measurement with a very slight but definite bend to put a *bound* on the effective uncertainty.

15.4 Procedure

15.4.1 Preparation (before the lab)

Pre-lab Tasks

PT1: Print off this section of the lab manual as it will not be provided. Also print Table A.1 and Table A.2 of Appendix A. Make sure you also print off the “template” sheet of the spreadsheet for this experiment and bring it to the lab. (*Alternatively, bring your laptop with both the lab manual and the spreadsheet*).

PT2: Look up the density of brass, steel, lead, aluminum, wood and cork. Explain any ambiguous information you find. Fill these values in Tables 15.1 and 15.2 and record the reference for them.

PT3: Rearrange Equation 15.1 to solve for g . In other words, complete the following:

$$g =$$

Copy the result into Table 15.3.

15.4.2 Investigation (in the lab)

Apparatus

- stopwatch
- bucket
- dense ball
- less dense ball
- tape measure

Method

Note about groups of 3: There is not really a difference between the roles in a group of 3 and a group of two. For the purpose of the experiment, anyone who is *not* the “dropper” is the “gofer”; the important thing to note is whether the person dropping the ball is the one timing it. Anyone not dropping the ball is functionally equivalent, regardless of which floor he or she is on.

While you are getting the bucket lined up, practice drops with a ping pong ball. Once you have things aligned, you should switch to the dense ball for the first set of measurements.

1. Measure h .
2. Use the denser ball, which is relatively unaffected by friction, and time one drop. Repeat this a few times to see how consistent your times are.
3. Once you have some consistency in your times, do a calculation to see whether or not this gives you a reasonable result for g .

Note: Although this is not usually *explicit* when doing a lab, whenever you collect data you should have a general idea of what you expect the data to look like. If the data you are getting are not in the expected ballpark, stop and try and figure out why not. Don't wait until you get home to check unexpected (and potentially incorrect) data.

- Drop the ball several (i.e. at least 5) times and record the fall times as recorded by both the dropper and the gofer in Table 15.5. Calculate the average fall time.

i	Times				Ball two ()
	Ball one ()				
	A dropping		B dropping		
	gofer(B)	dropper(A)	dropper(B)	gofer(A)	
1					

Figure 15.4: Section of Table 15.5

- Switch position, and repeat the previous steps.
- Calculate values for g , based on the average times. Determine which of the methods used gave the best results. Try to figure out why this might be the case. *Note that “best” has to consider both accuracy and consistency of data.*
- Use the “preferred method” to collect data for the ping pong ball, which should be more affected by friction. Drop the ball several times and record the fall times as before.
- Average the values for t , and calculate g for the second type of ball.

In-lab Tasks

Wherever you are asked to copy information into the template, you may use the appropriate spreadsheet(s) instead as long as you get them checked off. That means you can have a printout of the spreadsheet, or fill it in on your laptop in the lab.

IT1: Identify on the template which person (i.e. you or your partner) is person 'A' and which is person 'B'. *It would be wise to use the same designation of Person 'A' and Person 'B' as you did when determining your reaction time.*

Person A is: Alice
 Person B is: Bob

Figure 15.5: Section of template for **IT1**

IT2: Fill in all of the information in Table 15.5.

Instrument	
name (or reference)	
units	
precision measure	
	Times

Figure 15.6: Section of Table 15.5 for **IT2**

IT3: Record at least 2 experimental factors leading to uncertainty in h *other than the one given*. Provide at least 2 experimental factors *other than reaction time* leading to uncertainty in t . Record these in Table 15.4. Also record bounds and indicate whether these sources of uncertainty are random or systematic.

symbol	factor	bound	units
Sources of <i>systematic</i> error			
h	bend in tape measure	2	cm

Figure 15.7: Section of template for **IT3**

In-lab Questions

You may never see the lab equipment set up this way again, so make sure before leaving the lab that you have completed your determination of uncertainties in measurements and other factors affecting uncertainties.

IQ1: What is the *realistic* uncertainty in h , and what experimental factor(s) cause this uncertainty? This value should be one you feel you can defend as being neither extremely high nor extremely low. (There may be more than one contributing factor.)

IQ2: Was the technique which produced the most *accurate* time also the one which produced the most *precise* (i.e. consistent) time? How difficult is it to determine the “best” technique if the most accurate one is not the most precise one?

IQ3: Write up a *Title* and a *Purpose* for this experiment, which are more appropriate than the ones given here. Be sure to include both *quantitative* goals and *qualitative* ones in the “Purpose”.

IQ4: If a person delays starting the watch after the ball is dropped, but does not delay stopping the watch when the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

IQ5: If a person does not delay starting the watch when the ball is dropped, but delays stopping the watch after the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

IQ6: If a person delays starting the watch after the ball is dropped *and* delays stopping the watch after the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

IT4: Read over each of the inlab and postlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 15.6.

15.4.3 Analysis (after the lab)

Before learning how to analyze uncertainties, there may be few obvious conclusions to be drawn from any experiment, including this one. Some of the following steps will be completed in later exercises.

1. Calculate the standard deviation and standard deviation of the mean for the sets of times for each type of ball, and then determine the uncertainty in \bar{t} for each data set.
2. Given the values for the precision measure and σ , determine the optimum number of measurements for both balls using the preferred technique.
3. Determine the formula for the uncertainty in g , given the uncertainties in h and \bar{t} .
4. Calculate the uncertainty in your values of g using the uncertainties determined above.

Post-lab Discussion Questions

The following questions will be able to be answered after further exercises.

Answers to the following questions will form the basis of the *Discussion* and *Conclusions* sections of your lab report. Write these sections in paragraph form, with each individual answer underlined or highlighted. At the beginning of each question put the question number in super-script. The goal is to have a flow to the whole section, rather than to have the section appear as a series of statements of unrelated facts. *Be sure to include your numerical results to explain your answers.*

Q1: Were the times given by different methods for the same type of ball significantly different? In other words, did they agree within their uncertainties or not? (Include your actual calculated values in your explanation.)

Q2: Would a more precise stop watch reduce the uncertainty in t or not? Explain. (*Note: Timing technique may help, but that's a different matter!!*)

Q3: Calculate the optimal number of measurements need for each type of ball. Based on this calculation, would timing more drops help in reducing

the uncertainty of the average times? In this experiment would it be feasible to take the optimal number of measurements?

Q4: Were the times given by the preferred method for the first ball and the second ball significantly different? (Include your actual calculated values in your explanation.)

Q5: Measurements of reaction time (random, anticipated, or synchronization) will apply differently to the droppers and gofers. Which of the above situations would apply to the dropper and which would apply to the gofer? Was there any correlation between the reaction times you measured and the variation in times you had when dropping the ball? In other words, did your measurements of reaction time give any useful insight into the experiment?

Q6: Would a more precise device to measure h reduce the uncertainty in g or not? Explain.

Q7: Do either of the values for g determined using the preferred technique agree with the accepted value? Explain. (This question can be answered definitively based on your uncertainties.)

Remember that values agree if the difference between them is less than the sum of their uncertainties. If they do not agree then you should calculate the percent difference between them. (DON'T calculate the percent difference if they agree!! That's what "agreement" is all about!!)

Q8: Are any of your resulting values for g *higher* than expected? What could explain that? What bounds does that give on factors like reaction time? Explain. (Hint: Look at your answers to the in-lab questions.)

Q9: Do the two different values for g using the preferred technique suggest friction is significant or not? Explain. (As with the previous question, this can answered definitively based on your uncertainties.)

15.5 Recap

By the time you have finished this lab report, you should know how to :

- collect data and analyze it
- write a lab report which includes:
 - *title* which describes the experiment
 - *purpose* which explains the objective(s) of the experiment
 - *results* obtained, including data analysis
 - *discussion of uncertainties* explaining significant sources of uncertainty and suggesting possible improvements
 - *conclusions* about the experiment, which should address the original objective(s).

In addition, you should have some understanding about how *experimental technique* can influence your results, and what to do about it.

15.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	_____	0
In-lab Questions	6	_____	70
Post-lab Questions	9		(in report)
Pre-lab Tasks	3	_____	10
In-lab Tasks	4	_____	20
Post-lab Tasks	0	_____	0

15.7 Template

My name:
 My student number:
 My partner's name:
 My other partner's name:
 My lab section:
 My lab demonstrator:
 Today's date:

Person A is:
 Person B is:

The dense ball is made of:
 The other ball is :

quantity	symbol		single/ repeated/ constant
	given/ mine		
Not in equations			

Table 15.1: List of quantities

symbol	value	units	instrument			effective uncertainty
			name or reference (e.g. A.1)	precision measure	zero error	
Not in equations						

Table 15.2: Single value quantities

quantity	symbol	equation	uncertainty
acceleration due to gravity			

Table 15.3: Calculated quantities

symbol	factor	bound	units
Sources of <i>systematic</i> error			
h	bend in tape measure		
Sources of <i>random</i> error			

Table 15.4: Experimental factors responsible for effective uncertainties

Instrument					
name (or reference)					
units					
precision measure					
<i>i</i>	Times				
	Ball one ()				Ball two ()
	A dropping		B dropping		
	gofer(B)	dropper(A)	dropper(B)	gofer(A)	
1					
2					
3					
4					
5					
average					
<i>g</i>					
σ_t					
α_t					
$\Delta(\bar{t})$					

Table 15.5: Timing data

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Post-lab		
Hints		
	“think” “suggest” “explain” “how” “why” “what”	“agree” “equal” “do (did, does) ” “significantly different” “support” “verify”

Table 15.6: Lab Report Organization

Appendix A

Information about Measuring Instruments

Fill this in as you use new measuring instruments so you will have a reliable reference. Put frequently used instruments in Table A.1 and experiment-specific ones in Table A.2.

ref. #	measuring instrument	precision measure	range	units
A1	stopwatch			
A2	metre stick			
A3				
A4				
A5				
A6				

Table A.1: Measuring instrument information

ref. #	measuring instrument	precision measure	range	units
B1	spring scale A			
B2	spring scale B			
B3				
B4				
B5				
B6				
B7				
B8				
B9				
B10				
B11				
B12				
B13				
B14				
B15				

Table A.2: Measuring instrument information (continued)

Appendix G

Lab Checklist

This marking checklist will be used for lab reports this term. You need to print one off and attach it to each lab report you hand in. Lab reports will be marked as follows:

- Start with 90

For items *not* in italics

- Subtract 1 for each \sim .
- Subtract 2 for each $-$.

For items *in italics*

- Subtract 3 for each \sim .
- Subtract 6 for each $-$.

Note the importance of items in italics. These are very important in a report, and so are weighted accordingly.

The other 10 marks will be based on how well the post-lab discussion questions were answered within the text of the report. *Remember that the answers to these questions should be an integral part of the report, not merely an afterthought.*

Lab Format Checklist (V3.0ng)

A. General

1. Your own work _____
2. Complete _____
3. Clear and appropriate "Purpose" _____
4. Flows _____
5. Did not require help on or after due date _____
6. Correct grammar _____
7. Correct spelling _____
8. Complete sentences where required _____
9. Legible _____
10. Professionally presented _____
11. Properly identified (e.g. name, partner) _____
12. On time _____
13. Checklist included _____
14. Template included _____

B. *Plagiarism Avoidance*

1. *Data only shared with partner(s)* _____
2. *Individual choice of sample data* _____
3. *Individual formatting* _____
4. *Individual structure of text* _____

C. Data

1. Neat _____
2. Table column headings informative _____
3. Units given _____
4. Uncertainties given _____
5. Reasonable values _____
6. Reasonable uncertainties _____
7. Correct number of significant figures _____
8. Tables labeled (e.g. "Sample 1 Data") _____
9. Tables given numbers (e.g. "Table #2") _____

D. Calculations and Results

1. Any required derivations done correctly _____
2. Analysis explained where needed _____
3. Correct formulas used _____
4. Sample calculations shown where needed _____
5. All required values calculated _____
6. Uncertainties included _____
7. Units included _____
8. Correct number of significant figures _____
9. Appropriate use of standard form _____
10. Theoretical or reasonable value _____
11. Agreement of experiment with theory _____

E. *Error Discussion*

1. *Sources listed are significant* _____
2. *Sources are prioritized* _____
3. *Systematic error consequences* _____
4. *Evidence: i.e. test or bound* _____
5. *Reasonable suggestions for improvement* _____

F. *Conclusions*

1. *Relate to purpose* _____
2. *Major results stated* _____
3. *Comparisons made where appropriate* _____
4. *Agreement noted when found* _____
5. *% difference only when no agreement* _____

G. References

1. Source(s) of constants listed _____

