PC131 Lab Manual
Wilfrid Laurier University

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Chapter 1

Lab Manual Layout

1.1 This is a Reference

Just as the theory you learn in courses will be required again in later courses, the skills you learn in the lab will be required in later lab courses. *Save this manual as a reference: you will be expected to be able to do anything in it in later lab courses.* That is part of why the manual has been made to fit in a binder; it can be combined with later manuals to form a reference library.

1.2 Parts of the Manual

The lab manual is divided mainly into four parts: *background information*, *lab exercises*, *experiments*, and *appendices*.

New definitions are usually presented **like this** and words or phrases to be highlighted are *emphasized like this*.

1.3 Lab Exercises and Experiment Descriptions

Each experiment description is divided into several parts:

- **Purpose**

  The specific objectives of the experiment are given. These may be in terms of *theories to be tested* (see Theory below) or in terms of *skills* *to be developed* (see Introduction below).

- **Introduction**

  This section should explain or mention new measuring techniques or equipment to be used, data analysis methods to be incorporated, or other *skills* to be developed in this experiment. *Knowledge is cumulative; what you learn in one lab you will be assumed to know and use subsequently, in this course and beyond. As well, proficiency comes with practice; the only way to become comfortable with a new skill is to take every opportunity to use it. If you get someone else (such as your lab partner) to do something which you don't like doing, you will never be able to do it better, and will get more intimidated by it as time goes on.*

- **Theory**

  (Note: lab exercises and experiment descriptions will make slightly different use of the "theory" section. In a lab exercise, "theory" will refer to explanations and derivations of the *techniques* being taught. In experiment descriptions, "theory" will refer to the *physics* behind an experiment.)

  A physical theory is often expressed as a mathematical relationship between measurable quantities. Testing a theory involves trying to determine whether such a mathematical relationship *may* exist. All measurements have **uncertainties** associated with them, so we can only say whether or not any difference between our results and those given by the relationship (theory) can be accounted for by the *known* uncertainties or not. (There may be other factors affecting the results which were not accounted for.) We cannot conclude that a theory is "true" or "false", only whether our experiment "agrees" with or "supports" it. Experimentation in general is an **iterative** process; one sets up an experiment, performs it and takes measurements, analyzes the results, refines the experiment, and the process repeats. No experiment is ever "perfect", although it may at some point be "good enough", meaning that it demonstrates what was required *within experimental uncertainty.* The theory section for an experiment should give any mathematical relationship(s) pertinent to that experiment, along with any definitions, etc. which may be needed. *You don't have to understand a theory in depth to test it; inasmuch as it is a mathematical* *relationship between measurable quantities, all you need to understand is* how *to measure the quantities in question and* how *they are related.* This is why whether or not you understand the theory is irrelevant in the lab. (In fact, you may at times find the experiment helps you understand the theory, whether you do the lab before or after you cover the material in class.)

Consider the example of Table 1.1

qwertys	*poiuyts*
1	1
2	3
3	5

Table 1.1: Relationship between qwertys and poiuyts

If you were asked, "Does $p = q^2$?", you would say no. It is no different to be asked "Is the following theorem correct?"

Lab Practice 1 *In a first year physics laboratory, poiuyts always vary as the square of qwertys.*

As long as you can measure qwertys and poiuyts (or can calculate them from other things you *can* measure), then you can answer the question, even without knowing *why* there should be such a relationship.

Sometimes there is a *disadvantage* to knowing too much about what to expect. It is easy to overlook unexpected data because it is not "right"; (meaning it doesn't give you the result you expected.)

The data are always right!

If your data[1] are giving you a result you don't like, that is a message that either you have made a mistake or there is more going on than you have accounted for.

[1] Dat*um* is the singular term. Dat*a* is the plural term.

- **Procedure**

  This tells you what you are required to do to perform the experiment. Unless you are told otherwise, these instructions are to be followed precisely. If there are any changes necessary, you will be informed in the lab.

  Questions will need to be handed in; *tasks* will be checked off in the lab.

  Included in this section are three important subsections:

  - Preparation (including pre-lab questions and tasks)

    The amount of time spent in a lab can vary greatly depending on what has been done ahead of time. This section attempts to minimize wasted time in the lab .

  - Experimentation or Investigation

    (including in-lab questions and tasks)

    For some lab exercises, there won't be an "experiment" as such, but there will be things to be done in the lab. The in-lab questions can usually only be answered while you have access to the lab equipment, but the answers will be important for further calculations and interpretation. For computer labs, instead of questions there will often be tasks, consisting of points to be demonstrated while you are in the lab.

  - Analysis (usually for labs) or Follow-up (usually for exercises)

    (including post-lab questions and tasks)

    Many of the calculations for a lab can be done afterward, provided you understand clearly what you are doing in the lab, record all of the necessary data, and answer all of the in-lab questions. The post-lab questions summarize the important points which must be addressed in the lab report.

    Since most of the exercises are developing skills, the results can usually be applied immediately to labs either already begun or upcoming.

- **Bonus**

  Most experiments will have a bonus question allowing you to take on further challenges to develop more understanding, either of data analysis concepts or of the underlying theory. (Bonus questions will usually be worth an extra 5% or so on the lab if they are done correctly.)

1.4 Templates

Each experiment, and many exercises, include a **template**. This is to help you ensure that you are not missing any data when you leave the lab. Since you will perform many calculations outside the lab, you'll need to make sure you have everything you need before you leave.

1.4.1 Table format in templates and lab reports

The templates are set up to help you consistently record information. For that reason, the tables are very "generic". It would be more concise to create tables that are specific to each experiment, but that would not be as helpful for your education. *When you write a report, you should set up tables which are concise and experiment-specific, even if they look different than the ones in the template.*

> Don't automatically set up tables in your lab reports like the ones in the templates.

1.4.2 Template tables

- The first table contains information which you should record *every* time you do an experiment, and looks like this:

  My name:
  My student number:
  My partner's name:
  My other partner's name:
  My lab section:
  My lab demonstrator:
  Today's date:

1.4.3 Before the lab

- Read through the lab, and find (or create) symbols for each of the quantities in the experiment, and fill in Table 1.2.

quantity	symbol		single/ repeated/ constant
		given/ mine	
Not in equations			

Table 1.2: List of quantities

- For any quantities to be *calculated*, fill in the equations in Table 1.3.

quantity	symbol	equation	uncertainty
acceleration due to gravity			

Table 1.3: Calculated quantities

- For any *constants* to be used, either in calculations or to be compared with results, look up values and fill in Table 1.4.

1.4.4 In the lab

- For any *measuring instruments*, fill in the information in the appropriate tables.

symbol	value	units	instrument			effective uncertainty
			name	precision measure	zero error	
Not in equations						

Table 1.4: Single value quantities

- For any quantity measured *only once*, fill in Table 1.4. *There may be quantities which you do not need in your calculations, but which you would like to record for completeness. For that purpose there is a section in the table under the heading* Not in equations*.*
- If there is an effective uncertainty for the quantity in Table 1.4, then give specific information in Table 1.5. *Always include at least one source of systematic error, even if the bound you give is small enough to make it insignificant. This is so that you can show you understand how it would affect the results if it were big enough.*
- Sometimes there may be several parts to an experiment, in which case it may help to keep things straight by separating them in the table, as in Table 1.5.
- For repeated measurements, there will be experiment-specific tables, such as Table 1.6.
- For any quantity where measurement is *repeated*, fill in Table 1.6. *In this case, there is no place for an effective uncertainty, since that will be determined by statistical analysis.*

1.4.5 Spreadsheet Templates

For some labs and exercises, there will be spreadsheets set up to potentially help you do your calculations. *For experiment-specific tables, you*

symbol	factor	bound	units
For Part 1			
Sources of *systematic* error			
Sources of *random* error			
For Part 2			
Sources of *systematic* error			
Sources of *random* error			

Table 1.5: Experimental factors responsible for effective uncertainties

may choose to print and bring the spreadsheet template instead. This will make the most sense if you are going to use the spreadsheet to do your calculations.

Even when there is a spreadsheet, there will still be information in the lab to record in the manual template, which does not have corresponding sections in the spreadsheet.

Instrument	
reference (or name)	
units	
precision measure	
zero error	
Trial #	Angle
1	
2	
3	
4	
5	

Table 1.6: Example of experiment-specific table

- Figure 1.1 is an example of how the corresponding experiment-specific table might look in the spreadsheet. The extra cells at the bottom are for calculation results.

1.5 Steps in processing lab data

Any experiment involves a similar process, regardless of the details.

1. **Collect** data.
    - Record measurements.
2. **Transform** data.
    - Often the quantities measured are not the desired ones for calculations.. For instance, mass is measured but force is required.
3. **Combine** data.
    - Calculate mean, standard deviation, standard deviation of the mean, uncertainty in the mean

	Instrument
name	
units	degrees
precision	
measure	
zero error	
Data	
Trial	Angle
1	
2	
3	
4	
5	

Table 2a

Figure 1.1: Example of spreadsheet table

4. **Interpret** data.

    - Use combined data (mean and its uncertainty) to test general principle or determine global parameter(s).

Often additional calculations must be performed on the combined data before general interpretations can be made, but I have included that as part of the "interpretation" process.

Chapter 2

Goals for PC131 Labs

If you're going to be a cook, you have to read some cookbooks, but eventually you need to get into the kitchen and cook.

If you're going to be an artist, you can study art and go to galleries, but eventually you're going to need to go into a studio.

If you're going to become a programmer, you can read about computers, study operating systems and programming languages, but eventually you have to sit down and program.

If you're going to become a scientist, you can read about science, go to lectures and watch videos about science, but eventually you need to go into a lab and do some research.

In the lecture part of this course, you'll learn a lot of physics. But it's only in the lab that you can learn about how to do research, which is ultimately what science is all about.

The labs and exercises in this course are to teach you about how to collect, analyze, and interpret data, and how to report your results so that they can be useful to other researchers.

The labs and exercises are chosen to teach research principles, rather than to illustrate specific physics concepts. This means, (among other things), that the "theory" behind any particular lab may not be covered in great detail in class. What you need will be covered briefly in the lab (or the manual).

Chapter 3

Instructions for PC131 Labs

Students will be divided into sections, each of which will be supervised by a lab supervisor and a demonstrator. This lab supervisor should be informed of any reason for absence, such as illness, as soon as possible. (If the student knows of a potential absence in advance, then the lab supervisor should be informed in advance.) A student should provide a doctor's certificate for absence due to illness. Missed labs will normally have to be made up, and usually this will be scheduled as soon as possible after the lab which was missed *while the equipment is still set up for the experiment in question.*

It is up to the student to read over any theory for each experiment and understand the procedures and do any required preparation before the laboratory session begins. This may at times require more time outside the lab than the time spent in the lab.

Students are normally expected to complete all the experiments assigned to them, and to submit a written report of your experimental work, including raw data, as required.

You will be informed by the lab instructor of the location for submission of your reports during your first laboratory period. This report will usually be graded and returned to you by the next session. The demonstrator who marked a particular lab will be identified, and any questions about marking should first be directed to that demonstrator. *Such questions must be directed to the marker within one week of the lab being returned to the student if any additional marks are requested.*

Unless otherwise stated, all labs and exercises will count toward your lab mark, although they may not all carry equal weight. If you have questions about this talk to the lab supervisor.

3.1 Expectations

As a student in university, there are certain things expected of you. Some of them are as follows:

- You are expected to *come to the lab prepared.* This means first of all that you will ensure that you have all of the information you need to do the labs, *including answers to the pre-lab questions.* After you have been told what lab you will be doing, you should read it ahead and be clear on what it requires. You should bring the lab manual, lecture notes, etc. with you to every lab. (*Of course you will be on time* so you do not miss important information and instructions.)

- You are expected to *be organized* This includes recording raw data with sufficient information so that you can understand it, keeping proper backups of data, reports, etc., hanging on to previous reports, and so on. It also means starting work early so there is enough time to clarify points, write up your report and hand it in on time.

- You are expected to *be adaptable and use common sense.* In labs it is often necessary to change certain details (eg. component values or procedures) at lab time from what is written in the manual. You should be alert to changes, and think rationally about those changes and react accordingly.

- You are expected to *value the time of instructors and lab demonstrators.* This means that you make use of the lab time when it is scheduled, and try to make it as productive as possible. This means NOT arriving late or leaving early and then seeking help at other times for what you missed.

- You are expected to *act on feedback from instructors, markers, etc.* If you get something wrong, find out how to do it right and do so.

- You are expected to *use all of the resources at your disposal.* This includes everything in the lab manual, textbooks for other related courses, the library, etc.

- You are expected to *collect your own data.* This means that you perform experiments with your partner and *no one else.* If, *due to an* *emergency*, you are *forced* to use someone else's data, you *must* explain why you did so and explain *whose* data you used. Otherwise, you are committing *plagiarism*.

- You are expected to *do your own work*. This means that you prepare your reports with *no one else*. If you ask someone else for advice about something in the lab, make sure that anything you write down is based on your *own* understanding. If you are basically regurgitating someone else's ideas, even in your own words, you are committing *plagiarism*. (See the next point.)

- You are expected to *understand your own report*. If you discuss ideas with other people, even your partner, do not use those ideas in your report unless you have adopted them yourself. You are responsible for all of the information in your report.

- You are expected to *be professional* about your work. This means meeting deadlines, understanding and meeting requirements for labs, reports, etc. This means *doing what should be done*, rather than what you think you can get away with. This means proofreading reports for spelling, grammar, etc. before handing them in.

- You are expected to *actively participate* in your own education. This means that in the lab, you do not leave tasks to your partner because you do not understand them. This means that you try and learn *how* and *why* to do something, rather than merely finding out the *result* of doing something.

3.2 Workload

Even though the labs are each only worth part of your course mark, the amount of work involved is probably disproportionately higher than for assignments, etc. Since most of the "hands-on" portion of your education will occur in the labs, this should not be surprising. (*Note: skipping lectures or labs to study for tests is a very bad idea.* Good time management is a much better idea.)

3.3 Administration

1. Students will be required to have a binder to contain all lab manual sections and all lab reports which have been returned. (A 3 hole punch will be in the lab.)

2. Templates will be used in each experiment as follows:

   (a) The template must be checked and initialed by the demonstrator before students leave the lab.

   (b) No more than 3 people can use one set of data. If equipment is tight groups will have to split up. (i.e. Only as many people as fit the designated places for names on a template may use the same data.)

   (c) Part of the lab mark will be for the template.

   (d) The template *must* be included with lab handed in. penalty will be incurred if it is missing.) It must be the original, not a photocopy.

   (e) If a student misses a lab, and if space permits (decided by the lab supervisor) the student may do the lab in another section the same week without penalty. (However the due date is still for the student's own section.) In that case the section recorded on the template should be where the experiment was *done*, not where the student normally belongs.

3. In-lab tasks must be checked off before the end of the lab, and answers to in-lab questions must be handed in at the end of the lab. Students are to make notes about question answers and keep them in their binders so that the points raised can be discussed in their reports. Marks for answers to questions will be added to marks for the lab. For people who have missed the lab without a doctor's note and have not made up the lab, these marks will be forfeit. The points raised in the answers will still be expected to be addressed in the lab report.

4. Labs handed in after the due date incur a late penalty according to the lateness of the submission. After the reports for an experiment have been returned, any late reports submitted for that experiment cannot receive a grade higher than the lowest mark from that lab section for the reports which were submitted on time.

No labs will be accepted after the last day of classes.

3.4 Plagiarism

5. Plagiarism includes the following:
   - Identical or nearly identical wording in any block of text.
   - Identical formatting of lists, calculations, derivations, etc. which *suggests* a file was copied.
6. You will get one warning the first time plagiarism is suspected. After this any suspected plagiarism will be forwarded directly to the course instructor. With the warning you will get a zero on the relevant section(s) of the lab report. If you wish to appeal this, you will have to discuss it with the lab supervisor and the course instructor.
7. If there is a suspected case of plagiarism involving a lab report of yours, it does not matter whether yours is the original or the copy. The sanctions are the same.

3.5 Calculation of marks

8. The precise weightings of labs, exercises, and anything else will be discussed later in the lab manual.
9. The weighting of individual labs and exercises may depend on the quality of the work; i.e. if you do better work on some things they will count more toward your final grade. Details will be discussed in the lab.
10. There may be a lab test at the end of term.

Chapter 4

How To Prepare for a Lab

> "The theory section for an experiment should give any mathematical relationship(s) pertinent to that experiment, along with any definitions, etc. which may be needed. *You don't have to understand a theory in depth to test it; inasmuch as it is a mathematical relationship between measurable quantities all you need to understand is* how *to measure the quantities in question and* how *they are related.* This is why whether or not you understand the theory is irrelevant in the lab. (In fact, you may at times find the experiment helps you understand the theory, whether you do the lab before or after you cover the material in class.)"

1. *Check* the web page after noon on the Friday before the lab to make sure of what you need to bring, hand in, etc. (It is a good idea to check the web page the day of your lab, in case there are any last minute corrections to the instructions.)

2. *Read over* the lab write–up to determine what the physics is behind it. (Even without understanding the physics in detail, you can do all of the following steps.)

3. *Answer* all of the pre-lab questions and bring the answers with you to the lab.

4. *Complete* all of the pre-lab tasks and bring anything that needs to be checked off with you to the lab.

5. *Examine* the spreadsheet and/or template for the lab (if either of them exists) to be sure that you understand what all of the quantities, symbols, etc. mean. (If there is a spreadsheet, you can prepare any or all of the formulas before the lab to simplify analysis later.)

6. *Enter* any constants into the appropriate table(s) in the template.

7. *Highlight* all of the in-lab questions and tasks so you can be sure to answer them all in the lab.

8. *Check* the web page the day of the lab in case there are any last minute changes or corrections to previous instructions.

9. *Arrive* on time, prepared. Bring all previous labs, calculator, and anything else which might be of use. (If the theory is in your textbook, maybe it would be good to bring your textbook to the lab!)

Chapter 5

Plagiarism

5.1 Plagiarism vs. Copyright Violation

These two concepts are related, but may get confused. Both involve unethical re-use of one person's work by another person, but they are different because the victim is different in each case.

Copyright is the right of an author to control over the publication or distribution of his or her own work. A violation of copyright is, in effect, a crime against the *producer of the work*, since adequate credit and/or payment is not given.

Plagiarism is the presentation of someone else's work as one's own, and thus the crime is against the *reader or recipient of the work* who is being deceived about its source.

Putting these two together suggests that there is a great deal of overlap, since trying to pass off someone else's work *without that person's permission* as one's own is both plagiarism and a violation of copyright. However, copying someone else's work without permission, *even while admitting who produced it*, is still a violation of copyright. Conversely, presenting someone else's work as your own, *even with that person's permission*,is still plagiarism.

5.2 Plagiarism Within the University

The Wilfrid Laurier University calendar says: " plagiarism ... *is the unacknowledged presentation, in whole or in part, of the work of others as one's* *own, whether in written, oral or other form, in an examination, report, assignment, thesis or dissertation* ”

A search of the university web site for the word “plagiarism” turns up several things, among them the following:

- “Of course, under no circumstance is it acceptable to directly use an author’s words (or a variation with only a few words of a sentence changed) without giving that author credit; this is plagiarism!!!” (Psychology 229)

- “plagiarism, which includes but is not limited to: the unacknowledged presentation, in whole or in part, of the work of others as one’s own; the failure to acknowledge the substantive contributions of academic colleagues, including students, or others; the use of unpublished material of other researchers or authors, including students or staff, without their permission;” (Faculty Association Collective Agreement)

- “DO NOT COPY DOWN A SECTION FROM YOUR SOURCE VERBATIM OR WITH VERY MINOR CHANGES. This is PLAGIARISM and can lead to severe penalties. Obviously,no instructor can catch all offenders but, to paraphrase the great Clint Eastwood, “What you need to ask yourself is ‘Do I feel lucky today?’ ” (Contemporary Studies 100 Notes on Quotes)

- “Some people seem to think that if they use someone else’s work, but make slight changes in wording, then all they need to do is make reference to the “other” work in the standard way, i.e., (Smith, 1985), and there is no plagiarism involved. This is not true. You must either use direct quotes (with full references, including page numbers) or completely rephrase things in your own words (and even here you must fully reference the original source of the idea(s)).” (Psychology 306, quote from *Making sense in psychology and the life sciences: A student’s guide to writing and style* , by Margot Northey and Brian Timney (Toronto: Oxford University Press, 1986, pp. 32-33).)

- “Any student who has been caught submitting material that is not properly referenced, where appropriate, or submits material that is copied from another source (either a text or another student’s lab), will be subject to the penalties outlined in the Student Calendar." (Geography 100)

- "Paraphrasing means restating a passage of a text in your own words, that is, rewording the ideas of someone else. In such a case, proper reference to the author must be given, or it is plagiarism. Copying a passage verbatim (not paraphrased) also constitutes plagiarism if it is not placed in quotes and is not referenced. Plagiarism is the appropriation or imitation of the language, ideas, and thoughts of another author, and the representation of these as one's own." (Biology 100)

5.3 How to Avoid Plagiarism

Plagiarism is a serious offense, and will be treated that way, but often students are unclear about what it is. The above quotes should help, but here are some more guidelines:

- If you use the same data as anyone else, this should be clearly documented in your report, WHETHER THE DATA ARE YOURS OR THEIRS.
- Use different data values than the ones used by your partner for your sample calculations.
- If you copy any file, even if you modify it, it is plagiarism unless you clearly document it. (This does not mean you can copy whatever you like as long as it's documented; you still are expected to do your own work. However at least you're not plagiarizing if you document your sources properly.)
- You are responsible for anything in your report; if you answer a question about your report with, "I don't know, my partner did that part", you are guilty of plagiarism, because you are passing off your partner's work as your own.
- The purpose for working together is to help each other learn. If collaboration is done in order for one or more people to avoid having to learn and/or work, then it is very likely going to involve plagiarism, (and is a no-no for pedagogical reasons anyway.)

- If you *give* your data, files, etc. to anyone else and they plagiarize it *you* are in trouble as well, because you are aiding their attempt to cheat. *Do not give out data, files, or anything else* without express permission from the lab supervisor. This includes giving others your work to "look at"; if you give it to them, for whatever reason, and they copy it, *you* have a problem.

- If you want to talk over ideas with others, *do not* write while you are discussing; if everyone is on their own when they write up their reports, then the group discussion should not be a problem. However, as in a previous point, do not use group consensus as justification for what you write; discussion with anyone else should be to help you sort out your thoughts, not to get the "right answers" for you to parrot.

Look at the document from the writing centre, "How to Use Sources and Avoid Plagiarism".

Chapter 6

Lab Reports

A lab report is *personal*, in the sense that it explains what *you* did in the lab and summarizes *your* results, as opposed to an assignment which generally answers a question of some sort. On an assignment, there is (usually) a "right answer", and finding it is the main part of the exercise. In a lab report, rather than determining an "answer", you may be asked to *test* something. (Note that no experiment can ever *prove* anything; it can only provide evidence for or against; just like in mathematics finding a single case in which a theorem holds true does not prove it, although a single case in which it does not hold refutes the theorem. A **law** in physics is simply a theorem which has been tested countless times without evidence of a case in which it does not hold.) The point of the lab report, when testing a theorem or law, is to explain whether or not you were successful, and to give reasons why or why not. In the case where you are to produce an "answer", (such as a value for g), your answer is likely to be different from that of anyone else; your job is to describe how you arrived at yours and why it is reasonable under the circumstances.

6.1 Format of a Lab Report

The format of the report should be as follows:

6.1.1 Title

The title should be more specific than what is given in the manual; it should reflect some specifics of the experiment.

6.1.2 Purpose

The specific purpose of the experiment should be briefly stated. (Note that this is not the same as the goals of the whole set of labs; while the labs as a group are to teach data analysis techniques, etc., the specific purpose of one experiment may indeed be to determine a value for g, for instance.) *Usually, the purpose of each experiment will be given in the lab manual. However, it will be very general. As in the title, you should try and be a bit more specific.*

There should always be both **qualitative** and **quantitative** goals for a lab.

Qualitative

This would include things like *"see if the effects of friction can be observed"*. In order to achieve this, however, specific quantitative analyses will need to be performed.

Quantitative

In a scientific experiment, there will always be numerical results produced which are compared with each other or to other values. It is based on the results of these comparisons that the qualitative interpretations will be made.

6.1.3 Introduction

In general, in this course, you will not have to write an Introduction section.

An introduction contains two things: *theory* for the experiment and *rationale* for the experiment.

Theory

Background and theoretical details should go here. Normally, detailed derivations of mathematical relationships should not be included, but references must be listed. All statements, equations, and 'accepted' values must be justified by either specifying the reference(s) or by derivation if the equation(s) cannot be found in a reference.

Rationale

This describes why the experiment is being done, which may include references to previous research, or a discussion of why the results are important in a broader context.

6.1.4 Procedure

The procedure used *should not be described* unless you deviate from that outlined in the manual, or unless some procedural problem occurred, which must be mentioned. A reference to the appropriate chapter(s) of the lab manual is sufficient most of the time.

Ideally, someone reading your report and having access to the lab manual should be able to *reproduce* your results, within reasonable limits. (Later on we will discuss what "reasonable limits" are.) If you have made a mistake in doing the experiment, then your report should make it possible for someone else to do the experiment *without* making the same mistake. For this reason, lab reports are required to contain **raw data**, (which will be discussed later), and **explanatory notes**.

Explanatory notes are recorded to

- explain any changes to the procedure from that recorded in the lab manual,
- draw attention to measurements of parameters, values of constants, etc. used in calculations, and
- clarify any points about what was done which may otherwise be ambiguous.

Although the procedure need not be included, your report should be clear enough that the reader does not need the manual to understand your write–up.

(If you actually need to describe completely how the experiment was done, then it would be better to call it a "Methods" section, to be consistent with scientific papers.)

6.1.5 Experimental Results

There are two main components to this section; *raw data* and *calculations.*

Raw Data

In this part, the reporting should be done part by part with the numbering and titling of the parts arranged in the same order as they appear in the manual.

The *raw data* are provided so that someone can work from the actual numbers you wrote down originally before doing calculations. Often mistakes in calculation can be recognized and corrected after the fact by looking at the raw data.

In this section:

- Measurements and the names and precision measures of all instruments used should be recorded; in tabular form where applicable.
- If the realistic uncertainty in any quantity is bigger than the precision measure of the instrument involved, then the cause of the uncertainty and a bound on its value should be given.
- Comments, implicitly or explicitly asked for regarding data, or experimental factors should be noted here. This will include the answering of any given *in-lab* questions.

Calculations

There should be a clear path for a reader from raw data to the final results presented in a lab report. In this section of the report:

- Data which is modified from the original should be recorded here; in tabular form where applicable.
- Uncertainties should be calculated for all results, unless otherwise specified. The measurement uncertainties used in the calculations should be those listed as realistic in the raw data section.
- Calculations of quantities and comparisons with known relationships should be given. If, however, the calculations are repetitive, only one sample calculation, shown in detail, need be given. Error analysis should appear here as well.

- Any required graphs would appear in this part. (More instruction about how graphs should be presented will be given later.)
- For any graph, a table should be given which has columns for the data (including uncertainties) which are actually plotted on the graph.
- Comments, implicitly or explicitly asked for regarding calculations, observations or graphs, should be made here.

Sample calculations may be required in a particular order or not. If the order is not specified, it makes sense to do them in the order in which the calculations would be done in the experiment. If the same data can be carried through the whole set of calculations, that would be a good choice to illustrate what is happening.

> Printing out a spreadsheet with formulas shown does not count as showing your calculations; the reader does not have to be familiar with spreadsheet syntax to make sense of results.

> *Post-lab* questions should not be answered in a numbered list; rather the answers should be integrated in to the *Discussion* and *Conclusion* sections based on where they would be most appropriate.

6.1.6 Discussion

This section is where you explain the significance of what you have determined and outline the *reasonable limits* which you place on your results. (This is what separates a scientific report from an advertisement.) It should outline the major sources of random and systematic error in an experiment. *Your emphasis should be on those which are most significant, and on which you can easily place a numerical value. Wherever possible, you should try to suggest evidence as to why these may have affected your results, and include recommendations for how their effects may be minimized.* This can be accompanied by suggested improvements to the experiment.

Two extremes in tone of the discussion should be avoided: the first is the "sales pitch" or advertisement mentioned above, and the other is the "apology" or disclaimer (" *I wouldn't trust these results if I were you; they're* *probably hogwash.")* *Avoid whining* about the equipment, the time, etc. Your job is to explain briefly what factors most influenced your results, not to absolve yourself of responsibility for what you got, but to suggest changes or improvements for someone attempting the same experiment in the future. Emphasis should be placed on improving the experiment by changed *technique*, (which may be somewhat under your control), rather than by changed *equipment*, (which may not).

> *M*any of the in-lab questions are directed to things which ought to be discussed here. Like the post-lab questions, don't answer them in a list, but integrate them into the text.

This section is usually worth a large part of the mark for a lab so be prepared to spend enough time thinking to do a reasonable job of it.

> You must discuss at least one source of systematic error in your report, even if you reject it as insignificant, in order to indicate how it would affect the results.

6.1.7 Conclusions

Just as there are always both **qualitative** and **quantitative** goals for a lab, there should always be both **qualitative** and **quantitative** *conclusions* from a lab.

Qualitative

This would include things like *"see if the effects of friction can be observed"*. In order to achieve this, however, specific quantitative analyses will need to be performed.

Quantitative

In a scientific experiment, there will always be numerical results produced which are compared with each other or to other values. It is based on the results of these comparisons that the qualitative interpretations will be made.

General comments regarding the nature of results and the validity of relationships used would be given in this section. Keep in mind that these comments can be made with certainty based on the results of error calculations.

The results of the different exercises should be commented on individually. *Your conclusions should refer to your original purpose; eg. if you set out to determine a value for g then your conclusions should include your calculated value of g and a comparison of your value with what you would expect.*

While you may not have as much to say in this section, what you say should be clear and concise.

6.1.8 References

If an 'accepted' value is used in your report, then the value should be footnoted and the reference given in standard form. Any references used for the theory should be listed here as well.

6.2 Final Remarks

Reports should be clear, concise, and easy to read. Messy, unorganized papers never fail to insult the reader (normally the marker) and your grade will reflect this. A professional report, in quality and detail, is at least as important as careful experimental technique and analysis.

Lab reports should usually be typed so that everything is neat and organized. Be sure to spell check and watch for mistakes due to using words which are correctly spelled but inappropriate.

6.3 Note on Lab Exercises

Lab *exercises* are different than lab reports, and so the format of the write-up is different. Generally exercises will be shorter, and they will not include either a *Discussion* or a *Conclusion* section.

Computer lab exercises may require little or even no report, but will have points which must be demonstrated in the lab.

Chapter 7

Measurement and Uncertainties

> If it's green and wiggles, it's biology;
> If it stinks, it's chemistry;
> If it doesn't work, it's physics.[1]

The quote above is rather cynical, but depending on what is meant by "work", there may be some truth to it. In physics most of the numbers used are not exact but only *approximate.* These approximate numbers arise from two principal sources:

1. *uncertainties* in individual measurements

2. *reproducibility* of successive measurements of the same quantities.

The first of these cases will be discussed in the following section, while the second will be discussed somewhat here, and more later.

7.1 Errors and Uncertainties

When an experiment is performed, every effort is made to ensure that what is being measured is what is *supposed* to be measured. Factors which hinder this are called **experimental errors**, and the existence of these factors results in *uncertainty* in quantities measured.

[1] *The Physics Teacher* 11, 191 (1973)

7.2 Single Measurement Uncertainties

When a number is obtained as a measure of length, area, angle, or other quantity, its reliability depends on the **precision**[2] of the instrument used, the *repeatability* of the measurement, the *care* taken by the experimenter, and on the *subjectivity* of the measurement itself.

7.2.1 Expressing Quantities with Uncertainties

Consider a measured length that is found to be between 14.255 cm and 14.265 cm. A number like this should be recorded as 14.260± 0.005 cm, where the 0.005 cm is the **uncertainty** [3] in the length.

Note: Digits which are not *stated* are definitely uncertain. They are, in fact, unknown, and you can't get any more uncertain than that! For instance, it makes no sense to quote a value of 78.3±0.0003kg. Unless the next three digits after the decimal place are *known* to be zeroes, then the uncertainty due to those unknown digits is much bigger than 0.0003kg. If we actually measured a value of 78.3000kg, then those zeroes should be stated, otherwise our uncertainty is meaningless. (More will be said about significant figures later.)

> Remember: The uncertainty in a measurement should always be in the last digit quoted; i.e. the least significant digit recorded is the uncertain one.

7.2.2 Random and Systematic Errors

There are two main categories for *errors*, (i.e. sources of uncertainty), which can occur: *systematic* and *random*.

- *Systematic* errors are those which, if present, will skew the results in a particular direction, and possibly by a relatively consistent amount. For instance, If we need to calculate the volume of the inside of a tube, and we measure the outer dimensions of the tube, then the volume we

[2]This term will be discussed in detail later.

[3]The term *error* is also used for uncertainty, but it suggests the idea of mistakes, and so it will be avoided where possible, except to describe the experimental factors which lead to the uncertainty.

calculate will be a little higher than it should be. If we repeated the measurement a few times, we might get slightly different results, but they would all be high.

- *Random* errors, on the other hand, cannot be consistently predicted, in direction or size, outside of perhaps broad limits. For example, if you are trying to measure the average diameter of a sample of ball bearings, then if they are randomly chosen there is no reason to assume that the first one measured will be either above average or below.

One of the important differences between random and systematic errors is that systematic errors can be corrected for after the fact, if they can be bounded. (If we measured the thickness of the walls of the tube from the example above, we could use this to correct the volume.) Random errors can only be reduced by repeating the number of measurements. (This will be discussed later.)

It should be noted that a particular measurement may combine both types of errors; if the two above examples are combined, so that one is trying to determine the average inner volume of a bunch of tubes by measuring the outer dimensions, then there would be a systematic error, (due to the difference between inner and outer dimensions), and a random error, (due to the variation between the tubes), which would both affect the results.

7.2.3 Recording Precision with a Measurement

When taking measurements, one can usually estimate a reading to the nearest 1/2 of the smallest division marked on the scale. This quantity is known as the **precision measure** of the instrument. For a *digital* device, you can measure to the least significant digit.

So, for example, if a metre stick has markings every millimetre, then the precision measure is 0.5 mm, and all measurements should be to 10ths of millimetres. On the other hand, if a digital stopwatch measures to 1/100th of a second, the precision measure is 0.01s and measurements should be to hundredths of seconds.

> Determine the precision measure of an instrument *before* taking any measurements, not after. Since the number of digits you quote will depend on the precision measure, you cannot make them up after the fact, or assume them to be zero.

7.2.4 Realistic Uncertainties

Sometimes the precision you can actually achieve in a measurement is less than what is theoretically possible. In other words, your uncertainty is not determined by the precision measure of the instrument, but is somewhat *larger* because of other factors.

> The size of the uncertainty you quote should reflect the *real* range of possible values for the quantity measured. You should be prepared to defend any measurement within the uncertainty you give for it, so do not blithely quote the precision measure of the instrument as the uncertainty unless you are convinced that it is appropriate. The precision measure is the *best* that you can do with an instrument; the uncertainty you quote should be what you can *realistically* do. Your goal as you do the experiment is to try and reduce other factors as much as possible so that you can get as close to the precision measure as possible.

There are many possible *sources* for the uncertainty in a single quantity which all contribute to the total uncertainty. The magnitude of each uncertainty contribution can vary, and the uncertainty you quote with the measurement should take all of the sources into account and be realistic. For instance, suppose you measure the length of a table with a metre stick. The uncertainty in the length will come from several sources, including:

- the precision measure of the metre stick
- the unevenness of the ends of the table
- the unevenness of the top of the table (or the side, if you place the metre stick alongside the table to measure)
- the temperature of the room (a metal metre stick will expand or contract)
- the humidity of the room (a wooden table and/or metre stick will swell or shrink)

It is possible to come up with many other sources of uncertainty, but it should be clear that this does not make the uncertainty you use arbitrarily large. In this example, you'd probably ultimately believe your measurement to be within a cm or so, no matter what, and so that is the uncertainty you should use. (On the other hand, your uncertainty should not be unrealistically small. Even if the metre stick has a precision measure of say, one millimetre, your uncertainty is obviously bigger than that if the ends of the table have variations of 3 or 4 millimetres from one side of the table to the other.)

> Usually if the uncertainty in a quantity is bigger than the precision measure, it will be due mainly to a single factor, or perhaps a couple of factors. It is rare that there will be several errors equally contributing to the uncertainty in a single quantity.

Repeatability of Measurements

Whether we repeat a measurement or not, its *realistic* uncertainty should reflect how close we would be able to be if we attempted to repeat the experiment. This reflects many things, including the strictness of our definition of what we are measuring. A later section of the lab manual, Chapter 9, *"Repeated Independent Measurement Uncertainties"*, will discuss how to calculate the uncertainty if we are actually able to repeat a measurement several times.

> Here's a guideline for determining the size of the "realistic uncertainty" in a quantity: If someone was to try and repeat your measurement, with only the instructions you have written about how the measurement was made, how big a discrepancy could they reasonably have from the value you got?

Subjectivity

Suppose you are measuring the distance between two dots on a page with a ruler. If the ruler has a precision measure of 0.5mm, but the dots are non-uniform "blobs" which are several millimetres wide, then your *effective* uncertainty is going to be perhaps a few millimeters. The subjectivity in determining the centre of the blobs is responsible for this. When you find yourself in this situation, you should note *why* you must quote a larger uncertainty than might be expected, and *how* you have determined its value.

7.2.5 Zero Error

Some measuring instruments have a certain **zero error** associated with them. This is the *actual* reading of the instrument when the *expected* value would be zero. For instance, if a spring scale reads 5g with no weight hanging on it, then it has a zero error of 5g. Any measurement made will thus be 5g too high, and so 5g must be subtracted from any measurements. (If the zero error was *minus* 5g, then 5g would have to be *added* to every subsequent measurement. Always be sure to check and record the zero error of an instrument with its uncertainty. (Since the zero error is itself a measurement, then it has an uncertainty just like any other measurement.) Subsequent measurements with that instrument should be corrected by adding or subtracting the zero error as appropriate.

With some very sensitive digital instruments, there may be another factor: if the "zero" value of the scale fluctuates over time, then the fluctuation should be taken into account.

7.3 Precision and Accuracy

Two concepts which arise in the discussion of experimental errors are **precision** and **accuracy** which, in general, are not the same thing.

7.3.1 Precision

Precision refers to the number of significant digits and/or decimal places that can be reliably determined with a given instrument or technique. The precision of a quantity is revealed by its uncertainty.
Precision (or uncertainty) can be expressed as either *absolute* or *relative*. In the first case, it will have the same units as the quantity itself; in the latter, it will be given as a proportion or a percentage of the quantity.

Uncertainties may be expressed in the first manner, (i.e. having **units**), are called **absolute uncertainties**. Uncertainties be expressed as a *percentage* of a quantity are then called **percentage uncertainties**.

For example, the measurement of the diameter of two different cylinders with a meter stick may yield the following results:

$$d_1 = 0.10 \pm 0.05\text{cm}$$
$$d_2 = 10.00 \pm 0.05\text{cm}$$

Clearly, both measurements have the same absolute precision of 0.05 cm, i.e., the diameters can be determined reliably to within 0.5 millimeters, but the relative precisions are quite different. For d_1, the relative precision is

$$\frac{0.05}{0.10} \times 100\% = 50.0\%$$

whereas for d_2 it is

$$\frac{0.05}{10.00} \times 100\% = 0.5\%$$

so we could express these two quantities as

$$\begin{aligned} d_1 &= 0.10 \pm 50.0\% \\ d_2 &= 10.00 \pm 0.5\% \end{aligned}$$

An error which amounts to a half a percent in the overall diameter is probably not worth quibbling about, but a fifty percent error is highly significant. Consequently, we would say that the measurement of d_2 is *more precise* than the measurement of d_1. The relative precision tells us immediately that there is something wrong with the first measurement, namely, we are using the wrong instrument. Something more precise is needed, like a micrometer or vernier calipers, where the precision may be more like ± 0.0005 cm.

> When comparing quantities, the *more precise* value is the one with the smaller uncertainty.

7.3.2 Accuracy

Accuracy refers to how close the measured value is to the 'true' or correct value. Thus, if a steel bar has been carefully machined so that its length is 10.0000 ± 0.0005 cm, and you determine its length to be 11.00 ± 0.05 cm, your measurement is precise, but inaccurate. On the other hand, if your measurement is 10.0 ± 0.5 cm, it is accurate, but imprecise, and a measurement of the length which yields a value of 10.001 ± 0.005 cm can be considered to be accurate and precise. Errors in precision and errors in accuracy arise from very different causes, as we shall discuss in the next section.

If you do not know what value to "expect" for a quantity, then you cannot determine the accuracy of your result. This will sometimes be the case. However, even in these cases, you will often still be able to use common sense to determine whether a value is *plausible*. For instance, if you measured the mass of a marble to be 21kg, you should realize that is unreasonable. If you get a value which is unreasonable, you should try and figure out why. (In this case, it may be that the mass should be 21 *grams*; incorrect units are a common source of odd results.)

> When comparing quantities, the *more accurate* value is the one which is closer to the "correct" or expected value.

> Systematic errors affect the *accuracy* of a measurement or result, while random errors affect the *precision* of a measurement or result.

7.4 Significant Figures

The approximate number 14.26 is said to be *correct to* 4 *figures*, or to have 4 figure accuracy. Those figures that are known with reasonable accuracy are called **significant figures**. *It is permissible to retain only one estimated figure in a result and this figure is also considered significant.*

If three numbers are measured to be 327, 4.02, and 0.00268 respectively, they are *each* said to have three significant digits. Thus, in counting significant figures, the decimal place is disregarded. Zeros at the end of a number are significant *unless they are merely place holders*. If, for example, a mass is found to be 3.20 grams, the zero *is* significant. On the other hand, when the distance to the sun is given to be 150,000,000 km, this is considered to have only 2 significant digits. Note that there is some ambiguity about the significance of the trailing zeros in this case. This can be avoided by the use of **scientific notation**, which for the above measurement would be 1.5×10^8 km. (Note that in this case, zeros are never place holders, and so, if shown, are always significant.) The following rules tell us which digits are significant in an approximate number:

1. all digits other than zero are significant
2. zeros between non–zero digits are significant
3. leading zeros in a number are not significant

4. trailing zeros in a number may or may not be significant. Use the standard form when appropriate to avoid any confusion of this type.

7.4.1 Significant Figures in Numbers with Uncertainties

When quantities have uncertainties, they should be written so that the uncertainty is given to one significant digit, and the the least significant digit of the quantity is the uncertain one. Thus, if a mass is measured to be $152.1g$ with an uncertainty of $3.5g$, then the quantity should be written as

$$152 \pm 4\text{g}$$

(Note the "2" is the uncertain digit.)

> When using scientific notation, you should separate the power of ten from both the quantity and its uncertainty to make it easier to see that this rule has been followed. This is known as the **standard form**. Use the standard form when the quantities you are quoting have placeholder zeroes. When they don't, the standard form is unnecessary and a bit cumbersome. *If you are using the standard form correctly, it should allow you to present results with uncertainties more concisely. Any time that it would be shorter to present a result without using the standard form, it should not be used.*

For instance, if the speed of light was measured to be $2.94 \times 10^8 m/s$ with an uncertainty of $6.3 \times 10^6 m/s$, then it should be written as

$$c = (2.94 \pm 0.06) \times 10^8 m/s$$

Note that this makes the relative uncertainty easier to determine.

7.4.2 Rounding Off Numbers

Often it is necessary to round off numbers. The length 14.26 feet if correct to three figures is 14.3 feet, and if correct to two figures is 14 feet. Following is a list of rules used when rounding off numbers:

1. when the digit immediately to the right of the last digit to be retained is *more than* 5, the last digit retained is increased by one.

2. when the digit immediately to the right of the last digit to be retained is *less than* 5, the last digit retained is unchanged.

3. when the digit immediately to the right of the last digit to be retained *is* 5, the last digit to be retained remains unchanged if even and is increased by one if odd.

> This last rule exists so that, for instance, 12.345 rounds to 12.34, but 12.355 rounds to 12.36. If either of these were rounded *again*, they would round correctly. However if the first had been rounded to 12.35, then rounding again would make it 12.4, which is incorrect.

7.5 How to Write Uncertainties

There are different ways of expressing the same uncertainties; which method is used depends on the circumstances. A couple of these will be described below.

7.5.1 Absolute Uncertainty

An **absolute uncertainty** is expressed in the same units as the quantity. Note that uncertainties are always expressed as *positive* quantities. For example, in the quantity

$$123 \pm 4cm$$

"4" is the uncertainty (not "±4"), and both the 123 and the 4 are in cm.

7.5.2 Percentage Uncertainty

An uncertainty can be written as a percentage of a number, so in the above example we could write

$$123 \pm 4 = 123 \pm \left(\frac{4}{123}\right) \times 100\% = 123 \pm 3\%$$

Generally percentage uncertainties are not expressed to more than one or two significant figures.

7.5.3 Relative Uncertainty

Although uncertainties are not actually *presented* this way, they are often *used* this way in calculations, as you will see later. The relative uncertainty is simply the ratio of the uncertainty to the quantity, or the percent uncertainty divided by 100. So again, in the example above, the relative uncertainty is

$$\frac{4}{123}$$

7.6 Bounds on Uncertainty

Occasionally you will have to measure a quantity for which the uncertainty is unknown. In these cases, the uncertainty can be **bounded** by varying the quantity of interest by a small amount and observing the resulting effect. The uncertainty is the amount by which the quantity of interest can be varied with no *measurable* effect. *This is a case where trying to induce more error in an experiment may be desirable!* Suppose that you have a circuit, and you think that the resistance in the wires of the circuit may be affecting your results. You can test this hypothesis by *increasing* the lengths of the wires in your circuit; if your results do not get worse, then there is no evidence that the original length of wires caused a problem.

7.7 Recap

When performing an experiment,

- every measurement has an uncertainty
- every measuring instrument has a degree of uncertainty associated with it called its *precision measure*
- sometimes experimental factors produce an *effective uncertainty* which is bigger than the precision measure
- **precision** refers to the size of the uncertainty in a quantity; a number with a smaller uncertainty is more precise.

- **accuracy** refers to how closely a measurement or calculation is to its "correct" value; **Note: You can only determine accuracy if you have solid evidence for a "correct" value.**
- a *systematic* error is some experimental factor which will introduce an uncertainty into measurements *which will always be in the same direction*
- a *random* error is some experimental factor which will introduce an uncertainty into measurements *which will **not** always be in the same direction*
- uncertainties can be expressed in absolute or relative terms
- all measurements must be recorded with uncertainties

Chapter 8

Some Measuring Instrument Concepts and Examples

8.1 Before you come to the lab

8.2 Introduction

Doing experiments in science involves measuring. In order to take useful measurements, a few concepts need to be understood:

8.2.1 Range

Any instrument has a limit to the values it can measure. For instance, a metre stick can only measure lengths up to a metre. When you choose an instrument to measure something, you are probably making an estimate in your head of how big the things is you're going to measure to be sure that the instrument you use will work.

Examples of range

What is the largest measurement you can make with each of these?

1. Vernier caliper (approximately)
2. micrometer caliper (approximately)
3. spring scale A

4. spring scale B

Since you'll use these same instruments for several exercises and labs, it makes sense to have these tables in one place where you can simply reference them for other exercises and labs, rather than having to record the information each time.

Why have multiple instruments for one quantity?

Since not all instruments measuring one quantity, (such as length), have the same range, why wouldn't you just use the instrument with the biggest range for all measurements? (e.g. Why wouldn't you just use a meter stick for all length measurements?)

8.2.2 Precision

If you want to compare two objects to see if they are the same in some way, such as whether two marbles have the same mass, you need to measure them with some instrument. The **precision** of an instrument refers to *how close two measurements can be and still be distinguished*. Usually instruments with a large range don't have as much precision, (or, "are not as precise"), as instruments with a small range.

Digital instruments

If you have a digital clock, which shows hours and minutes, how close can two times be and still be different? Obviously, if they are at least 1 minute apart, then they are different. What about a stopwatch that measures to hundredths of seconds? Times that are at least one hundredth of a second apart will be distinct. Since these times are much closer than the times which the digital clock can distinguish, we say the stopwatch is *more precise* than the clock.

We call this *smallest difference between two measurements which can be distinguished* the **precision measure** of an instrument. A smaller precision measure indicates a more precise instrument.

For a digital instrument, the precision measure is the distance between the value you measure and the next possible value. (If the instrument "autoranges", then the precision measure will change when the range changes. Watch out for that.)

Scales and analog instruments

Many measuring instruments are not digital; they are **analog**. This means that rather than giving an unambiguous distinct value, they show a continuous range of possible values. Consider the scale reading in Figure 8.1.

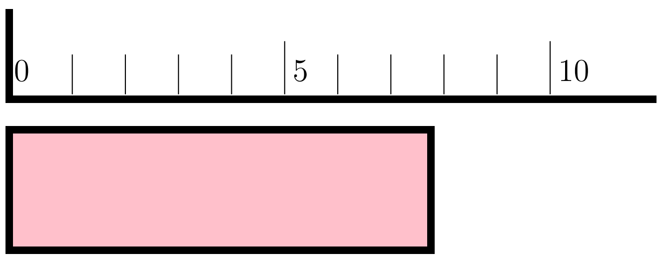


Figure 8.1: An analog scale

First of all, the left edge of the object is lined up with the zero of the scale, so we should be able to read the length of the object from the scale at the right edge of the object. It's pretty clear that that the object ends between the '7' and the '8' of the scale. Now take a look at another object measured with the same scale in Figure 8.2.

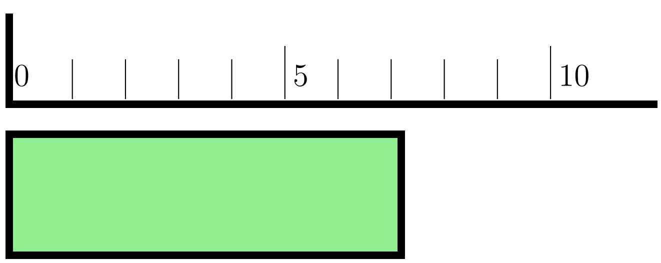


Figure 8.2: Different measurement?

This object also ends between the '7' and the '8' of the scale. The question is: *Can these two objects be distinguished?*

The first object is closer to the '8' than the '7', while the second object is closer to the '7' than the '8', so we might say they can *possibly* be distinguished. What scientists usually do in this situation is to **estimate** one more digit than they know for sure. So, for the first one, I might estimate it to be 7.7 units. (Someone else might estimate it to be 7.8 units, but that's fine.) The second one I estimate to be 7.2 units. (Someone else might estimate it to be 7.3 units, but that's also fine.) The *precision measure* of a scale like this would be said to be *one half of the smallest spacing on the scale*. Since this scale has spacings 1 unit apart, the precision measure would be 0.5 units.

Examples of analog instrument precision measure

In order to use instruments, you need to understand how to operate and read them. Here are instructions on how to use two important ones.

Micrometer Caliper

A **micrometer caliper** is shown in Fig. 8.3, and a reading from a micrometer scale is shown in Fig. 8.4. As with other non-digital instruments, you must estimate one more digit than you know. The uncertainty is one half of the smallest division, as it would be for any non-digital instrument.[1]

> Always use the ratchet to close a micrometer caliper; never use the thimble because that would allow you to apply enough force to damage the caliper.

Figure 8.4 gives an example of a reading from a micrometer scale.

While the micrometer scale is linear, it is a little different than a regular scale because there are actually two distinct scales which must be read to determine the measurement. There is a horizontal scale on the barrel, which counts rotations of the thimble, and a vertical scale on the thimble, which gives the fractional part of the reading. (Usually the numbers on the thimble will go up to 50, meaning that each complete rotation of the thimble represents a change of 0.5 of the units of the barrel scale.)

[1] The reason a micrometer is so named is that when the main scale is in millimetres, the digit estimated will be in thousandths of millimetres; i.e. in "micrometres".

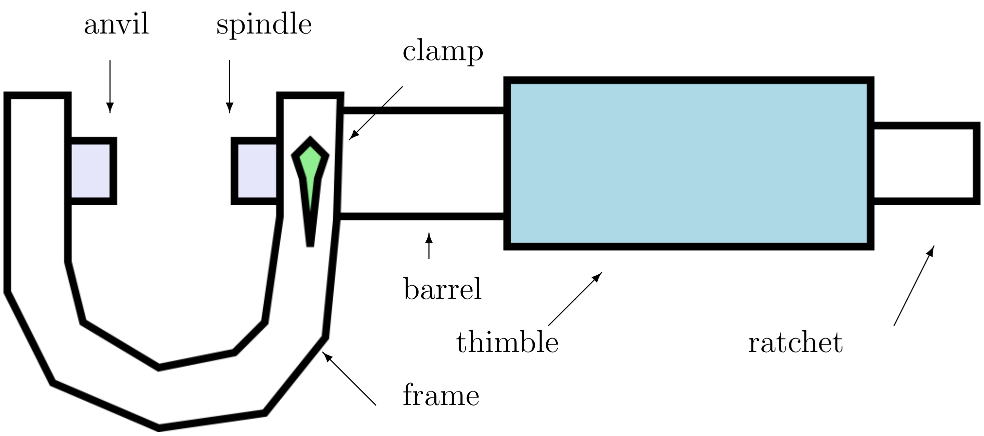


Figure 8.3: Micrometer Caliper

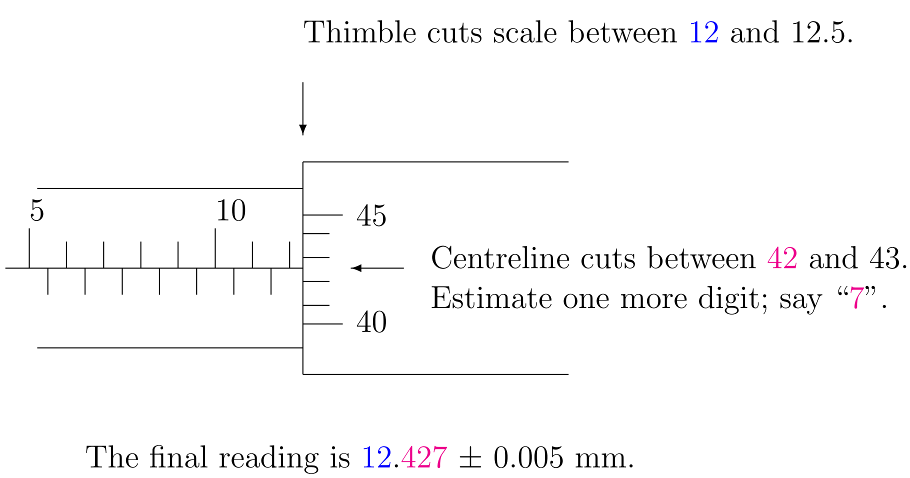


Figure 8.4: Reading a Micrometer Scale

Vernier Caliper

A **Vernier caliper** is shown in Fig. 8.5. A reading from a vernier scale is shown in Fig. 8.6. The example shown is fairly simple. It is possible to have vernier scales which are more complicated, but the principle of operation is the same. The important thing to understand is the purpose of the two scales, and how to tell which is which.

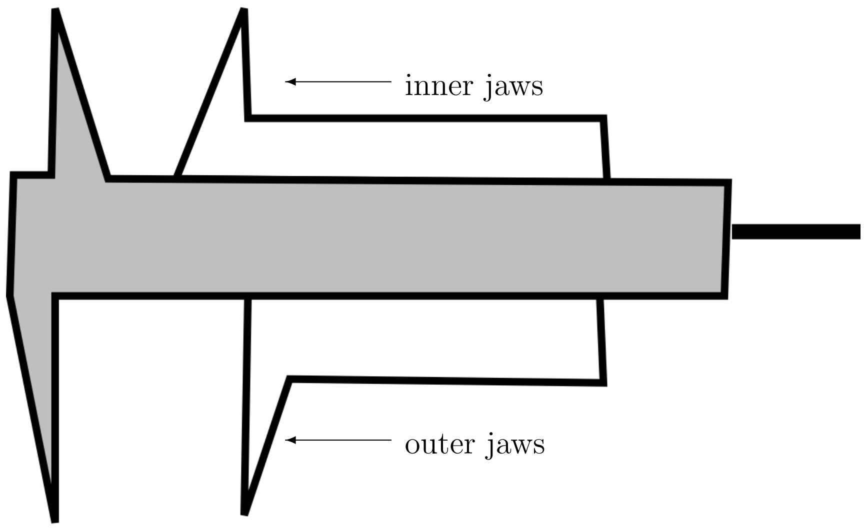
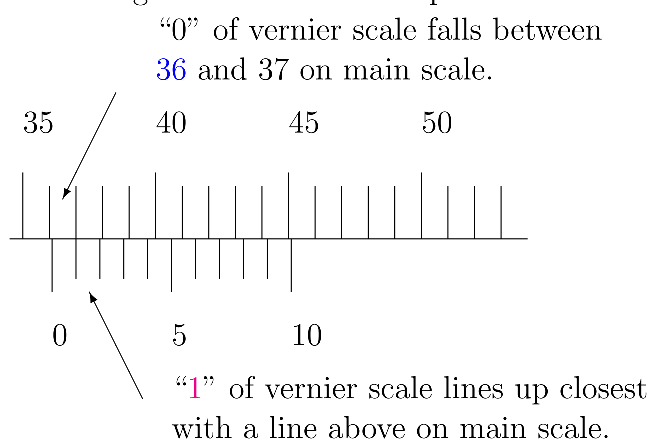


Figure 8.5: Vernier Caliper



The final reading is 36.1 ± 0.1 (of whatever appropriate units).

Figure 8.6: Reading a Vernier Scale

Remember that the precision measure is the smallest difference between two measurements which can be distinguished, so that if the difference between two measurements is less than the precision measure they are the same. (Or, to be more correct, we say that they are *"the same within experimental uncertainty"*).

8.2.3 Precision and Measurement Uncertainty

Since the precision measure is the smallest difference between two measurements which can be distinguished, then we can say that a measurement made with a given instrument will have an **uncertainty** equal to the precision measure. This means that another measurement would have to be either smaller or larger than this one by *at least* the precision measure to be distinguished from it. For this reason, the way that we usually record a measurement is

$$\textit{value} \quad \pm \quad \textit{precision measure}$$

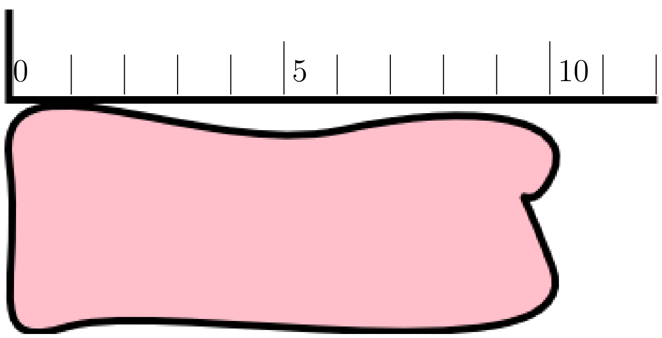


Figure 8.7: Measuring a realistic object

Data with Unknown Precision Measure

Sometimes you may have to work with data from someone else where you are not told the precision measure. In this case you may have to infer the precision measure from the data.

Each of the columns in Table 8.1 hints at probable values for the precision measure for each of the three variables, X, Y, and Z.

- Note that all of the values in the X column end in either 0 or 5. This suggests that 5 is the smallest increment between the values, and so we can infer a precision measure of 5 for X.

- In the Y column, note that the least significant digit of each value is even. This suggests that 0.02 is the smallest increment between the values, and so we can infer a precision measure of 0.02 for Y.

point number	X	Y	Z
1	25	0.86	3.6
2	30	0.92	4.2
3	25	0.68	2.7
4	5	0.74	5.1
⋮	⋮	⋮	⋮
N	10	0.80	1.9

Table 8.1: Data with unknown precision measure

- The values in the Z column don't exhibit any obvious pattern, but they all go to only one place after the decimal, so we can infer a precision measure of 0.1 for Z.

8.2.4 Effective Uncertainties

Reality is not always precise

Many times when we take measurements the precision measure of the instrument doesn't really matter, since we're trying to measure something "fuzzy". Look at the object in Figure 8.7. Since it doesn't have straight edges, measuring the length is a bit problematic. In fact, what we want to measure may depend on why we want to measure it. Consider these questions:

1. If we used this to prop open a window, how big a window opening could we have?

2. If we wanted to determine the area of the object, what length would we want?

3. If we wanted to use the edge for drawing straight lines on paper, what length of line could we draw?

In a situation where the precision measure isn't really the limitation on the precision of a measurement, we *estimate* a **realistic** or **effective** uncertainty based on whatever sort of limits make sense.

> If we're going to give a realistic (or effective) uncertainty, it's going to have to be bigger than the precision measure, because the precision measure is the best we can do with the instrument. (Remember that if the difference between two measurements is less than the precision measure they are the same.)

8.2.5 Accuracy

The **accuracy** of an instrument refers to *how close a measurement is to the true value of the quantity being measured.* Usually if an instrument is inaccurate it is due to one of two factors:

- It doesn't read zero when it should.
- Readings that should be something other than zero are off by amounts that depend on the readings themselves.

The first of these is known as **zero error** and the second is known as *calibration error* or **linearity error**.

Zero error

Some measuring instruments have a certain **zero error** associated with them. This is particularly true of micrometer calipers, as used in this experiment. Take note of the *actual* reading of the instrument when the *expected* value would be zero. (For example, close the calipers and record the reading you get with its uncertainty.) Subsequent measurements should be corrected by adding or subtracting the zero error as appropriate. For some instruments, zero error will be easy to determine. For others, it may be very difficult.

> Note that since the zero error is a reading taken from the instrument, it has an uncertainty equal to the precision measure, like any other measurement would.

- Spring Scale

  If you look at the portion of a spring scale near zero, you may see zero error. Remember to record it with its uncertainty, and to state whether the zero error should be added or subtracted to subsequent readings.

Be sure to hang the scale vertically to determine the zero error, and explain why this is important.

- Micrometer Caliper

  If you look at the portion of a micrometer caliper near zero, you may see zero error. Remember to record it with its uncertainty, and to state whether the zero error should be added or subtracted to subsequent readings. *Be sure to use the friction screw when closing the micrometer to avoid damaging it.*

Calibration Error (or linearity error)

A good example of a linearity error would be seen in a metal ruler. Since metal expands when heated, if the ruler was used at very high or very low temperatures the readings would be correspondingly high or low. (On the other hand, a wooden ruler may absorb humidity from the air and expand, or dry out and contract.)

Testing the linearity of a spring scale Over time, it is possible for the spring in a spring scale to get stretched out so that its response is not what it was originally. For the spring scale, one could

1. Check the reading with no load and record the zero error, if any. (Be sure to include the uncertainty.)
2. Check the reading with a known mass that should give the full-scale reading, or close to it, and record it. (As always, be sure to include the uncertainty.)
3. Adjust for the zero error from the full-scale (or whatever you used) reading. Record the value. (Call the the *measured range.*)
4. Subtract the known mass from the *measure range* and record the absolute value of the result. (Call this the *range difference.*)

If the range difference was less than twice the precision measure of the scale, then you have no linearity error. If not, then you do. If a device has linearity error then remember to indicate whether it will tend to read higher or lower than it should.

While the precision measure of an instrument can usually be obtained by simply looking at the instrument, the accuracy of an instrument can only be determined by using it to measure known reference quantities.

8.3 Recap

Some specific rules about uncertainties:

- the precision measure of a digital instrument is the distance between one measurement and the next possible measurement
- the precision measure of an analog instrument is one half of the distance between the scale markings
- a measurement should always be recorded with enough digits that its last digit is in the same decimal place as the precision measure of the instrument
- if some experimental factor creates an effective uncertainty bigger than the precision measure, the this experimental factor must be noted and a bound for the effective uncertainty must be determined

Chapter 9

Repeated Independent Measurement Uncertainties

If a quantity is measured several times, it is usually desirable to end up with *one* characteristic value for the quantity. (Quoting all the data may be more complete, but will not result in the kind of general conclusion that is desired.) The uncertainty in a quantity measured several times is the *bigger of* the uncertainty in the individual measurements and the uncertainty related to the reproducibility of the measurements. Most of this write-up deals with how to determine the latter quantity.

Sometimes an attempt at a single measurement will give multiple values. For instance, a very sensitive digital balance may produce a reading which fluctuates over time. Recording several values over a period of time will produce a more useful result than simply picking one at random. There are 3 common values which are extracted from data distributions which may be considered "characteristic" in certain circumstances. They are the **mean**, the **median**, and the **mode**. The mean is simply the *average*, with which you are familiar. The median is the "middle" value; *the value which has an equal number of measurements above and below*[1]. (StatsCan often reports the median income, rather than the average. The average wealth of people in Redmond, Washington, where Bill Gates lives, is huge, but it doesn't affect most people.) The mode is the *most commonly occurring value*. (If there is a continuous range of data values, then the data may be grouped into

[1] If there are an even number of points, the median is the average of the two central ones.

smaller "bins" so that a mode of the bins may be defined. Tax brackets for Revenue Canada are an example of these bins.) Depending on the *reason* for the experiment, the choice of a characteristic answer may change.

In a Gaussian, or **normal** distribution, the above decision is simplified by the fact that the mean, median, and mode all have the same value. Thus, *if* the data are expected to fit such a distribution, then an average will probably be a good choice as a quantity characteristic of all of the measurements. The uncertainty in this characteristic number will reflect the distribution of the data. Since the variations in the observations are governed by chance, one may apply the laws of statistics to them and arrive at certain definite conclusions about the magnitude of the uncertainties. No attempt will be made to derive these laws but the ones we need will simply be stated in the following sections.

9.1 Arithmetic Mean (Average)

Note: (In the following sections, each measurement x_i can be assumed to have an uncertainty Δx_i due to measurement uncertainty. How this contributes to the uncertainty in the average, etc. will be explained later.)

The **arithmetic mean** (or average) represents the best value obtainable from a series of observations from "normally" distributed data.

$$\begin{aligned} \textit{Arithmetic mean} &= \overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \\ &= \frac{x_1 + x_2 + \cdots + x_n}{n} \end{aligned}$$

9.2 Deviation

The difference between an observation and the average is called the **deviation** and is defined as

$$\textit{Deviation} = \delta x_i = |x_i - \overline{x}|$$

9.2.1 Average Deviation

The **average deviation**, which is a measure of the uncertainty in the experiment due to reproducibility, is given by

$$\textit{Average Deviation} = \delta = \frac{\sum_{i=1}^{n} |x_i - \overline{x}|}{n}$$

9.2.2 Standard Deviation

The **standard deviation** of a number of measurements is a more common measurement of the uncertainty in an experiment due to reproducibility than the average deviation. The standard deviation is given by

$$\begin{aligned} Standard\ Deviation = \sigma &= \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \\ &= \frac{1}{\sqrt{n-1}}\sqrt{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}} \end{aligned}$$

(One of the main advantages of using the standard deviation instead of the average deviation is that it can be expressed in the second form above which can be simply re–evaluated each time a new observation is made.) With random variations in the measurements, about 2/3 of the measurements should fall within the region given by $\overline{x} \pm \sigma$, and about 95% of the measurements should fall within the region given by $\overline{x} \pm 2\sigma$. (If this is not the case, then either uncertainties were not random or not enough measurements were taken to make this statistically valid.)

This occurs because the value calculated for $\overline{x}$, called the **sample mean**, may not be very close to the "actual" **population mean**, μ, which one would get by taking an infinite number of measurements. (For example, if you take 2 measurements of a quantity and get values of 1 and 2 respectively, should you guess that the "actual" value is 1, 1.5, 2, or something else?) Because of this, there is an uncertainty in the calculated mean due to the random variation in the data values. This uncertainty will be discussed further in the following section.

> Rule of thumb: For normally distributed data, an order of magnitude approximation for the standard deviation is 1/4 the range of the data. (In other words, take the difference between the maximum and minimum values and divide by 4 to get an approximate value for the standard deviation.)

9.3 Standard Deviation of the Mean

(In some texts this quantity is called the "standard *error* of the mean".) Once a number of measurements have been taken, and a mean calculated, one may calculate the uncertainty in the calculated mean *due to the scatter of the data points*, (i.e. reproducibility). Or to be more precise, one can calculate an *interval* around the *calculated* mean, $\overline{x}$, in which the *population* mean, μ, can be reasonably assumed to be found. This region is given by the *standard deviation of the mean*,

$$Standard\ deviation\ of\ the\ mean = \alpha = \frac{\sigma}{\sqrt{n}}$$

and one can give the value of the measured quantity as $\overline{x} \pm \alpha$. (In other words, μ should fall within the range of $\overline{x} \pm \alpha$.)

If possible, when doing an experiment, enough measurements of a quantity should be taken so that the uncertainty in the measurement due to instrumental precision *is greater than or equal to* α. *This is so that the random variations in data values at some point become less significant than the instrument precision. (In practice this may require a number of data values to be taken which is simply not reasonable, but sometimes this condition will not be too difficult to achieve.)*

> In any case, the uncertainty used in subsequent calculations should be the *greater* of the uncertainty of the individual measurements and α.

In mathematical terms[2],

$$\Delta\overline{x} = max\,(\alpha, \text{p.m.})$$

since p.m., the precision measure of the instrument, would be the uncertainty in the average due to the measurement uncertainties alone.

(Note that you need not calculate uncertainties when calculating the average deviation, the standard deviation, and the standard deviation of the mean, since these quantities are used to determine *the uncertainty in the data due to random variations.)*

[2] This is only strictly true if the precision measure is the uncertainty in each of the individual measurements. It is possible that there would be different uncertainties in different measurements, in which case the result should be written $\Delta\overline{x} = max\left(\alpha, \overline{(\Delta x_i)}\right)$, where Δx_i is the uncertainty in measurement i.

i	x_i	x_i^2
1	1.1	1.21
2	1.4	1.96
3	1.3	1.69
4	1.2	1.44
n	$\sum x_i$	$\sum x_i^2$
4	5.0	6.3

Table 9.1: Sample Data

9.4 Preferred Number of Repetitions

Since the the uncertainty of the average is the *greater* of the uncertainty of the individual measurements and α, and α decreases with each additional measurement, then there is a point at which *alpha* will equal the precision measure. In this case, the experiment is "optimized" in the sense that in order to improve it (i.e. reduce the uncertainty in the result), one would have to get a more precise instrument *and* take more measurements. This situation occurs when

$$\alpha = \text{p.m.}$$

so if we set

$$\text{p.m.} = \frac{\sigma}{\sqrt{N_{optimal}}}$$

and solve for $N_{optimal}$, then the result will be the optimal number of repetitions. (Keep in mind that σ does not change much after a few measurements, so it can be calculated and used in this equation.)

9.5 Sample Calculations

Following is an example of how the mean, standard deviation, and standard deviation of the mean are calculated. (The x_i values represent a set of data; x_1 is the first value, x_2 is the second, etc.)

Therefore

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{5.0}{4} = 1.25$$

and so

$$\begin{aligned}
\sigma &= \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \\
&= \frac{1}{\sqrt{n-1}}\sqrt{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}} \\
&= \frac{1}{\sqrt{4-1}}\sqrt{(6.3) - \frac{(5.0)^2}{4}} \\
&= 0.129099
\end{aligned}$$

thus

$$\alpha = \frac{\sigma}{\sqrt{n}} = \frac{0.129099}{\sqrt{4}} = 0.06455$$

The uncertainty which should be quoted with the average above will be the *bigger* of the uncertainties in the individual measurements and the standard deviation of the mean. So, if the above x_i values all had an uncertainty of 0.05, then since 0.05 is less than α, we would write

$$\overline{x} = 1.25 \pm 0.06$$

If, on the other hand, the x_i had an uncertainty of 0.07 units, then we would write

$$\overline{x} = 1.25 \pm 0.07$$

since 0.07 is greater than α.

Note that in both of these cases, the uncertainties have been rounded to one significant digit, and the average is rounded so that its last significant digit is the uncertain one, as required.

9.6 Simple Method; The Method of Quartiles

There is a way to get values very close to those given by calculating the mean and standard deviation of the mean with very little calculation. (This will be true if the data have a Gaussian[3] distribution.) The method involves dividing the data into *quartiles*. The first quartile is the value which is above

[3] or "normal"

1/4 of the data values; the second quartile is the value which is above 1/2 of the data values[4] and so on. The second quartile gives a good estimate for the average, and the third quartile minus the first quartile gives a good estimate[5] for the standard deviation. Thus,

$$\bar{x} \pm \alpha \approx Q_2 \pm \frac{(Q_3 - Q_1)}{\sqrt{n}}$$

If you use a number of data values which is a perfect square, such as 16, then the only calculation is *one* division!

9.7 Recap

When dealing with a set of numbers, calculate

- the average, $\bar{x}$
- the standard deviation, σ
- the standard deviation *of the mean*, α
- the uncertainty in the average, which is the bigger of the precision measure of the instrument and the the standard deviation of the mean

The range of the set of numbers (biggest minus smallest) is about four times the standard deviation.

[4] which is also the median
[5] Actually the inter-quartile distance or $IQR \approx 1.35\ \sigma$ for normally distributed data.

Chapter 10

Uncertain Results

10.1 The most important part of a lab

The "Discussion of Errors" (or Uncertainties) section of a lab report is where you outline the *reasonable limits* which you place on your results. If you have done a professional job of your research, you should be prepared to defend your results. In other words, you should expect anyone else to get results which agree with yours; if not, you suspect theirs. In this context, you want to discuss sources of error which you have reason to believe are significant.

10.2 Operations with Uncertainties

10.2.1 Determining Uncertainties in Functions by Algebra

Uncertainties in Functions of a Single Variable Algebraically

Consider a function as shown in Figure 10.1. If we want the know the uncertainty in $f(x)$ at a point x, what we mean is that we want to know *the difference between* $f(x + \Delta x)$ *and* $f(x)$.

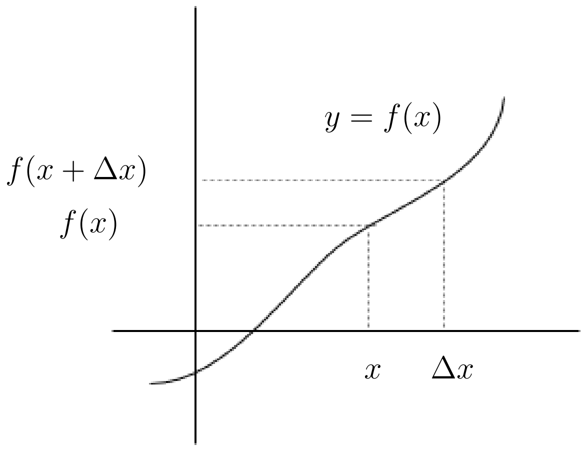


Figure 10.1: Uncertainty in a Function of x

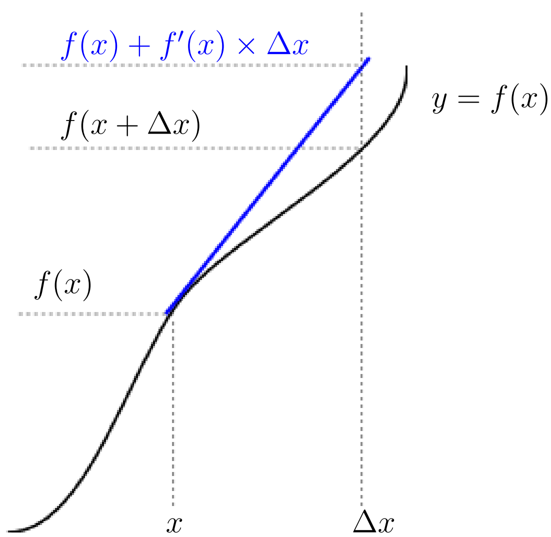


Figure 10.2: Closer View of Figure 10.1

If we take a closer look at the function, like in Figure 10.2, we can see that if Δx is small, then the difference between the function and its tangent line will be small. We can then say that

$$f(x) + f'(x) \times \Delta x \approx f(x + \Delta x)$$

or

$$\Delta f(x) \approx f'(x) \times \Delta x$$

For a function with a negative slope, the result would be similar, but the sign would change, so we write the rule with absolute value bars like this

$$\Delta f(x) \approx |f'(x)| \, \Delta x \tag{10.1}$$

to give an uncertainty which is positive.[1] Remember that uncertainties are usually rounded to one significant figure, so this approximation is generally valid.

Example: Marble volume Here is an example. Suppose we measure the diameter of a marble, d, with an uncertainty Δd, then quantities such as the volume derived from d will also have an uncertainty. Since

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

then

$$V' = 2\pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{2}d^2$$

and so

$$\Delta V \approx \left|\frac{\pi}{2}d^2\right| \Delta d$$

If we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.157$ cm^3 by this method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3.

[1]Now our use of the Δ symbol for uncertainties should make sense; in this example it has been used as in calculus to indicate “a small change in”, but for experimental quantities, “small changes” are the result of uncertainties.

Uncertainties when Combining Variables Algebraically

When numbers, some or all of which are approximate, are combined by addition, subtraction, multiplication, or division, the uncertainty in the results due to the uncertainties in the data is given by the *range of possible calculated values based on the range of possible data values.*

> *Remember:* Since uncertainties are an indication of the imprecise nature of a quantity, uncertainties are usually only expressed to one decimal place. (In other words, it doesn't make sense to have an extremely *precise* measure of the *imprecision* in a value!)

For instance, if we have two numbers with uncertainties, such as $x = 2 \pm 1$ and $y = 32.0 + 0.2$, then what that means is that x can be as small as 1 or as big as 3, while y can be as small as 31.8 or as big as 32.2 so adding them can give a result $x+y$ which can be as small as 32.8 or as big as 35.2, so that the uncertainty in the answer is the sum of the two uncertainties. If we call the uncertainties in x and y Δx and Δy, then we can illustrate as follows:

Adding

$$
\begin{array}{ccccccccc}
 & x & \pm & \Delta x & = & 2 & \pm & 1 \\
+ & y & \pm & \Delta y & = & 32.0 & \pm & 0.2 \\
 & (x+y) & \pm & ? & = & 34.0 & \pm & 1.2 \\
\hline
 & & & & = & (x+y) & \pm & (\Delta x + \Delta y)
\end{array}
$$

Thus

$$\Delta(x+y) = \Delta x + \Delta y \tag{10.2}$$

Thus $x+y$ can be between 32.8 and 35.2, as above. (Note that we should actually express this result as 34 ± 1 to keep the correct number of significant figures.)

> *Remember:* Uncertainties are usually only expressed to one decimal place, and quantities are written with the last digit being the uncertain one.

Subtracting

If we subtract two numbers, the same sort of thing happens.

$$\begin{array}{rcccccc} & x & \pm & \Delta x & = & 45.3 & \pm & 0.4 \\ - & y & \pm & \Delta y & = & -18.7 & \pm & 0.3 \\ & (x-y) & \pm & ? & = & 26.6 & \pm & 0.7 \\ \hline & & & & = & (x-y) & \pm & (\Delta x + \Delta y) \end{array}$$

Thus

$$\Delta(x-y) = \Delta x + \Delta y \tag{10.3}$$

Note that we still *add* the uncertainties, even though we *subtract* the quantities.

Multiplying

Multiplication and division are a little different. If a block of wood is found to have a mass of 1.00 ± 0.03 kg and a volume of 0.020 ± 0.001 m^3, then the **nominal** value of the density is $\frac{1.00\text{kg}}{0.020\text{m}^3} = 50.0\text{kg/m}^3$ and the uncertainty in its density may be determined as follows:

The mass given above indicates the mass is known to be *greater than or equal to* 0.97 kg, while the volume is known to be *less than or equal to* 0.021 m^3. Thus, the *minimum* density of the block is given by $\frac{0.97\text{kg}}{0.021\text{m}^3} = 46.2\text{kg/m}^3$. Similarly, the mass is known to be *less than or equal to* 1.03 kg, while the volume is known to be *greater than or equal to* 0.019 m^3. Thus, the *maximum* density of the block is given by $\frac{1.03\text{kg}}{0.019\text{m}^3} = 54.2\text{kg/m}^3$.

Notice that the above calculations do not give a symmetric range of uncertainties about the nominal value. This complicates matters, but if uncertainties are small compared to the quantities involved, the range is approximately symmetric and may be estimated as follows:

$$
\begin{array}{rrcll}
 & x \pm \Delta x & = & 1.23 \pm 0.01 & = & 1.23 \\
 & & & & \pm & (0.01/1.23 \times 100\%) \\
\times & y \pm \Delta y & = & \times 7.1 \pm 0.2 & = & \times 7.1 \\
 & & & & \pm & (0.2/7.1 \times 100\%) \\
 & (x \times y) \pm ? & = & 8.733 \pm ? & \approx & 8.733 \\
 & & & & \pm & ((0.01/1.23 + 0.2/7.1) \times 100\%) \\
 & & & & \approx & 8.733 \\
 & & & & \pm & ((0.01/1.23 + 0.2/7.1) \times 8.733) \\
 & & & & \approx & 8.733 \\
 & & & & \pm & 0.317 \\
\hline
 & & & & \approx & (x \times y) \\
 & & & & \pm & \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \times y)
\end{array}
$$

Thus

$$\Delta(x \times y) \approx \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \times y) \tag{10.4}$$

So rather than adding *absolute* uncertainties, we add *relative* or *percent* uncertainties. (To the correct number of significant figures, the above result would be

$$x \times y \approx 8.7 \pm 0.3$$

with one figure of uncertainty and the last digit of the result being the uncertain one.)

If you're a purist, or if the uncertainties are not small, then the uncertainty in the density can then be estimated in two obvious ways;

1. the *greater of the two* differences between the maximum and minimum and the accepted values

2. (or the maximum and minimum values can both be quoted, which is more precise, but can be cumbersome if subsequent calculations are necessary.)

(In the previous example, the first method would give an uncertainty of 4.2 kg/m^3.)

Dividing

Division is similar to multiplication, as subtraction was similar to addition.

$$\begin{array}{rcrcl}
x \pm \Delta x & = & 7.6 \pm 0.8 & = & 7.6 \\
 & & & \pm & (0.8/7.6 \times 100\%) \\
\div y \pm \Delta y & = & \div 2.5 \pm 0.1 & = & \div 2.5 \\
 & & & \pm & (0.1/2.5 \times 100\%) \\
(x \div y) \pm ? & = & 3.04 \pm ? & \approx & 3.04 \\
 & & & \pm & ((0.8/7.6 + 0.1/2.5) \times 100\%) \\
 & & & \approx & 3.04 \\
 & & & \pm & ((0.8/7.6 + 0.1/2.5) \times 3.04) \\
 & & & \approx & 3.04 \\
 & & & \pm & 0.4416 \\
\hline
 & & & = & (x \div y) \\
 & & & \pm & \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \div y)
\end{array}$$

Thus

$$\Delta(x \div y) \approx \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)(x \div y) \tag{10.5}$$

(To the correct number of significant figures, the above result would be

$$x \div y \approx 3.0 \pm 0.4$$

with one figure of uncertainty and the last digit of the result being the uncertain one.)

Summary of Algebraic Rules

To summarize, the uncertainty in results can *usually* be calculated as in the following examples (if the percentage uncertainties in the data are small):

$$\begin{array}{cc}
(a) & \Delta(A + B) = (\Delta A + \Delta B) \\
(b) & \Delta(A - B) = (\Delta A + \Delta B) \\
(c) & \Delta(A \times B) \approx |AB| \left(\left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta B}{B}\right|\right) \\
(d) & \Delta(\frac{A}{B}) \approx \left|\frac{A}{B}\right| \left(\left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta B}{B}\right|\right) \\
(e) & \Delta f(A \pm \Delta A) \approx |f'(A)| \, \Delta A
\end{array}$$

Note that the first two rules above *always* hold true.

To put it another way, when *adding* or *subtracting*, you *add absolute* uncertainties. When *multiplying* or *dividing*, you *add percent* or *relative* uncertainties. Note that for the last rule above that angles and their uncertainties must be in *radians* for the differentiation to be correct! (In the examples above, absolute value signs were omitted since all positive quantities were used.) (Some specific uncertainty results can be found in *Appendix I.*)

Remember that a quantity and its uncertainty should always have the same units, so you can check units when calculating uncertainties to avoid mistakes.

Three important corollaries: constants, inverses, and powers The above rules can be used to derive the results for three very common situations;

- multiplying a quantity with an uncertainty by a constant
- inverting a quantity with an uncertainty
- raising a quantity with an uncertainty to a power

In the first case, a constant can be thought of as a number with *no* uncertainty. The product rule above is

$$\Delta(A \times B) \approx |AB| \left(\left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta B}{B} \right| \right)$$

If A is a constant, then $\Delta A = 0$, so

$$\Delta(A \times B) \approx |AB| \left(\left| \frac{\cancel{\Delta A}}{A} \right| + \left| \frac{\Delta B}{B} \right| \right) = |A\cancel{B}| \left(\left| \frac{\Delta B}{\cancel{B}} \right| \right) = |A\Delta B| = |A|\,\Delta B$$

In the second case, the inverse of a number can use the quotient rule with '1' being a constant; i.e. having no uncertainty. So we have:

$$\Delta(\frac{1}{B}) \approx \left| \frac{1}{B} \right| \left(\left| \frac{\cancel{\Delta 1}}{1} \right| + \left| \frac{\Delta B}{B} \right| \right) == \left| \frac{1}{B} \right| \left| \frac{\Delta B}{B} \right|$$

so the **proportional uncertainty** in the *inverse* of a quantity is the same as the **proportional uncertainty** in the *quantity itself*.

In the third case, the function rule is:

$$\Delta f(A \pm \Delta A) \approx |f'(A)|\, \Delta A$$

and so if

$$f(A) = A^n$$

then

$$f'(A) = nA^{n-1}$$

and so

$$\Delta(A \pm \Delta A)^n \approx \left|nA^{n-1}\right| \Delta A$$

10.2.2 Determining Uncertainties in Functions by Inspection

Note: In the following section and elsewhere in the manual, the notation Δx is used to mean *"the uncertainty in x"*.

When we have a measurement of 2.0 ± 0.3 *cm*, this means that the *maximum* value it can have is $2.0+0.3cm$. The uncertainty is the difference between this maximum value and the *nominal* value (i.e. the one with no uncertainty). We could also say that the *minimum* value it can have is $2.0-0.3$ *cm*, and the uncertainty is the difference between the nominal value and this minimum value. Thus if we want to find the uncertainty in a function, $f(x)$, we can say that

$$\Delta f(x) \approx f_{max} - f \tag{10.6}$$

or

$$\Delta f(x) \approx f - f_{min} \tag{10.7}$$

where f_{max} is the same function with x replaced by *either* $x+\Delta x$ or $x-\Delta x$; whichever makes f bigger, and f_{min} is the same function with x replaced by *either* $x+\Delta x$ or $x-\Delta x$; whichever makes f *smaller*. (The approximately equals sign is to reflect the fact that these two values may not be quite the same, depending on the function f.) For instance, if

$$f(x) = x^2 + 5$$

then clearly, if x is positive, then replacing x by $x + \Delta x$ will make f a maximum.

$$f_{max} = f(x + \Delta x) = (x + \Delta x)^2 + 5$$

and so

$$\Delta f(x) \approx f_{max} - f = f(x + \Delta x) - f(x) = \left((x + \Delta x)^2 + 5\right) - \left(x^2 + 5\right)$$

On the other hand, if we wanted to find the uncertainty in

$$g(t) = \frac{1}{\sqrt{t}}$$

then, if t is positive, then replacing t by $t - \Delta t$ will make g a maximum.

$$g_{max} = g(t - \Delta t) = \frac{1}{\sqrt{(t - \Delta t)}}$$

and so

$$\Delta g(t) \approx g_{max} - g = g(t - \Delta t) - g(t) = \left(\frac{1}{\sqrt{(t - \Delta t)}}\right) - \left(\frac{1}{\sqrt{t}}\right)$$

If we had a function of two variables,

$$h(w, z) = \frac{\sqrt{w}}{z^2}$$

then we want to replace *each* quantity with the appropriate value in order to maximize the total, so if w and z are both positive,

$$h_{max} = \frac{\sqrt{(w + \Delta w)}}{(z - \Delta z)^2}$$

and thus

$$\Delta h \approx h_{max} - h = \frac{\sqrt{(w + \Delta w)}}{(z - \Delta z)^2} - \frac{\sqrt{w}}{z^2}$$

Notice in each of these cases, it was necessary to restrict the range of the variable in order to determine whether the uncertainty should be added or subtracted in order to maximize the result. In an experiment, usually your data will automatically be restricted in certain ways. (For instance, masses are always positive.)

Example:Marble volume Using the above example of the volume of a marble,

$$\Delta V \approx V(d + \Delta d) - V(d)$$

Since

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

then

$$\Delta V \approx \frac{4}{3}\pi\left(\frac{d+\Delta d}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

If we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.173$ cm^3 by this method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3 as the value to be quoted.

Example: Marble volume If we have a value of $d = 1.0 \pm 0.1$ cm, as used previously, then $\Delta V = 0.157$ cm^3 by this method.

> Mathematically, this result and the previous one are equal if $\Delta d << d$. You can derive this using the **binomial approximation**, which simply means multiplying it out and discarding and terms with two or more Δ terms multiplied together; for instance $\Delta A \Delta B \approx 0$

Determining Uncertainties by Trial and Error

For a function $f(x, y)$, the uncertainty in f will be given by the *biggest* of

$$|f(x + \Delta x, y + \Delta y) - f(x, y)|$$

or

$$|f(x - \Delta x, y + \Delta y) - f(x, y)|$$

or

$$|f(x + \Delta x, y - \Delta y) - f(x, y)|$$

or

$$|f(x - \Delta x, y - \Delta y) - f(x, y)|$$

Note that for each variable with an uncertainty, the number of possibilities doubles. In most cases, common sense will tell you which one is going to be the important one, but things like the sign of numbers involved, etc. will matter a lot! For example, if you are adding two positive quantities, then the first or fourth above will give the same (correct) answer. However, if one quantity is negative, then the second and third will be correct.

> The advantage of knowing this method is that it always works. Sometimes it may be easier to go through this approach than to do all of the algebra needed for a complicated expression, especially if common sense makes it easy to see which combination of signs gives the correct answer.

10.2.3 Choosing Algebra or Inspection

> Since uncertainties are usually only expressed to one decimal place, then small differences given by different methods of calculation, (i.e. inspection or algebra), do not matter.

Example: Marble volume Using the previous example of the marble, if we have a value of $d = 1.0 \pm 0.1$ cm, then $\Delta V = 0.173$ cm^3 by the inspection method. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3 as the value to be quoted. By the algebraic method, $\Delta V = 0.157$ cm^3. Rounded to one significant figure gives $\Delta V \approx 0.2$ cm^3, which is the same as that given by the previous method. So in this example a 10% uncertainty in d was still small enough to give the same result (to one significant figure) by both methods.

10.2.4 Sensitivity of Total Uncertainty to Individual Uncertainties

When you discuss sources of uncertainty in an experiment, it is important to recognize which ones contributed most to the uncertainty in the final result. In order to determine this, proceed as follows:

1. Write out the equation for the uncertainty in the result, using whichever method you prefer.

2. For each of the quantities in the equation which have an uncertainty, calculate the uncertainty in the result *which you get if all of the other uncertainties are zero.*

3. Arrange the quantities in descending order based on the size of the uncertainties calculated. The higher in the list a quantity is, the greater its contribution to the total uncertainty.

The sizes of these uncertainties should tell you which factors need to be considered, remembering that only quantities contributing 10% or more to the total uncertainty matter. For example, from before we had a function of two variables,

$$h(w,z) = \frac{\sqrt{w}}{z^2}$$

so by inspection, its uncertainty is given by

$$\Delta h \approx h_{max} - h = \frac{\sqrt{(w+\Delta w)}}{(z-\Delta z)^2} - \frac{\sqrt{w}}{z^2}$$

So we can compute

$$\Delta h_w \approx \frac{\sqrt{(w+\Delta w)}}{z^2} - \frac{\sqrt{w}}{z^2}$$

and

$$\Delta h_z \approx \frac{\sqrt{w}}{(z-\Delta z)^2} - \frac{\sqrt{w}}{z^2}$$

Note that in the first equation, all of the Δz terms are gone, and in the second, all of the Δw terms are gone. By the algebraic method,

$$\Delta h \approx \frac{\sqrt{w}}{z^2}\left(\frac{\Delta w}{2w} + \frac{2\Delta z}{z}\right)$$

and so

$$\Delta h_w \approx \frac{\sqrt{w}}{z^2}\left(\frac{\Delta w}{2w}\right)$$

and

$$\Delta h_z \approx \frac{\sqrt{w}}{z^2}\left(\frac{2\Delta z}{z}\right)$$

Note that *until you plug values into these equations, you can't tell which uncertainty contribution is larger.*

In the above example, if we use values of $w = 1.00 \pm 0.01$ and $z = 2.00 \pm 0.02$, then the proportional uncertainties in both w and z are the same, 1%. However, using either inspection or the algebraic method, $\Delta h = 0.006$, and $\Delta h_w = 0.001$ while $\Delta h_z = 0.005$; in other words, the uncertainty in the *result* due to Δz is five times the uncertainty due to Δw! (As you get more used to uncertainty calculations, you should realize this is because z is raised to a higher power than w, and so its uncertainty counts for more.) In order to improve this experiment, it would be more important to try and reduce Δz than it would be to try and reduce Δw.

10.2.5 Simplifying Uncertainties

Uncertainty calculations can get quite involved if there are several quantities involved. However, since uncertainties are usually only carried to one or two significant figures at most, there is little value in carrying uncertainties through calculations if they do not contribute significantly to the total.

> You do not need to carry uncertainties through if they do not contribute more than 10% of the total uncertainty, since uncertainties are usually only expressed to one decimal place. (However, be sure to give bounds for these uncertainties when you do this.)

Note that this shows a difference between doing calculations by hand versus using a spreadsheet. If you are doing calculations by hand, it makes sense to drop insignificant uncertainties like this.

> If you're using a spreadsheet in order to allow you to change the data and recalculate, it may be worth carrying all uncertainties through in case some of them may be more significant for different data.

10.2.6 Uncertainties and Final Results

When an experiment is performed, it is crucial to determine whether or not the results *make sense*. In other words, do any calculated quantities fall within a "reasonable" range?

The reason for doing calculations with uncertainties is so that uncertainties in *final answers* can be obtained. If, for instance, a physical constant was measured, the calculated uncertainty determines the range around the calculated value in which one would expect to find the "theoretical" value.

If the theoretical value falls within this range, then we say that our results *agree* with the theory *within our experimental uncertainty.*

For instance, if we perform an experiment and get a value for the acceleration due to gravity of $g = 9.5 \pm 0.5\text{m/s}^2$ then we can say that we say that our values agrees with the accepted value of $g = 9.8\text{m/s}^2$ *within our experimental uncertainty.*

If we have two values to compare, such as initial and final momentum to determine whether momentum was conserved, then we see if the ranges given by the two uncertainties overlap. In other words, if there is a value or range of values common to both, then they agree within experimental uncertainty.

So if an experiment gives us a value of $p_i = 51.2 \pm 0.7$ kg-m/s and $p_f = 50.8 \pm 0.5$ kg-m/s, then we would say the values agree within experimental uncertainty since the range from 50.5 kg-m/s $\rightarrow$ 51.3 kg-m/s is common to both. Since what we were studying was the conservation of momentum, then we would say that in this case momentum was conserved within experimental uncertainty. Note that if both uncertainties were 0.1 kg-m/s, then our results would *not* agree and we would say that momentum was *not* conserved within experimental uncertainty.

Mathematically, if two quantities a and b, with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if

$$|a - b| \leq \Delta a + \Delta b \tag{10.8}$$

A constant given with no uncertainty given can usually be assumed to have an uncertainty of zero.

If we need to compare 3 or more values this becomes more complex.

If two quantities agree within experimental error, this means that the discrepancy between experiment and theory can be readily accounted for on the basis of measurement uncertainties which are known. If the theoretical value does not fall within this range, then we say that our results *do not agree* with the theory within experimental uncertainty. In this situation, we cannot account for the discrepancy on the basis of measurement uncertainties alone, and so some other factors must be responsible.

If two numbers do not agree within experimental error, then the *percentage difference* between the experimental and theoretical values must be calculated as follows:

$$Percent\ Difference = \left| \frac{theoretical - experimental}{theoretical} \right| \times 100\% \quad (10.9)$$

Remember: Only calculate the percent difference if your results do *not* agree within experimental error.

In our example above, we would *not* calculate the percentage difference between our calculated value for the acceleration due to gravity of $g = 9.5 \pm 0.5\text{m/s}^2$ and the accepted value of $g = 9.8\text{m/s}^2$ since they agree within our experimental uncertainty.

Often instead of comparing an experimental value to a theoretical one, we are asked to test a law such as the Conservation of Energy. In this case, what we must do is to compare the initial and final energies of the system in the manner just outlined.[2] If the values agree, then we can say that energy was conserved, and if the values don't agree then it wasn't. In that case we would calculate the percentage difference as follows:

$$Percent\ Difference = \left| \frac{initial - final}{initial} \right| \times 100\% \quad (10.10)$$

Significant Figures in Final Results

Always express final answers with absolute uncertainties rather than percent uncertainties. Also, always quote final answers with one significant digit of uncertainty, and round the answers so that the least significant digit quoted is the uncertain one. This follows the same rule for significant figures in measured values.

Even though you want to round off your final answers to the right number of decimal places, don't round off in the middle of calculations since this will introduce errors of its own.

[2] There is another possibility which you may consider. Suppose you compare the *change in energy* to its expected value of zero. In that case, *any* non-zero change would result in infinite percent difference, which is mathematically correct but not terribly meaningful physically.

10.3 Discussion of Uncertainties

In an experiment, with each quantity measured, it is necessary to consider all of the possible sources of error *in that quantity*, so that a realistic uncertainty can be stated for that measurement. The "Discussion of Uncertainties" (or "Discussion of Errors") is the section of the lab report where this process can be explained.

Discussions of sources of error should always be made as concrete as possible. That means they should be use specific numerical values and relate to specific experimental quantities. For instance, if you are going to speak about possible air currents affecting the path of the ball in the *"Measuring 'g'"* experiment, you must reduce it to a finite change in either the fall *time* or the *height*.

10.3.1 Relative Size of Uncertainties

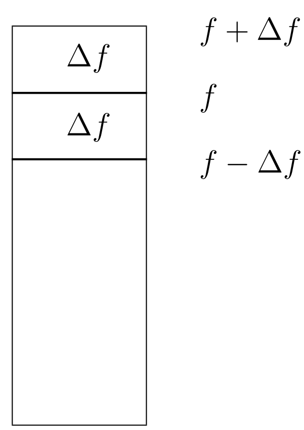


Figure 10.3: Relative Size of Quantity and its Uncertainty

The uncertainties which matter most in an experiment are those which contribute most to the uncertainty in the final result. Consider Figure 10.4, which may be seen as a magnification of one of the bands in Figure 10.3.

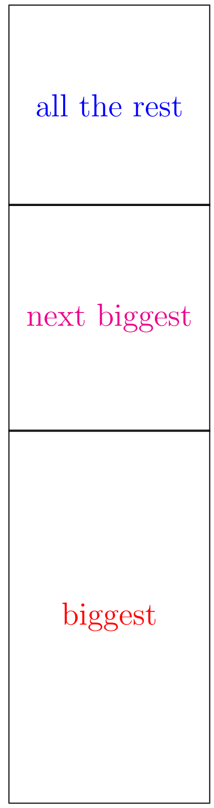


Figure 10.4: Contributions of Various Sources to Total Uncertainty

If the big rectangle represents the uncertainty in the final result, and the smaller rectangles inside represent contributions to the total from various sources, then one source contributes almost half of the total uncertainty in the result. The first two sources contribute about 75% of the total, so that all of the other sources *combined* only contribute about 25%. If we want to improve the experiment, we should try to address the factors contributing most. Similarly, in discussing our uncertainties, the biggest ones deserve most attention. In fact, since uncertainties are rounded to one decimal place, any uncertainty contributing less than 10% to the final uncertainty is basically irrelevant. The only reason to discuss such uncertainties is to explain why they are not significant.

10.3.2 Types of Errors

There are 3 major "categories" of sources of errors, in order of importance;

1. **Measurable uncertainties**-these are usually the biggest. The precision measure of each instrument used must always be recorded with every measurement. If "pre-measured" quantities are used, (such as standard masses), then there will usually be uncertainties given for these as well. If physical constants are given they may have uncertainties given for them, (such as the variation in the acceleration due to gravity by height above sea level, latitude, etc.) Where the realistic uncertainty in a quantity comes from any of these, (which will often be the case), you do not usually need to refer to them in your discussion. However, if there are any which contribute *greatly* to the uncertainty in your results, you should discuss them. For example, when you measure the mass of an object with a balance, then if the precision measure is the uncertainty used in your calculations, you don't need to discuss it, *unless* it is one of the *biggest* uncertainties in your calculations. *Keep in mind that without these values being given, it is impossible to tell whether any of the following sources of error are significant or not.*

2. **Bounded uncertainties**-these are things *which you observed*, and *have put limits on* and *usually* are much smaller than those in the group above. (Remember that since uncertainties are ultimately rounded to one significant digit, any which contribute less than 10% to the total uncertainty can be ignored.) Since you have observed them, you can give some estimate of how much effect they may have. For instance, suppose you measure the length of a table with a metre stick, and notice that the ends of the table are not exactly smooth and straight. If you can find a way to measure the variation in the length of the table due to this, then you can incorporate this into your uncertainty (if it is big enough) and discuss it.

A *plausible* error is one which can be tested. If you cannot figure out how to test for an error, it is not worth discussing. (Putting a bound on an error implies some method of testing for its existence, even if you are not able to do it at the present time.)

3. **Blatant filler**-these are things you may be tempted to throw in to sound more impressive. *Don't!!! If you did not observe them, don't discuss them.* If you suggest the gravitational pull of Jupiter is affecting your results, you'd better be prepared to show evidence (such as getting consistently different results at different times of day as the Earth rotates and so changes the angle of Jupiter's pull.) Do you even *know* in which direction the pull of Jupiter would be???

If you are going to discuss a source of uncertainty, then you must either have included it in your calculations, or given some reasonable bounds on its size. If you haven't done either of those, forget it!

You must discuss at least one source of systematic error in your report, even if you reject it as insignificant, in order to indicate how it would affect the results.

10.3.3 Reducing Errors

Whenever errors are discussed, you should suggest how they may be reduced or eliminated. There is a "hierarchy" of improvements which should be evident in your discussion. The following list starts with the best ideas, and progresses to less useful ones.

1. Be smart in the first place. You should never suggest you may have done something wrong in the lab; a professional who recognizes a mistake goes back and fixes it before producing a report. If you find yourself making a mistake which would seem likely to be repeated by other people, you may want to mention it in your report so that instructions may be clarified for the future.

2. Repeat the measurements once or twice *to check for consistency*. Repetition is a very *good* thing to do if your data are inconsistent or scattered. If certain values appear to be *incorrect*, you may want to repeat them to make sure. If this seems to be true, and you feel a measurement was wrong, you should still include it in your report but explain why it was not used in your calculations. (This is probably similar to the previous one; *if* you think your data may be messed up, you should try to repeat it *before* you write your report, so this is not something you should be *suggesting* in your own report, although you should explain that you did it if you felt it was necessary.)

3. Change technique. It may be that a different way of doing things, using the *same equipment*, could (potentially) improve your results. If so, this should be explained.

   One example of this which may sound odd at first is to try and increase *the error and see what change is produced. For instance, if you neglected the mass of something in an experiment, you could increase that mass and then repeat the experiment. If the results do not change, then it is unlikely that the original mass had a significant effect.*

   Question: How big a change in the quantity in question (such as the mass just mentioned) should you try? Explain.

4. Make more *types* of observations. In some cases, monitoring certain things during the experiment may ensure they do not affect the results. This may be relevant in the case of "bounded uncertainties" above. It should be possible with equipment available in the lab. (For instance, if you are measuring the speed of sound, and the expected value is given at 25° C, then you might explain a discrepancy by the temperature being different. However, in this case, if you think the temperature may have affected your results, then you should check a thermometer to get the actual temperature during the experiment to suggest whether or not that was likely to have caused an effect.)

5. Repeat the measurements *to average the results*. While it is always good to repeat measurements, there is a law of diminishing returns. (In other words, repeating measurements a few times will give you a lot of information about how consistent your results are; repeating them many *more* times will not tell you as much. That is why the standard

deviation of the mean decreases as $1/\sqrt{n}$, where n is the number of measurements; as n gets bigger, the change happens more slowly.) In fact, depending on the uncertainties involved, repetition at some point is of no value. (That is when the standard deviation of the mean gets smaller than the uncertainty in the individual measurements. At that point you cannot improve without using a more precise instrument, no matter how many times you repeat the experiment.)

6. Change equipment; this is a *last resort*. Since this in essence means doing a different experiment, it is least desirable, and least relevant. Your goal is to produce the best results possible *with the equipment available*.

10.3.4 Ridiculous Errors

Certain errors crop up from time to time in peoples' reports without any justification. The point of your discussion is to *support* your results, placing *reasonable* bounds on them, not to absolve yourself of responsibility for them. Would you want to hire people who did not have faith in their own research? Including errors merely to "pad" your report is not good; one realistic source of error with justification is better than a page full of meaningless ones. Following are some commonly occurring meaningless ones.

- *"..human error..."*

  This is the *most irritating* statement you can make; you should have read over the instructions beforehand until you knew what was required, and then performed the experiment to the best of your ability. If you didn't you were being unprofessional and are wasting the reader's time. After doing your calculations, you should be able to tell from your results if they make sense. If not, you should go back and correct your errors. (Note something like reaction time does *not* fall into this category, because it is well-defined and can easily be measured. Vague, undefined errors are the big no-no.)

- *"..parallax..."*

  Parallax is the error you get from looking at a scale like a speedometer or a clock from the side; the position of the hands will appear different depending on your angle. With just about any scale I've seen, I'd be hard pressed to get an error of more than $5 \rightarrow 10\%$ from parallax (and the latter very rarely). Even that would only occur if I was deliberately *trying* to observe off-axis. Unless there is some reason that you cannot eliminate it, don't ascribe any significant error to it.

- *"..component values may not have been as stated..."*

  Usually people say this about masses, etc. I'm tempted to say *"Well, DUH! "* but I won't. *Of course* if given values are incorrect then calculations will be in error, but unless you have *evidence* for a specific value being wrong, (which should include some bounds on *how* wrong it could be), then it is just wild speculation. (You may allow *reasonable* uncertainties for these given values if you justify them.) Of course, suggesting equipment was damaged or broken is in this same category. If you understand what is going on, you should be able to tell if the equipment is functioning correctly. If it isn't, you should fix it or replace it (unless it's not working because you are not using it correctly; in that case, see "human error" above.) If it's possible you have broken it, you should bring this to the attention of the lab demonstrator, and be *very sure* you know how to use it properly before trying again with new equipment.

10.3.5 A Note on Human Errors

By now you are probably wondering why human error is so bad, even though humans have to make judgments in experiments, which will certainly contribute to uncertainties in the results. The problem is vague unspecified "human error" which is more of a disclaimer than a real thoughtful explanation. *If* you had to judge the time when an object stopped moving, for instance, you *can* discuss the judgment required, but in that case you should be able to determine concrete bounds for the uncertainties introduced, rather than suggesting some vague idea that your results may be meaningless.

> A rule of thumb to follow in deciding whether a particular type of "human error" is valid is this; if it is something which *you* may have done wrong, that is not valid. If it is a limitation which *anyone* would have doing the experiment, then it is OK, provided you bound it. (But don't call it "human error"; be specific about what judgment is involved.)

10.4 Recap

When performing calculations,

- calculate the uncertainties in all results
- determine which measurements contributed most to the uncertainty in the final results
- if possible, compare results to expectations to see if they agree *within uncertainties.*

When writing a *Discussion*

- focus on quantities that contributed most to the uncertainties in the final results
- suggest how uncertainties can be reduced, *preferably without having to change equipment used in the experiment*

Chapter 11

Exercise on Repeated Measurements

11.1 Purpose

The purpose of the exercise is to measure your reaction time and to see how it compares to that of your partner, as it is a source of uncertainty in many experiments. You will attempt to measure it two different ways to see if they are equivalent. In doing so, you will be introduced to important concepts in statistics so that you can determine uncertainties in averages. You'll see how to determine the precision of a measuring instrument, and you will also learn to show how two quantities compare by using a graph.

11.2 Introduction

Lots of experiments involve measurements which are repeated. Repeating measurements allows the experimenter to be more accurate and precise in conclusions drawn from the experiment. This exercise will illustrate how to place bounds on a source of uncertainty which can be used in many experiments. Many of these results can be illustrated with bar graphs for comparison purposes. To illustrate many of the concepts involved, we'll do an experiment. The question we want to answer is:

What is a reasonable value for human reaction time?

This will give important insight into analyzing the *"Measuring 'g'"* experiment.

11.3 Theory

There are actually three different topics in this lab:

1. human reaction time
2. comparing quantities graphically
3. analyzing data with repeated measurements

The first two will be discussed briefly here; the third will be discussed as part of the *Procedure* section.

11.3.1 What is Human Reaction Time?

There are many different situations where *human reaction time* is important, but there are several different *types* of reactions which may be considered, and thus may have different times. Some of the possibilities are:

1. The time it takes to react to a *random* event: this includes situations like hitting the brakes in a vehicle when a child runs onto the street.
2. The time it takes to react to an *anticipated* event: this includes situations like batting a baseball as it crosses home plate.

There are other related things to consider, such as how closely in time people may *synchronize* their actions with other events and/or people, and the possibility of changes to these times with practice.

Random events

Since one cannot anticipate a random event, by definition, there will be a finite time required for a reaction after the event happens.

Anticipated events

When an event is anticipated, "reaction" does not necessarily *follow* the event. In the example of hitting a baseball, it is possible for a batter to swing *before* the ball crosses the plate, (and in fact success requires the swing to begin early), as easily as it is to swing *after* the ball crosses the plate.

Synchronization

In some situations, people must work together at a task, and thus they must try to *synchronize* their actions. In this case, there is a limitation in their ability to act together, and this also reflects a collective reaction time. An example of this is musicians in an orchestra playing in time with the conductor's baton. A small time difference (either positive or negative) between the motion of the baton and the playing of different instruments turns "music" into "noise".

Repeatability

No matter which situation occurs involving reaction time, it may be possible for experience to lead to an improvement. Drivers, batters, and musicians can all perform better as they develop focus and experience.

11.3.2 Precision Measure

The **precision** of an instrument refers to *how close two measurements can be and still be distinguished*. Usually instruments with a large range don't have as much precision, (or, "are not as precise"), as instruments with a small range.

Precision measure of a digital instrument

If you have a digital clock, which shows hours and minutes, how close can two times be and still be different? Obviously, if they are at least 1 minute apart, then they are different. What about a stopwatch that measures to hundredths of seconds? Times that are at least one hundredth of a second apart will be distinct. Since these times are much closer than the times which the digital clock can distinguish, we say the stopwatch is *more precise* than the clock.

We call this *smallest difference between two measurements which can be distinguished* the **precision measure** of an instrument. A smaller precision measure indicates a more precise instrument.

> For a digital instrument, the precision measure is the distance between the value you measure and the next possible value. (If the instrument "auto-ranges", then the precision measure will change when the range changes. Watch out for that.)

Data with unknown precision measure

Sometimes you may have to work with data from someone else where you are not told the precision measure. In this case you may have to infer the precision measure from the data.

point number	X	Y	Z
1	25	0.86	3.6
2	30	0.92	4.2
3	25	0.68	2.7
4	5	0.74	5.1
⋮	⋮	⋮	⋮
N	10	0.80	1.9

Table 11.1: Data with unknown precision measure

Each of the columns in Table 11.1 hints at probable values for the precision measure for each of the three variables, X, Y, and Z.

- Note that all of the values in the X column end in either 0 or 5. This suggests that 5 is the smallest increment between the values, and so we can infer a precision measure of 5 for X.
- In the Y column, note that the least significant digit (or last digit) of each value is an even number. This suggests that 0.02 is the smallest increment between the values, and so we can infer a precision measure of 0.02 for Y.

- The values in the Z column don't exhibit any obvious pattern, but they all go to only one place after the decimal, so we can infer a precision measure of 0.1 for Z.

11.3.3 Expressing Quantities with Uncertainties

Mathematically, the uncertainty in a quantity is usually expressed using the symbol Δ. So in other words, if mass has the symbol m, then the symbol Δm should be interpreted as *"the uncertainty in m"*. In that case you would write

$$m \pm \Delta m$$

to mean the mass with its uncertainty. Uncertainty is always given as a positive value, but it can be added or subtracted from the quantity to which it belongs.

Comparing quantities with uncertainties

Quantities with uncertainties are said to **agree** if the ranges given by the uncertainties for each overlap. For instance, if I estimated the length of the athletic complex as *"between 60 and 90 metres"* which I could state as *"75±15 metres"*, and I estimated the length of the science building as 100±20 metres, then I would say that the lengths of the two building agree since the ranges overlap. In other words, they *may* be the same; without more careful measurement I couldn't say for sure that they are different.

11.3.4 Displaying Comparisons Graphically

Sometimes you need to compare two or more values of one *numerical* variable, (such as mass), where the values are differentiated by some non-numerical parameter,[1], where each of the numerical values has an uncertainty. For instance, in an election poll, you might show percentage of voters favouring each party, (a numerical variable), broken down by party, (a non-numerical parameter). In this case, probably the easiest way to view the comparison is by a **bar graph**, where each bar is for a different value of the non-numeric parameter. [2] Such data are shown in Table 11.2. Note that along with the percent support for each party is the uncertainty in the percentage.

	Party			
Support	Conservative	NDP	Liberal	Green
Percent	32	25	10	4
$\Delta\%$	5	4	2	1

Table 11.2: Mythical Poll Results

From Table 11.2, it's clear that

$$Conservative\ Support = 32 \pm 5\%$$

and

$$NDP\ Support = 25 \pm 4\%$$

These two quantities agree within experimental uncertainty, since both include the range from $27\% \rightarrow 29\%$. We can show this comparison graphically by using a **stacked bar graph**.

To turn this into a stacked bar graph[3], we need to modify the data slightly, by making a third row, which is a duplicate of the second, and changing the items in the first row by subtracting the values from the second row. This is shown in Figure 11.3. In this way, the minimum value for each row, (i.e. the nominal value minus the uncertainty), is given by the top of the lowest bar,

[1] In statistics, these are called **"categorical variables"**, since they divide data into categories, rather than distinguish by a numerical value.

[2] If the numerical values are percentages, a pie chart may be more useful.

[3] This may also be called a stacked *column* graph, depending on whether the bars are drawn horizontally or vertically. The principle is the same.

the nominal value is given by the top of the middle bar, and the maximum value, (i.e. the nominal value plus the uncertainty), is given by the top of the top bar. The modified data can then be plotted to show how the different

Support	Party			
	Conservative	NDP	Liberal	Green
Percent - Δ%	27	21	8	3
Δ%	5	4	2	1
Δ%	5	4	2	1

Table 11.3: Modified Data

quantities compare.

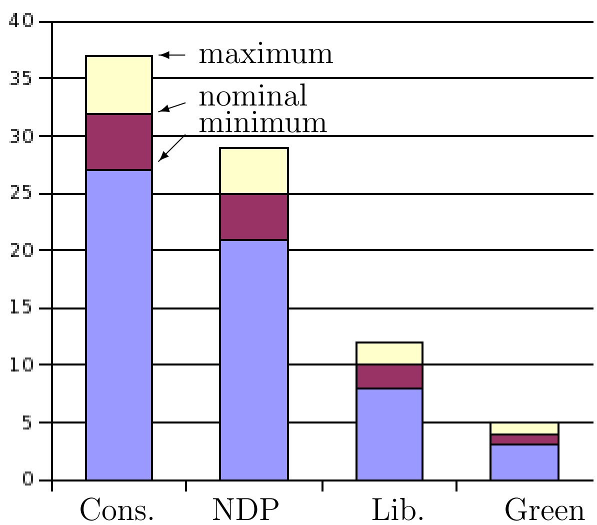


Figure 11.1: Comparing items

From Figure 11.1 we can see that support for Conservatives and NDP is the same, within experimental uncertainties, since a horizontal line can be drawn which passes through the uncertainty range for both. For any other combination, this is not the case.

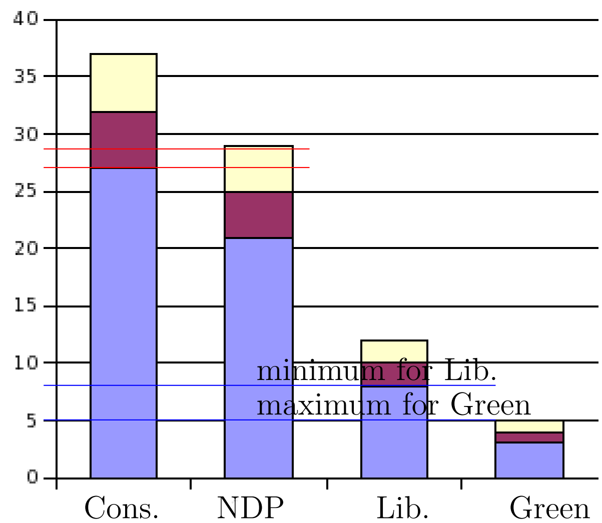


Figure 11.2: Range of overlap

Any line between the two red ones shown in Figure 11.2 crosses the error bars of both the Conservatives and the NDP, so that they are equal within experimental uncertainty. (Note that the top line of the bar graph is at 29, and the bottom one is at 27. This is the same range of values that was calculated earlier.)

> *If the values agree within experimental uncertainty, then they are the same.*

11.4 Procedure

> Much of this exercise can be done outside the lab, (except for the parts requiring the "in-lab" measurements of reaction time or your lab partner), using the reaction timer indicated on the website. In exercises like this, you may do as much as you can on your own. Then bring your answers to the in-lab questions and completed in-lab tasks to the lab.

11.4.1 Preparation (before the lab)

Most experiments and exercises will have requirements which must be completed *before* the lab and then presented *at the beginning* of the lab period.

11.4.2 Investigation (in the lab)

Reality check: Without doing any calculations, but using your head: Would you expect your reaction times to be in the ballpark of 1 second, 10 seconds, 0.1 second, or 0.01 second?

In-lab Questions

In this exercise, the in-lab questions are included with each part. Often they will be collected together at the end of the experiment or exercise.

Answer in-lab questions in a sentence or two. Be sure to state any data or results to back up your answer. Don't assume the reader knows the question or has access to your data. Make your responses complete and self-contained.

In-lab Tasks

Whenever you are asked to copy information into the template, you may use the appropriate computer spreadsheet(s) instead so long as you can show them to the IA who can check them off.

Data collection

There are going to be times in the lab when there is more data than there are data collection stations, therefore you will rotate through the stations whenever they become available. The order of collecting data does not matter. Make sure you record the data in the correct table in the template.

In-lab anticipated event measurement

1. Do several (at least 5) trials of the anticipated event test and record the times in the "self" column of Table 11.5. *Negative times are educated guesses and should be included.*

2. Get the reaction times for anticipated events from your partner. Put them in the "partner" column of Table 11.5.

3. Fill in the information about the instrument you used to measure reaction time; include specifically

   (a) the name

   (b) the units

   (c) the precision measure

4. Calculate the average of the values in each column and fill it in.

5. Find the value in each column that is in the middle; (i.e. two values are as big or bigger and two are as small or smaller), and add this information to the row labeled "median value".

6. Identify the largest and the smallest values in each column and add this information to the rows identified as "maximum value" and "minimum value".

7. Find the **range** of the values in each column, by taking the difference between the maximum and minimum values. Add this information to the table.

8. In the last row, put in the value of the range divided by 4.

IT1: Fill in Table 11.5 with the information required above.

measure	
Trial #	self
1	46
2	78
3	68
4	126
5	144
average	92.4
median value	78
minimum value	46
maximum value	144
range	98
range/4	24.5

Figure 11.3: Sample of data in Table 11.5

Mean and median

The mean and median are two useful quantities that can be calculated from a set of repeated measurements. The "middle value" you found is the **median**. It has the same number of values above and below. (If you have an even number of measurements, the median is the average of the two in the middle.) The average value you calculated is also known as the **mean**. ("mean" = "average") For *normally distributed* data, the median should be a reasonable approximation of the mean. In other words, they will probably be the same to one or two significant figures.

> For many sets of data, the mean and the median will be similar. So, since you can find the median with no calculations, it is a simple way to *estimate* the average.

IQ1: If several trials produced the exact same measurement for reaction time, such as shown in Table 11.4, is it likely that the next digit would be the same if an instrument with one more decimal place were used? Are there any two values in one of *your* sets of data which are the same? If yes, what are they?

Trial #	time (s)
1	0.3
2	0.3
3	0.3
4	0.3
5	0.3

Table 11.4: Sample reaction times measured with a different instrument

Range of the data

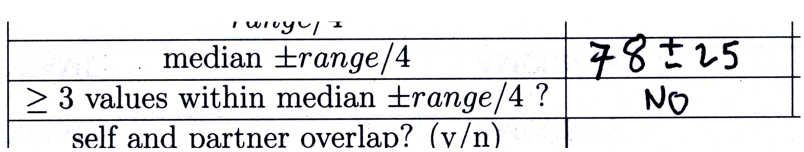
Suppose you have five measurements of reaction time from one person, such as in your sample. Probably you will find that the five reaction times are not all the same. Because of this, it makes comparing times from different people a bit tricky; you have to know what *range* of values are possible *for each person*.

IT2: Copy the appropriate results from Table 11.5 into Table 11.7.

Summarizing the information The median and the *range*/4 give us an easy way to summarize the information in the table. We could write it as follows:

$$\text{range of reaction times} \approx \textit{median value} \pm \textit{range}/4$$

With only 5 points, this may not seem much shorter than simply stating all five values, but if we had 10 (or 100!) measurements, presenting the results this way would be much more concise that stating all of the values.
Look at the median, and the value calculated for the *range*/4. Most of the values should fall between the median *minus* the *range*/4 and the median *plus* the *range*/4. Did this occur? In other words, did at least 3 values for each of you fall in the range of *median* ± *range*/4 ?



median ±*range*/4	78 ± 25
≥ 3 values within median ±*range*/4 ?	NO
self and partner overlap? (y/n)	

Figure 11.4: Sample of data in Table 11.7

IT3: Fill in the last two rows of Table 11.7. This is to allow comparing you and your partner for *this method*.

IQ2: State the *approximate range of* in-lab anticipated event reaction times for you and your partner using the form

$$\text{range of reaction times} \approx \textit{median value} \pm \textit{range}/4$$

Based on the times, does it seem like *you and your partner* are getting similar results for reaction time? In other words, do the two ranges of times for you and your partner overlap?

Online anticipated event measurement

Now look at the data for anticipated events using the online method.

1. Do several (at least 5) trials of the anticipated event test and record the times in the "self" column of Table 11.6.
2. Get the reaction times for anticipated events from your partner. Put them in the "partner" column of Table 11.6.

   Negative times are educated guesses and should be included.
3. Fill in the information about the instrument you used to measure reaction time; include specifically
   (a) the name
   (b) the units
   (c) the precision measure
4. Calculate the average of the values in each column and fill it in.

IT4: Fill in Table 11.6 with the information required above.

Synchronization

This part must be done using the reaction timer on the website.

Usually synchronization involves one person being the *initiator* and the other being the *responder* (like the conductor and a musician).

1. Figure out a method that allows you to be synchronized with your partner.

2. Do several trials of the synchronization test and record the times in Table 11.8.

3. Repeat and reverse the roles of "initiator" and "responder".

IT5: Fill in Table 11.8. Don't worry about the greyed out rows now. Did your results fit with your "reality check" at the beginning of the "Investigation" section?

IQ3: How did your synchronization times compare to your anticipated times? Did they overlap, or was one greater than the other? What does this suggest about how easy it is to synchronize compared to responding to anticipated events individually?

IT6: Identify on the template which person (i.e. you or your partner) is person 'A' and which is person 'B'.

Comparing data sets using statistics

In question **IQ2** above, all we could ask was whether it *looked* like the reaction times were different, since we had no way of determining how much variation would be small enough to ignore. In statistics, there is a quantity which can be calculated for an average to determine whether some other measurement is far enough away that it should be considered "different". That quantity is called the **standard deviation**, and it has the following properties:

- About 2/3 of the measurements should fall between the average of your measurements *minus* one standard deviation and the average *plus* one standard deviation.

- About 95% of the measurements should fall between the average of your measurements minus *two* standard deviations and the average plus *two* standard deviations.

- To turn the previous point around, the standard deviation will be about 1/4 of the difference between the biggest and smallest measurements. (It will actually be a little less than 1/4 of the difference.)

Calculating the standard deviation Table 11.9 is set up to help you calculate the standard deviation for the data for one of the sets of times you chose. (Figure 11.5 shows this table with sample data.)

The standard deviation is calculated by

$$\textbf{standard deviation} = \sqrt{\frac{\sum (time - \ average)^2}{n-1}}$$

where n is the number of data points; in this case, 5.

1. Copy the data from Table 11.5 to fill in the first column and calculate the average. Don't round the average off!

2. Subtract the average from each time to fill in the second column.

3. Square the second column values to fill in the third column. Don't round the numbers off! Always keep at least two significant figures.

4. Add up the third column values and fill in the appropriate cell.

5. Use the formula in the table to calculate σ, the standard deviation.

measure			
Trial # i	time t_i	time-average $t_i - \bar{t}$	$(t_i - \bar{t})^2$
1	46	-46.4	2152.96
2	78	-14.4	207.36
3	68	-24.4	595.36
4	126	33.6	1128.96
5	144	51.6	2662.56
	calculations		online calculator
average	$\bar{t}$	92.4	92.4
sum	$\sum (t_i - \bar{t})^2$	6747.2	N/A
andard deviation	$\sigma = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n-1}}$	41.07	41.07
dev. of the mean	$\alpha = \frac{\sigma}{\sqrt{n}}$	18.37	18.37

Figure 11.5: Sample of data in Table 11.9

IT7: Fill in your results in Table 11.9. Don't worry about the last row for now.

Now you can check the three points listed above to see if they apply in your case.

1. Highlight the rows in the table where the time is *within* $(average \pm \sigma)$. That should be about 2/3 of the values.

2. Highlight the rows in the table where the time is *within* $(average \pm 2\sigma)$. That should be about 95% of the values. Are there any that are outside of this range?

IQ4: State the value you calculated for the standard deviation, as well as the previously calculated value of $range/4$ from Table 11.5. Was your calculated value for the standard deviation *similar* to the $range/4$; i.e. was the smaller of the two more than half of the larger of the two? (If they are not similar, check to see if you calculated σ correctly.)

Using the online statistical calculator There is a calculator online to do these statistical calculations. Use it to check your results.

IT8: Use the statistical calculator to verify that your values for the standard deviation and the standard deviation of the mean are correct in Table 11.9. *Write in the calculator answers beside the cells where you calculated the results yourself.*

IT9: Repeat the previous process to fill in Table 11.10 with your partner's data from Table 11.5. (Again, don't worry about the last row.) **You can use the statistical calculator or do the calculations by hand if you prefer. Whichever you do, just leave the extra columns blank.**

Comparing averages: step one The reason for calculating the standard deviation is so that we can compare different averages of similar data sets. In our case, we want to compare the average anticipated event reaction times for the two partners. We're *almost* ready to do that. If we knew we had a *representative sample* of times from each partner that we used for our average, we'd be in great shape. However, we can't be sure our samples are "representative". (For instance, you may have taken a few tries to get familiar with the test.) If we have a lot of times from one person, than the sample will be more representative than if we only have a few. What we need is some quantity that reflects that. The quantity that we're looking for is called the **standard deviation of the mean**, or the **standard error of the mean**, and is calculated by

$$\textbf{standard deviation of the mean} = \frac{\textit{standard deviation}}{\sqrt{n}}$$

The usual symbol used for the standard deviation of the mean is α, so this is usually written as

$$\alpha = \frac{\sigma}{\sqrt{n}} \tag{11.1}$$

Basically, the standard deviation of the mean is a measure of the range around the mean (i.e. average) from our sample which should contain the average of a "representative" sample.

Since the standard deviation of the mean has $\sqrt{n}$ in the denominator, it will get smaller as the amount of data gets larger, which is what we'd expect. (A larger sample should, by definition, be more representative.)

> The **standard deviation** indicates how far the *data values* vary *from each other*. The **standard deviation of the mean** indicates how far the *average* is expected to vary from *the average from a very large set of measurements*.

IT10: Now fill in the last rows of Table 11.9 and Table 11.10.

Comparing averages: step two This part may sound pretty obvious, but it's important. We only have a hope of comparing two measurements if the instrument we used to measure them is precise enough to show a difference! For example, if we used a timer that only weighed to the nearest tenth of a second, *all* of the times might look the same. So to determine the uncertainty in the average of several measurements, we need to consider *both* the standard deviation of the mean *and* the precision measure of the instrument. This leads to the following rule:

> The uncertainty in the average of several measurements is the *larger of* the **standard deviation of the mean** and the **precision measure**.

(If you use the statistical calculator, note that it does this step automatically; i.e. if you have typed in the precision measure, it will calculate the uncertainty in the average.)

So, now we can be precise and write

$$\text{reaction time} = \bar{t} \pm \Delta\bar{t}$$

> Whenever you calculate the uncertainty in a final result, round the uncertainty to one significant figure. Then round the quantity to the same decimal place, so the last digit in the quantity is in the same decimal place as the uncertainty.

person	self
average ($\bar{t}$)	92.4
standard deviation (σ)	41.1
std. deviation of the mean (α)	18.4
uncertainty in average ($\Delta\bar{t}$)	18.4
$\bar{t} \pm \Delta\bar{t}$	92 ± 18
agree? (y/n)	

Figure 11.6: Sample of data in Table 11.11

IT11: Complete Table 11.11. Don't worry about the last row for now.

Comparing the times

Now that you've determined the uncertainty in the average for the two partners, you can state whether or not the averages *agree* within their uncertainties, or in this case, whether the average anticipated event reaction time for you and your partner were the same or not.

> Mathematically, if two quantities a and b, with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if
>
> $$|a - b| \leq \Delta a + \Delta b \tag{11.2}$$
>
> A constant given, *with no uncertainty given*, usually can be assumed to have an uncertainty of zero.

For example, suppose we have two sets of times:

$$T_A = 320 \pm 50ms$$

and

$$T_B = 240 \pm 60ms$$

Do they agree?

Clearly, the two ranges of values overlap; i.e. 270-300 ms is common to both, so we would say they agree. Mathematically,

$$T_A - T_B = 320 - 240 = 70ms$$

$$\Delta T_A + \Delta T_B = 50 + 60 = 110ms$$

Since

$$|T_A - T_B| = 70ms \leq \Delta T_A + \Delta T_B = 110ms$$

they agree. (The absolute value sign covers the case where T_B and T_A are interchanged.)

IT12: Fill in the last row of Table 11.11.

IQ5: If any two reaction times agreed, would that mean they are identical? State both you and your partner's times with their uncertainties. Based on your calculations, do the average times for you and your partner agree or not?

How many trials should you use? Since all of our statements about whether the times are the same or not depend on the the number of times in the sample we collected, is there any way of knowing whether we have a sufficiently large sample or whether we should obtain more?

It turns out that we can measure "enough" times to be confident. In fact, "enough" may not be that many in some cases. Look again at Equation 11.1. Since α will get smaller as we take more measurements, it seems like there's no limit to the number of useful measurements. However, remember that *"The uncertainty in the average of several measurements is the* larger of *the standard deviation of the mean and the precision measure."* Since α will keep getting smaller as more measurements are taken, there will *always* come a point where it will be smaller than the precision measure. After that point, the uncertainty will stay constant, no matter how many more measurements are taken, and so taking more measurements becomes mostly pointless.

> The optimum number of measurements has been taken when the standard deviation of the mean and the precision measure are *equal.*

Once you have a few measurements, you can calculate σ and then use it to determine how many measurements would be optimal by rearranging Equation 11.3 to solve for $N_{optimal}$.

$$precision\ measure = \frac{\sigma}{\sqrt{N_{optimal}}} \tag{11.3}$$

Since the standard deviation of the mean gets smaller as more measurements are taken, if the precision measure is *greater than* the standard deviation of the mean then no more measurements are needed.

IQ6: For one of the sets of times, determine the optimum number of trials to take. (Include your calculations.) Would this number of trials be feasible to collect and use in the lab? Explain. (**Hint:** If $pm > \alpha$, (the *precision measure* is greater than the *standard deviation of the mean*), then $N_{optimal}$ will be *less* than what you already have. If $pm < \alpha$, then $N_{optimal}$ will be *greater.*)

(If you use the statistical calculator, note that it does this step automatically as well; i.e. if you have typed in the precision measure, it will calculate the optimal number of measurements.)

Bonus: Random events *If you have time, you can do this in the lab. If not, you can do it using the online reaction time tester.*

1. Do several trials of the random event test and record the times in Table 11.15. *Any negative times are basically "wild guesses" and should be ignored.*
2. Repeat with your partner.

11.4.3 Analysis (after the lab)

Post-lab Discussion Questions

Read over each of the inlab and postlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 11.14.

Answer the following questions in paragraph form, with each individual question answer underlined or highlighted, At the beginning of each question put the question number is a super-script. The goal is to have a flow to the whole paragraph, rather than to have it appear as a series of statements of unrelated facts. *Be sure to include your numerical results to explain your answers.*

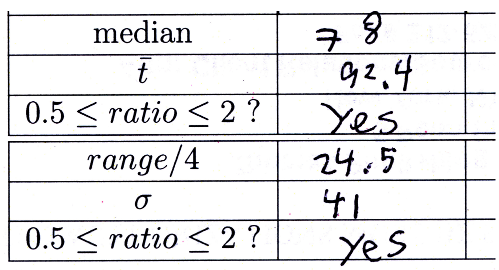
Comparing the two methods (inlab and online)

1. Use the statistical calculator to complete Table 11.12. This is to allow comparing both *methods* for one *person*, i.e you.

Q1: Did your times agree for the two methods, inlab and online? What would it mean if your times didn't agree for the different methods? Does that mean they are measuring different things, or could there be anything else that could account for the difference?

Checking the validity of the approximations Let's take a quick look back to see how much the "simple" results look like the "statistical" results.

1. Complete Table 11.13.



median	7 8	
$\bar{t}$	92.4	
$0.5 \leq ratio \leq 2$?	Yes	
$range/4$	24.5	
σ	41	
$0.5 \leq ratio \leq 2$?	yes	

Figure 11.7: Sample of data in Table 11.13

Q2: Based on your results in Table 11.13, did the median turn out to be a decent approximation to the average? Did the range/4 turn out to be a decent approximation to the standard deviation?

Repeatability

This applies to all of the previous situations.

1. Look at each of the sets of data and note if the times are changing according to a pattern.

Q3: Was there any indication that something was changing systematically as the trials went on? (For instance, were times getting shorter, longer, or staying the same?) What insight does this give you about doing an experiment such as this?

Q4: Did the in-lab reaction times for *anticipated events* for you and your partner agree within experimental uncertainty?

Q5: Determine the optimum number of trials for *the other method*; (i.e. the *online* measurement for anticipated events). Make sure to include your calculations with your answer. Did the optimal number of times depend on the device you used? Explain.

Q6: As in Figure 11.1, sketch a stacked bar graph to illustrate graphically whether the anticipated event reaction times for you and your partner agree or not. If they do, draw a horizontal line which shows that they agree. If they don't, draw a horizontal line that shows that they don't agree (i.e. one that falls below the higher value and above the lower one.). In this case, the partner name is a categorical variable.

Q7: Was either person's anticipated event reaction time equal to zero within experimental uncertainty by any method?

Q8: Did the two synchronization times for the two situations agree within experimental uncertainty?

Q9: Was either synchronization time equal to zero within experimental uncertainty? How did your synchronization times compare to your anticipated times?

Q10: Did your in-lab and online reaction times for *anticipated events* agree within experimental uncertainty? What does this suggest about how reaction times measured in the lab relate to the speed of your reactions when you are actually performing a task?

11.5 Bonus

Bonuses can be done in or outside of lab time depending on what equipment is needed to do the bonus questions. The bonus questions that are completed outside of lab time are to be handed in with the post-lab questions.

11.5.1 Bonus: Random Event Reaction Time

Using data collected earlier, or using the online reaction time tester, answer the following. (Be sure to include a copy of your data.)

Did the two reaction times for *random events* for the two people agree within experimental uncertainty?

Was either person's reaction time for *random events* equal to zero within experimental uncertainty?

11.5.2 Bonus: Other Factors to Study

Choose any one of the following events to test, depending on the equipment available to you:

1. See if it matters whether a button is pushed or released to stop the time.
2. See if the time taken between repetitions makes a difference.
3. Determine whether the *mode* of stimulus matters (e.g. sight versus sound).
4. Determine whether practice does indeed make perfect by doing many repetitions of one test. You need to do at least 30 repetitions to reach a valid conclusion.
5. Figure out whether the length of the sequence for a random event matters.
6. Figure out whether the speed of the sequence for a random event matters. (In other words, if you change the delay time between the steps of the sequence, does it make a difference?)
7. See if you can find a better method of synchronization by testing different approaches.
8. In the lab, check to see whether any person's reaction times are significantly better or worse for the random event.
9. In the lab, check to see whether any group's reaction times are significantly better or worse for a synchronized event.

10. Check to see whether a student's musical ability influences synchronization.

11. Check to see whether a student's athletic ability influences synchronization.

12. *(For musicians)* Given the suggested tempo and time signature of a piece of music, determine the maximum synchronization delay allowable for a performance.

11.5.3 Bonus: Using Calculator Built-in Functions

Note:If you don't have the manufacturer's manual for your calculator you can check the links on the lab web page.

If your calculator has built–in functions to calculate averages and standard deviations, test them out by using your data. Does your calculator calculate the correct value for σ, the standard deviation? If not, how can you correct it?

Write out the instructions needed to do this on your calculator. Include how to clear an old data set before entering a new one. For each step include the calculator's display. (Record the make and model of calculator you have.) *After you have verified the operation of the statistical functions on your calculator, you can use these functions whenever a lab task requires them.*

11.6 Recap

By the end of this exercise, you should understand the following terms, and be able to determine:

- precision measure of a digital instrument
- median of a set of values
- range of a set of values

You should also be able to calculate:

- mean; *the median is a good estimate of the mean (average)*

- standard deviation; *the range/4 is a good estimate of the standard deviation*
- standard deviation of the mean
- uncertainty in the average
- optimal number of measurements

11.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	6	________	35
Post-lab Questions	10	________	35
Pre-lab Tasks	0	________	0
In-lab Tasks	12	________	30
Post-lab Tasks	0	________	0
Bonus		________	5

11.8 Template

My name:
My student number:
My partner's name:
My other partner's name:
My lab section:
My lab demonstrator:
Today's date:

Person A name:
Person B name:

Instrument		
name (or reference)		
units		
precision measure		
Trial #	self	partner
1 2 3 4 5		
average		
median value		
minimum value		
maximum value		
range		
range/4		

Table 11.5: In-lab anticipated event reaction test

Instrument		
name (or reference)		
units		
precision measure		
Trial #	self	partner
1 2 3 4 5		
average		

Table 11.6: Other (online) method for measuring anticipated event reaction time

Instrument		
name (or reference)		
units		
precision measure		
person	self	partner
average		
median value		
$range/4$		
median $\pm range/4$		
≥ 3 values within median $\pm range/4$?		
self and partner overlap? (y/n)		

Table 11.7: Summary of results for first (in-lab) method

Instrument		
name (or reference)		
units		
precision measure		
	Responder	
trial i	Person A t_A	Person B t_B
1 2 3 4 5		
$\bar{t}$		
σ		
α		
$\Delta\left(\bar{t}\right)$		
agree?		

Table 11.8: Synchronization data

Instrument			
name (or reference)			
units			
precision measure			
Trial # i	time t_i	time-average $t_i - \bar{t}$	$(t_i - \bar{t})^2$
1 2 3 4 5			

calculations			online calculator
average	$\bar{t}$		
sum	$\sum (t_i - \bar{t})^2$		N/A
standard deviation	$\sigma = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n-1}}$		
std. dev. of the mean	$\alpha = \frac{\sigma}{\sqrt{n}}$		

Table 11.9: Standard deviation for self using first (in-lab) method

Instrument			
name (or reference)			
units			
precision measure			
Trial # i	time t_i	time - average $t_i - \bar{t}$	$(t_i - \bar{t})^2$
1 2 3 4 5			

calculations			online calculator
average	$\bar{t}$		
sum	$\sum (t_i - \bar{t})^2$		N/A
standard deviation	$\sigma = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n-1}}$		
std. dev. of the mean	$\alpha = \frac{\sigma}{\sqrt{n}}$		

Table 11.10: Standard deviation for partner using first (in-lab) method

Instrument		
name (or reference)		
units		
precision measure		
person	self	partner
average ($\bar{t}$)		
standard deviation (σ)		
std. deviation of the mean (α)		
uncertainty in average ($\Delta\bar{t}$)		
$\bar{t} \pm \Delta\bar{t}$		
agree? (y/n)		

Table 11.11: Comparing two partners (in-lab method)

Anticipated event reaction time		
units		
person	self	
method	in-lab	online
precision measure		
average ($\bar{t}$)		
σ		
α		
$\Delta\,(\bar{t})$		
agree? (y/n)		

Table 11.12: Summary of results for two methods

Anticipated event reaction time		
units		
method	in-lab	
person	self	partner
median		
$\bar{t}$		
$0.5 \leq ratio \leq 2$?		
$range/4$		
σ		
$0.5 \leq ratio \leq 2$?		

Table 11.13: Summary of results for approximate and precise calculations

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Post-lab		
Hints		
	"think" "suggest" "explain" "how" "why" "what"	"agree" "equal" "do (did, does) " "significantly different" "support" "verify"

Table 11.14: Lab Report Organization

Instrument		
name (or reference)		
units		
precision measure		
trial i	Person A t_A	Person B t_B
1 2 3 4 5		
$\bar{t}$		
σ		
α		
$\Delta(\bar{t})$		
agree?		

Table 11.15: Bonus Question data: Random events

Chapter 15

Measuring 'g'

15.1 Purpose

The purpose of this experiment is to measure the acceleration due to gravity and to see if the effects of air resistance can be observed by dropping various balls and recording fall times.

15.2 Introduction

This experiment will introduce the concept of using uncertainties to compare numbers.

> Actually this experiment is really intended to illustrate the *process* you will go through in each lab. The physics involved is extremely basic, so you should be able to focus on how to analyze the results and prepare the report.

This lab will actually be broken into parts, so you will spend several weeks to produce the report. After you know how to do this, you will be able to produce reports much more quickly.

The schedule will be somewhat like this:

- Collect data, including uncertainties in the raw data.
- Calculate numerical results, including uncertainties.
- Interpret the results and draw quantitative and qualitative conclusions.

- Learn how to write up “Discussion of Uncertainties” and “Conclusions” in your report.

After that, you will hand in the lab.

15.3 Theory

15.3.1 Physics Behind This Experiment

For a body falling from rest under the influence of gravity, assuming no air resistance, the height fallen in time t will be given by

$$h = \frac{1}{2}gt^2 \tag{15.1}$$

15.3.2 Uncertainties in measurement

Much of the “theory” about uncertainties for this experiment has been covered in Chapter 7; *“Measurement and Uncertainties”*.

Summary of rules for uncertainties

Uncertainties for all Measurements Make sure that any value you record has an uncertainty. You should record the precision measure of the instrument used. Also, list any other factors which may increase the actual uncertainty. Give your rationale for the size of the realistic uncertainty.

Expressing Quantities with Uncertainties Mathematically, the uncertainty in a quantity is usually expressed using the symbol Δ. So in other words, if mass has the symbol m, then the symbol Δm should be interpreted as *“the uncertainty in m”*. In that case you would write

$$m \pm \Delta m$$

to mean the mass with its uncertainty. Uncertainty is always given as a positive value, but it can be added or subtracted from the quantity to which it belongs.

Precision measure The **precision** of an instrument refers to *how close two measurements can be and still be distinguished*. Usually instruments with a large range don't have as much precision, (or, "are not as precise"), as instruments with a small range.

> For a digital instrument, the precision measure is the distance between the value you measure and the next possible value. (If the instrument "auto-ranges", then the precision measure will change when the range changes. Watch out for that.)

> The *precision measure* of an analog instrument would be *one half of the smallest spacing on the scale*. So if the scale has spacings 1 unit apart, the precision measure would be 0.5 units.

Effective Uncertainties In a situation where the precision measure isn't really the limitation on the precision of a measurement, we *estimate* a **realistic** or **effective** uncertainty based on whatever sort of limits make sense.

Comparing Quantities with Uncertainties Quantities with uncertainties are said to **agree** if the ranges given by the uncertainties for each overlap.

> *If the values agree within experimental uncertainty, then they are the same.*

15.3.3 About Experimentation in General

In order to learn anything useful from an experiment, it is critical to collect *meaningful* data.

There are a few things to consider:

1. Data must be correct. (This means values must be recorded accurately, along with their corresponding units.)

2. Data must be consistent. (Where you have repeated measurements, they should be similar.)

3. Data must be reproducible. (If you or someone else were to come back and do this later, the data should be similar to what you got the first

time. This is more determined by the clarity of your note-taking than the actual data recorded.)

Collaboration

If you are working with a partner, it is important that you both understand ahead of time what has to be done. It is easy to overlook details, but if two people are both looking at the material then it's much less likely that something important will be missed.

> Keep in mind that there may be some individual quantities which must be known when doing an experiment which can change with time. If you do not record them at the time, you may have to redo the experiment completely. Make sure you record all the variables that could influence the data. If you don't do this you may end up having to redo the entire experiment.

Technique

How you collect the data may have a huge effect on the usefulness of the data. Always try to use the best method of collecting data.

Preliminary Calculations

Before you leave the lab you need to do preliminary calculations of important results to see if they are in the right ballpark. This should prevent you from making scale errors (such as using wrong units) and forgetting to record time-sensitive values as mentioned above.

Well-Documented Raw Data

If your raw data are too messy or incomplete for you to understand later, you will have to redo the experiment. Always record

- Date
- Experimenters' names and student ID numbers
- Lab section

- Experiment name
- For each type of measurement,
  - Name of device used
  - Precision measure
  - Zero error (if applicable)
  - Other factors in measurement making realistic uncertainty bigger than the precision measure, and bound on uncertainty.
  - Notes about how the measurement was taken or defined.
- For each table of data,
  - Title
  - Number
  - Units for each column
  - Uncertainties for each column; (If uncertainties change for data values in a column, make a column for the uncertainties.)
- For each question asked in the manual,
  - Question number
  - Answer

In the labs and exercises, questions are often grouped together to try and develop a "big picture" of what is going on, and so the goal is to write explanations which address a group of questions, rather than handling each one individually. This is the approach which you are to take in writing your *"Discussion of Uncertainties"* in a lab. *Remember that wherever possible, you want to answer questions from your experimental observations rather than from theories.*

15.4 Procedure

15.4.1 Preparation (before the lab)

Pre-lab Tasks

PT1: Print off this section of the lab manual as it will not be provided. Make sure you also print off the “template” sheet of the spreadsheet for this experiment and bring it to the lab. *(Alternatively, bring your laptop with with both the lab manual and the spreadsheet).*

PT2: Look up the density of steel. Fill this value in Tables 15.1 and 15.2 and record the reference for it.

PT3: Look up the diameter and mass of a regulation ping pong (table tennis) ball. Use these values to claculate the effective density of a ping pong ball. Fill these values in Tables 15.1 and 15.2 and record the references for them.

PT4: Rearrange Equation 15.1 to solve for g. In other words, complete the following:

$$g =$$

Copy the result into Table 15.3.

15.4.2 Investigation (in the lab)

Apparatus

- stopwatch
- bucket
- dense ball
- less dense ball
- tape measure

Method

Note about groups of 3: There is not really a difference between the roles in a group of 3 and a group of two. For the purpose of the experiment, anyone who is *not* the "dropper" is the "gofer"; the important thing to note is whether the person dropping the ball is the one timing it. Anyone not dropping the ball is functionally equivalent, regardless of which floor he or she is on.

While you are getting the bucket lined up, practice drops with a ping pong ball. Once you have things aligned, you should switch to the dense ball for the first set of measurements.

1. Measure h. Make sure you know which scale on the tape measure has the correct units!

2. Use the denser ball, which is relatively unaffected by friction, and time one drop. Repeat this a few times to see how consistent your times are.

3. Once you have some consistency in your times, do a calculation to see whether or not this gives you a reasonable result for g.

> Note: Although this is not usually *explicit* when doing a lab, whenever you collect data you should have a general idea of what you expect the data to look like. If the data you are getting are not in the expected ballpark, stop and try and figure out why not. Don't wait until you get home to check unexpected (and potentially incorrect) data.

4. Drop the ball several (i.e. at least 5) times and record the fall times as recorded by both the dropper and the gofer in Table 15.5. Calculate the average fall time.

5. Switch position, and repeat the previous steps.

6. Calculate values for g, based on the average times. Determine which of the methods used gave the best results. Try to figure out why this might be the case. *Note that "best" has to consider both accuracy and consistency of data.*

	Times				
	Ball one ( )				Ball two ( )
	A dropping		B dropping		
i	gofer(B)	dropper(A)	dropper(B)	gofer(A)	
1					

Figure 15.1: Section of Table 15.5

7. Use the “preferred method” to collect data for the ping pong ball, which should be more affected by friction. Drop the ball several times and record the fall times as before.

8. Average the values for t, and calculate g for the second type of ball.

In-lab Tasks

> Wherever you are asked to copy information into the template, you may use the appropriate spreadsheet(s) instead as long as you get them checked off. That means you can have a printout of the spreadsheet, or fill it in on your laptop in the lab.

IT1: Identify on the template which person (i.e. you or your partner) is person 'A' and which is person 'B'. *It would be wise to use the same designation of Person 'A' and Person 'B' as you did when determining your reaction time.*

Person A is: Alice
Person B is: Bob

Figure 15.2: Section of template for **IT1**

IT2: Fill in all of the information in Table 15.5.

Instrument	
name	
units	
precision measure	
	Times

Figure 15.3: Section of Table 15.5 for **IT2**

IT3: Record at least 2 experimental factors leading to uncertainty in h *other than the one given.* Provide at least 2 experimental factors *other than reaction time* leading to uncertainty in t. Record these in Table 15.4. Also record bounds and indicate whether these sources of uncertainty are random or systematic.

symbol	factor	bound	units
Sources of *systematic* error			
h	bend in tape measure	2	cm

Figure 15.4: Section of template for **IT3**

In-lab Questions

You may never see the lab equipment set up this way again, so make sure before leaving the lab that you have completed your determination of uncertainties in measurements and other factors affecting uncertainties.

IQ1: What is the *realistic* uncertainty in h, and what experimental factor(s) cause this uncertainty? This value should be one you feel you can defend as being neither extremely high nor extremely low. (There may be more than one contributing factor.)

IQ2: Was the technique which produced the most *accurate* time also the one which produced the most *precise* (i.e. consistent) time? How difficult is it to determine the "best" technique if the most accurate one is not the most precise one?

IQ3: Write up a *Title* and a *Purpose* for this experiment, which are more appropriate than the ones given here. Be sure to include both *quantitative* goals and *qualitative* ones in the "Purpose".

IT4: Read over each of the inlab and postlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 15.6.

15.4.3 Analysis (after the lab)

Before learning how to analyze uncertainties, there may be few obvious conclusions to be drawn from any experiment, including this one. Some of the following steps will be completed in later exercises.

1. Calculate the standard deviation and standard deviation of the mean for the sets of times for each type of ball, and then determine the uncertainty in $\bar{t}$ for each data set.

2. Given the values for the precision measure and σ, determine the optimum number of measurements for both balls using the preferred technique.

3. Determine the formula for the uncertainty in g, given the uncertainties in h and $\bar{t}$.

4. Calculate the uncertainty in your values of g using the uncertainties determined above.

Post-lab Discussion Questions

Answers to the following questions will form the basis of the *Discussion* and *Conclusions* sections of your lab report. Write these sections in paragraph form, with each individual answer underlined or highlighted. At the beginning of each question put the question number in super-script. The goal is to have a flow to the whole section, rather than to have the section appear as a series of statements of unrelated facts. *Be sure to include your numerical results to explain your answers.*

Q1: If a person delays starting the watch after the ball is dropped, but does not delay stopping the watch when the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

Q2: If a person does not delay starting the watch when the ball is dropped, but delays stopping the watch after the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

Q3: If a person delays starting the watch after the ball is dropped *and* delays stopping the watch after the ball hits the ground, what will be the effect on the average time? What will be the effect on the value of g calculated?

Q4: Are any of your resulting values for g *higher* than expected? What could explain that? What bounds does that give on factors like reaction time? Explain. (Hint: Look at your answers to the three previous questions.)

Q5: Measurements of reaction time (random, anticipated, or synchronization) will apply differently to the droppers and gofers. Which of the above situations would apply to the dropper and which would apply to the gofer? Was there any correlation between the reaction times you measured and the variation in times you had when dropping the ball? In other words, did your measurements of reaction time give any useful insight into the experiment?

Q6: Were the times given by different methods for the same type of ball significantly different? In other words, did they agree within their uncertainties or not? (Include your actual calculated values in your explanation.)

Q7: Would a more precise stop watch reduce the uncertainty in t or not? Explain.*(Note: Timing* technique *may help, but that's a different matter!!)*

Q8: Calculate the optimal number of measurements need for each type of ball. Based on this calculation, would timing more drops help in reducing the uncertainty of the average times? In this experiment would it be feasible to take the optimal number of measurements?

Q9: Were the times given by the preferred method for the first ball and the second ball significantly different? (Include your actual calculated values in your explanation.)

Q10: Would a more precise device to measure h reduce the uncertainty in g or not? Explain.

Q11: Do either of the values for g determined using the preferred technique agree with the accepted value? Explain. (This question can be answered definitively based on your uncertainties.)

> Remember that values agree if the difference between them is less than the sum of their uncertainties. If they do not agree then you should calculate the percent difference between them. (DON'T calculate the percent difference if they agree!! That's what "agreement" is all about!!)

Q12: Do the two different values for g using the preferred technique suggest friction is significant or not? Explain. (As with the previous question, this can answered definitively based on your uncertainties.)

15.5 Recap

By the time you have finished this lab, you should know how to:

- determine the precision measure of an analog instrument
- identify experimental factors that create effective uncertainties in measurements and place bounds on those uncertainties

By the time you have finished this lab *report*, you should know how to:

- collect data and analyze it
- write a lab report which includes:
  - *title* which describes the experiment
  - *purpose* which explains the objective(s) of the experiment
  - *results* obtained, including data analysis
  - *discussion of uncertainties* explaining significant sources of uncertainty and suggesting possible improvements
  - *conclusions* about the experiment, which should address the original objective(s).

In addition, you should have some understanding about how *experimental technique* can influence your results, and what to do about it.

15.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	3	________	50
Post-lab Questions	12		(in report)
Pre-lab Tasks	4	________	10
In-lab Tasks	4	________	40
Post-lab Tasks	0	________	0

15.7 Template

My name:
My student number:
My partner’s name:
My other partner’s name:
My lab section:
My lab demonstrator:
Today’s date:

Person A is:
Person B is:

The dense ball is made of:
The other ball is :

quantity	symbol		single/ repeated/ constant
		given/ mine	
Not in equations			

Table 15.1: List of quantities

symbol	value	units	instrument			effective uncertainty
			name (e.g. A.1)	precision measure	zero error	
Not in equations						

Table 15.2: Single value quantities

quantity	symbol	equation	uncertainty
acceleration due to gravity			

Table 15.3: Calculated quantities

symbol	factor	bound	units
Sources of *systematic* error			
h	bend in tape measure		
Sources of *random* error			

Table 15.4: Experimental factors responsible for effective uncertainties

Instrument					
name					
units					
precision measure					
	Times				
	Ball one ( )				Ball two ( )
	A dropping		B dropping		
i	gofer(B)	dropper(A)	dropper(B)	gofer(A)	
1					
2					
3					
4					
5					
average					
σ_t					
α_t					
$\Delta(\bar{t})$					
g					

Table 15.5: Timing data

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Post-lab		
Hints		
	“think” “suggest” “explain” “how” “why” “what”	“agree” “equal” “do (did, does) ” “significantly different” “support” “verify”

Table 15.6: Lab Report Organization

Chapter 16

Exercise on Processing Uncertainties

16.1 Purpose

The purpose of the exercise is to develop skills in calculating with uncertainties, which will include calculating uncertainties in calculated values of 'g', recognizing significant sources of error, and writing a "Discussion". Many of these results will be used in future labs, so be sure not to lose this report.

16.2 Introduction

This exercise should help you become familiar with calculations involving uncertainties, and how to address them in lab reports.

The "Discussion of Errors" (or Uncertainties) section of a lab report is where you outline the *reasonable limits* which you place on your results. If you have done a professional job of your research, you should be prepared to defend your results. In other words, you should expect anyone else to get results which agree with yours; if not, you suspect theirs. In this context, you want to discuss sources of error which you have reason to believe are significant.

16.3 Theory

This has been covered previously. (See Chapter 10, *"Uncertain Results"*, in the manual.)

16.3.1 Summary of Rules for algebra (in the form of equations)

Uncertainties in functions of a single variable

$$\Delta f(x) \approx |f'(x)|\,\Delta x \tag{16.1}$$

Also, remember three important corollaries

$$\begin{aligned}
(e)\quad & \Delta\left(c \times (A \pm \Delta A)\right) = |c|\,(\Delta A) \\
(f)\quad & \Delta\left(\frac{1}{(A \pm \Delta A)}\right) \approx \left|\frac{1}{A}\right| \left|\frac{\Delta A}{A}\right| \\
(g)\quad & \Delta(A \pm \Delta A)^n \approx \left|nA^{n-1}\right| \Delta A
\end{aligned}$$

Rules for combining multiple variables

The uncertainty in results can *usually* be calculated as in the following examples (if the percentage uncertainties in the data are small):

$$\begin{aligned}
(a)\quad & \Delta(A + B) = (\Delta A + \Delta B) \\
(b)\quad & \Delta(A - B) = (\Delta A + \Delta B) \\
(c)\quad & \Delta(A \times B) \approx |AB| \left(\left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta B}{B}\right| \right) \\
(d)\quad & \Delta\left(\frac{A}{B}\right) \approx \left|\frac{A}{B}\right| \left(\left|\frac{\Delta A}{A}\right| + \left|\frac{\Delta B}{B}\right| \right)
\end{aligned}$$

Note that the first two rules above *always* hold true.

16.3.2 Summary of Rules for inspection (in the form of equations)

There's really only one rule for inspection.

$$\Delta f(x) \approx f_{max} - f \tag{16.2}$$

or

$$\Delta f(x) \approx f - f_{min} \tag{16.3}$$

16.3.3 Interpretation and Expression of Uncertainties

Sensitivity of Total Uncertainty to Individual Uncertainties

If $f = f(x, y)$, then to find the proportion of Δf due to each of the individual uncertainties, Δx and Δy, proceed as follows:

- To find Δf_x, let $\Delta y = 0$ and calculate Δf.
- To find Δf_y, let $\Delta x = 0$ and calculate Δf.

Simplifying Uncertainties

You do not need to carry uncertainties through if they do not contribute more than 10% of the total uncertainty, since uncertainties are usually only expressed to one decimal place. (However, be sure to give bounds for these uncertainties when you do this.)

Uncertainties and Final Results

Mathematically, if two quantities a and b, with uncertainties Δa and Δb are compared, they can be considered to agree within their uncertainties if

$$|a - b| \leq \Delta a + \Delta b \tag{16.4}$$

A value with no uncertainty given can be assumed to have an uncertainty of zero.

If two numbers do not agree within experimental error, then the *percent difference* between the experimental and theoretical values must be calculated as follows:

$$\textit{Percent Difference} = \left| \frac{\textit{theoretical} - \textit{experimental}}{\textit{theoretical}} \right| \times 100\% \tag{16.5}$$

Remember: Only calculate the percent difference if your results do *not* agree within experimental error.

Significant Figures in Final Results

Always quote final answers with one significant digit of uncertainty, and round the answers so that the least significant digit quoted is the uncertain one.

For instance, suppose an experiment determined a value for the speed of light

$$c = 2.953 \times 10^8 m/s$$

and from that experiment the uncertainty was

$$\Delta c = 4.26 \times 10^6 m/s$$

First round the uncertainty to one significant figure; i.e.

$$\Delta c = 4 \times 10^6 m/s$$

Rewrite this value so that it uses the same power of 10 as the value for c. In other words,

$$\Delta c = 0.04 \times 10^8 m/s$$

Since the uncertainty digit is the second one after the decimal, round the value for c to two places after the decimal.

$$c = 2.95 \times 10^8 m/s$$

So putting those together, we end up with

$$c = (2.95 \pm 0.04) \times 10^8 m/s$$

In other words, the uncertainty has only one significant digit, and the last digit we show for c is the uncertain one.

16.3.4 Discussion of Uncertainties

- Spend most of your time discussing the factors which contribute the most to the uncertainties in your results.

- Always give a measured value or a numerical bound on an uncertainty.
- State whether any particular factor leads to a systematic uncertainty or a random one. If it's systematic, indicate whether it would tend to increase or decrease your result.

Types of Errors

- Measurable uncertainties
- Bounded uncertainties
- Blatant filler

Don't use *"human error"* as it's far too vague.

Reducing Errors

1. Avoid mistakes.
2. Repeat for consistency, if possible.
3. Change technique.
4. Observe other factors as well including ones which you may have assumed were not changing or didn't matter.
5. Repeat and do a statistical analysis.
6. The *last resort* would be to change the equipment.

Ridiculous Errors

Anything which amounts to a mistake is not a valid source of error. A serious scientist will attempt to ensure no mistakes were made before considering reporting on results.

16.4 Procedure

16.4.1 Preparation (before the lab)

Pre-lab Tasks

PT1: Copy the data from the results of *"Measuring 'g'"* into Table 16.2.

PT2: Rearrange Equation 15.1 to solve for g as required in **IQ1** and fill it in the appropriate line on the first page of the template. *(If you're not using a printed template, write it out on a piece of paper and bring it with you to the lab.)*

PT3: What was the precision measure of the stopwatch from *"Measuring 'g'"*? Copy this value into the template. *(If you're not using a printed template, write it out on a piece of paper and bring it with you to the lab.)*

PT4: What were the height and the realistic uncertainty in the height from *"Measuring 'g'"*, and what factor(s) caused the uncertainty? Copy these values into the template. *(If you're not using a printed template, write it out on a piece of paper and bring it with you to the lab.)*

PT5: Use the online *statistics calculator*, or do the calculations by hand, to fill in the rows for $\bar{t}$ and $\Delta(\bar{t})$ in Table 16.2.

PT6:

Fill in the following results, based on Chapter 10, *"Uncertain Results"*, in the manual.

$$\begin{array}{rrcl} & 24.2 & \pm & 0.1 \\ + & 1.03 & \pm & 0.02 \\ \hline & & \pm & \end{array}$$

$$\begin{array}{rrcl} & 81.2 & \pm & 0.4 \\ - & 29.4 & \pm & 0.3 \\ \hline & & \pm & \end{array}$$

16.4.2 Investigation (in the lab)

This exercise can be done outside the lab if you wish. Alternative instructions are boxed like this. For your lab period bring your answers to the in-lab questions and the completed in-lab tasks to the lab.

In-lab Tasks

Complete the in-lab tasks as you encounter them.

Part 1: Finding the uncertainty by two methods

1. Find the equation for Δg by both methods; (algebra and inspection).

    - To find the equation for Δg **by algebra**:
        (a) Since the equation for g has two variables, h and t, find which of the rules for multiple variables looks most like the equation for g.
        (b) Rewrite the equation, replacing A and B with the appropriate quantities from the equation for g.
        (c) You will have a place where you need to determine the uncertainty in a *function* of h, and a place where you need to determine the uncertainty in a *function* of t. Identify these.
        (d) For the two places just mentioned, use the function rule to determine the two uncertainties.
        (e) Replace the results from the function rule in the equation for the uncertainty.
        (f) Simplify the equation where possible, but don't try to rearrange it in any other way.
        (g) If you've done things correctly, you should now have an equation for Δg that only involves h, Δh, t, and Δt, so you can plug in values.
    - To find the equation for Δg **by inspection**:
        (a) The equation for g has two variables, h and t. If you replace h by either $h + \Delta h$ or $h - \Delta h$, which would make g bigger? If you replace t by either $t + \Delta t$ or $t - \Delta t$, which would make g bigger?
        (b) Make the two substitutions above in the equation for g *and call this* g_{max}.
        (c) The uncertainty in g is given by subtracting g from g_{max}. Write this out.
        (d) Don't try to simplify the equation or rearrange it in any other way.
        (e) If you've done things correctly, you should now have an equation for Δg that only involves h, Δh, t, and Δt, so you can plug in values.

2. Using your actual data, and the results from previous exercises, find the numerical values ofΔg *for one data set* by both methods. The two methods should give the same answer when rounded to one significant figure. If they are very different, you have probably made an error somewhere.

3. Using either method, determine the equations for Δg_h and Δg_t.

4. Using your actual data, find the numerical values of the sensitivity of Δg to each of the quantities involved, (i.e. Δg_h and Δg_t), based on your data *for the same data set.*

IT1: Copy the equations for Δg by inspection and by algebra into the template.

Part 2: Using the online uncertainty calculator

1. For one of the values of $\bar{t}$ and $\Delta(\bar{t})$ from Table 16.2, use the uncertainty calculator to calculate $(\bar{t})^2$ and $\Delta\left((\bar{t})^2\right)$. *(Hint: This will use a* ***function*** *on the calculator.* You may want to do this for the other values of $\bar{t}$ and $\Delta(\bar{t})$ while you're at it.

2. Take your values of h and Δh from **PT4** and use the uncertainty calculator to calculate $2h$ and $\Delta(2h)$. *(Hint: This will use an* ***operator*** *on the calculator.*

IT2: Fill in these results in Table 16.1. *If you're using the online version of the manual, write the results on a sheet of paper.*

$\bar{t}$	$\Delta(\bar{t})$	$(\bar{t})^2$	$\Delta\left((\bar{t})^2\right)$
h	Δh	$2h$	$\Delta(2h)$

Table 16.1: Using the uncertainty calculator

In-lab Questions

In the last two steps of **IQ1** , you can use the online uncertainty calculator instead of doing the calculations by hand.

IQ1: (From *"Measuring 'g'"*)

- State the equation for Δg by both methods; (algebra and inspection). State which method you preferred and why.

  (Note: you will first have to rearrange the equation.)

- State the numerical values you found for ofΔg *for one data set* by both methods. Do the two methods give the same answer when rounded to one significant figure? If not, do they give consecutive values?

- State the equations you determined for Δg_h and Δg_t.

- State the numerical values you found for the sensitivity of Δg to each of the quantities involved, (i.e. Δg_h and Δg_t), based on your data *for the same data set.*

For the *"Measuring 'g'"* experiment, answer the following questions. *Don't discuss an uncertainty unless you have given a measured value or an estimated bound for it.*

IQ2: From your numerical results for the uncertainty in g, which quantity, (h or t), produced the biggest proportional effect? In other words, which was bigger, Δg_h or Δg_t?

IQ3: From the answer to question 2, what *experimental factor* from those discussed when you collected the data made the biggest contribution to the uncertainty in the quantity mentioned?

IQ4: From the answer to question 3, what steps might be taken to reduce the uncertainty, keeping in mind the guidelines about the best ways to reduce uncertainties from earlier in this exercise?

IQ5: For the quantity *not* used in question 2, (in other words, the quantity that produced the *smaller* proportional effect), what *factor* from those discussed when you collected the data made the biggest contribution to the uncertainty in the quantity? What steps might be taken to reduce the uncertainty, keeping in mind the guidelines about the best ways to reduce uncertainties?

IT3: Read over each of the inlab questions, and decide where the answers should appear in your lab report. (Note that some questions may have parts of the answers in each section.) Fill in the results in Table 16.3.

16.4.3 Analysis (after the lab)

All of the in-lab questions from this exercise should be incorporated into your *Discussion* for *"Measuring 'g'"*. This should also be the case for all of the exercises which you used when completing the *"Measuring 'g'"* lab report.

16.5 Bonus

Do either one.

16.5.1 Bonus: Examining the uncertainty calculator

From your results determined for **IT2:** Explain whether the uncertainty calculator uses inspection or algebra. How can you figure this out? *(Hint: Figure out the equation for the uncertainty in* t^2 *by both methods.)*

16.5.2 Bonus: Proof of earlier result; uncertainty in marble volume

This question is from the *"Uncertain Results"* section of the manual.
For a sphere,

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

and so by inspection

$$\Delta V \approx \frac{4}{3}\pi\left(\frac{d+\Delta d}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

Algebraically,

$$V' = 2\pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{2}d^2$$

and so

$$\Delta V \approx \left|\frac{\pi}{2}d^2\right| \Delta d$$

Show that these two methods give the same results if uncertainties are small; i.e. if $\Delta d << d$. Remember that uncertainties in final results are usually only expressed to one decimal place, so you can usually discard terms with two or more Δ terms multiplied together; for instance $\Delta A \Delta B \approx 0$

16.6 Recap

By the end of this exercise, you should know how to calculate the uncertainty for :

- quantities which are added
- quantities which are subtracted
- quantities which are multiplied
- quantities which are divided
- functions of quantities, by
  - algebraic method

– inspection

In addition, you should understand

- how to use the online uncertainty calculator

16.7 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	5	________	40
Post-lab Questions	0	________	0
Pre-lab Tasks	6	________	20
In-lab Tasks	3	________	40
Post-lab Tasks	0	________	0
Bonus		________	5

16.8 Template

My name:
My student number:
My partner's name:
My other partner's name:
My lab section:
My lab demonstrator:
Today's date:

g has units of:
The equation for g is:

h has a value (with units) of:
The realistic uncertainty in h is:

The precision measure of the stopwatch (including units) was:

The equation for Δg *by inspection* is:

The equation for Δg *by algebra* is:

<table>
<tr><td rowspan="4">i</td><td colspan="5">Times (seconds)</td></tr>
<tr><td colspan="4">Ball one</td><td rowspan="3">Ball two</td></tr>
<tr><td colspan="2">A dropping</td><td colspan="2">B dropping</td></tr>
<tr><td>gofer(B)</td><td>dropper(A)</td><td>dropper(B)</td><td>gofer(A)</td></tr>
<tr><td>1
2
3
4
5</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>$\bar{t}$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>$\Delta(\bar{t})$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>g</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Δg</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Δg_h</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Δg_t</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Which is most significant, Δh or Δt?</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Table 16.2: Uncertainties for g

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Hints		
	"think" "suggest" "explain" "how" "why" "what"	"agree" "equal" "do (did, does) " "significantly different" "support" "verify"

Table 16.3: Lab Report Organization

Chapter 17

Torque and the Principle of Moments

17.1 Purpose

The object of this experiment is to study and understand the concepts of torque and static equilibrium of a body.

17.2 Introduction

This experiment should help the student become familiar with simple calculations with uncertainties.

17.3 Theory

In order for a rigid body to remain in equilibrium, the following two conditions must be satisfied:

1. the resultant external force acting on the body must be zero.
2. the sum of the moments (torques) acting in a counter-clockwise direction about *any* point must equal the sum of the clockwise moments about the same point. (This is the **Principle of Moments**.

The body is in *static* equilibrium if it is at rest with respect to its frame of reference (which in this case would be the lab table).

The **moment of a force**, or **torque**, is a measure of the force's tendency to cause rotation. It is defined as *the product of the **magnitude** of the force and the perpendicular distance from the axis of rotation to the **line of action** of the force.*
In the case of a **lever**, the axis of rotation is called the **fulcrum**.
In Figure 17.1, the moment of force, $\vec{F}$ about an axis of rotation through A is given by Fd. (Of course, the *units* of torque are those of force $\times$ distance).

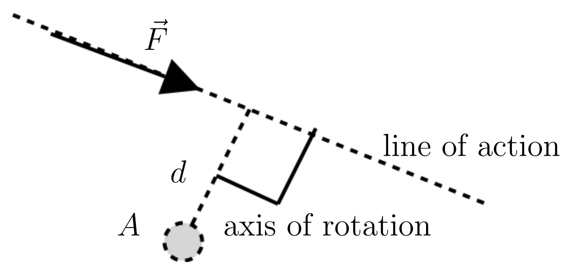


Figure 17.1: Definition of Torque

Consider the system of Figure 17.2. If the horizontal bar is a stick of uniform cross-section and density balanced at its centre of mass, then this system is subjected to *two* torques about a point at the centre of mass of the stick;

1. the force $W_a = m_a g$ at perpendicular distance a counterclockwise.

2. the force $W_b = m_b g$ at perpendicular distance b clockwise.

The second condition of static equilibrium (The Principle of Moments) is satisfied if

$$W_a a = W_b b \tag{17.1}$$

which simplifies to

$$m_a a = m_b b \tag{17.2}$$

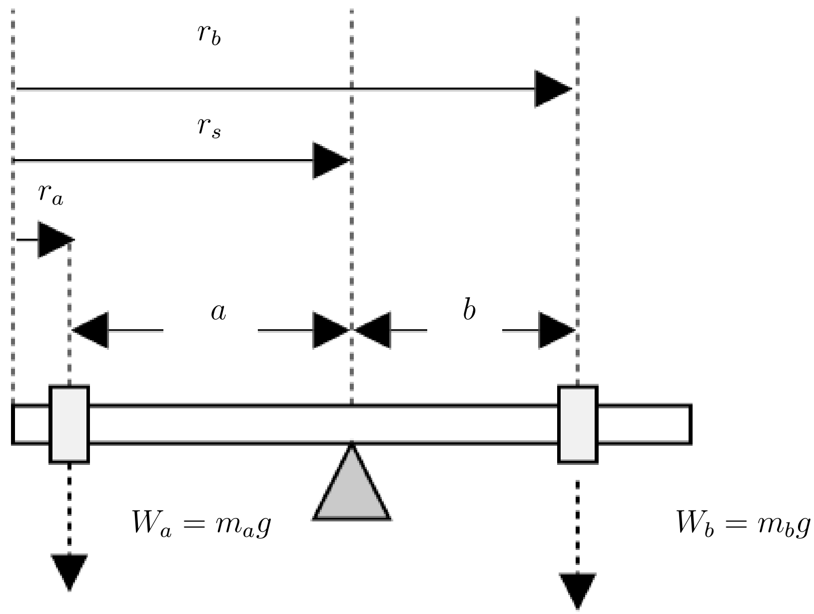


Figure 17.2: System in Static Equilibrium (Part 1)

17.4 Procedure

17.4.1 Preparation (before the lab)

Pre-lab Tasks

PT1: For the system in Figure 17.2, determine the equations for $\Delta\tau_{cw}$ and $\Delta\tau_{ccw}$, given a, b, m_a, m_b, Δa, Δb, Δm_a and Δm_b. (τ_{cw} and τ_{ccw} are the clockwise torque and the counter-clockwise torque, respectively.) Copy the equations into Table 17.7

17.4.2 Investigation (in the lab)

Apparatus

Uniform and non-uniform meter sticks, 3 meter stick clamps with knife-edge load supports, a spring scale, a set of hooked weights, a laboratory balance and a support stand.

In-lab Questions

In this experiment, the in-lab questions are included with each part.

Method

Part 1: Testing the Principle of Moments

1. Place one metre stick clamp near the centre of the *uniform* metre stick and suspend this combination from the suspension point so that the zero cm mark of the metre stick is at the *left* end.

2. Find the reading at the centre of mass of the metre stick by sliding it in its clamp[1] until it lies horizontally at rest. Record this position, r_s.

3. Adjust the clamp slightly to determine how far you can move it until the stick is noticeably not horizontal. Use this to determine the uncertainty Δr_s.

4. Weigh the two remaining clamps and record their masses with their uncertainties.

5. With the metre stick as above, suspend a weight from a hanger on each side of the centre of mass, as in Figure 17.2. Use $m_b \geq 2m_a$ so that you get very different distances for a and b. When adjusting the position of the second mass for equilibrium, use the same procedure used earlier for Δr_s to determine the uncertainty in the position of the second mass.

6. Record r_a, the reading at position a, and r_b, the reading at position b with their uncertainties.

IT1: Fill in the results in Table 17.1.

IQ1: Why do you not need to experimentally find the uncertainty in the positions of *both* masses?

Note that m_a and m_b include the masses of their respective hangers.

[1]In this experiment, the terms *"hanger"* and *"clamp"* will both be used to describe the same piece of equipment. This is because the function is slightly different in each case.

Part 2: Mass and Center of Mass Determination from a Single Equilibrium Setup

1. Check to see if the spring scale reads zero when unloaded. If it doesn't, be sure to record the zero error with its uncertainty for use later in this experiment. Hang the spring scale from the system support.

2. Suspend a 200g mass from a hanger between the 10 and 20 cm marks of the metre stick as in Figure 17.3 and record the metre stick reading at the position of the hanger (r_a in the diagram).

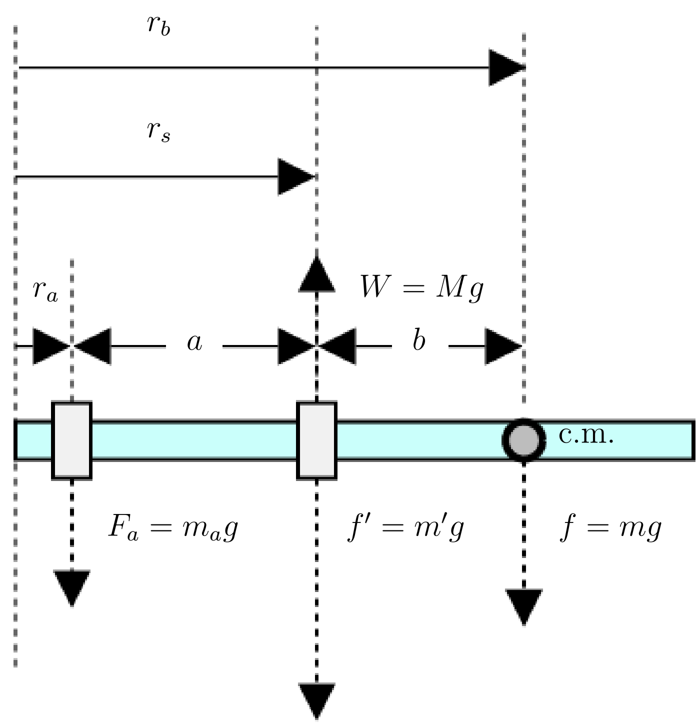


Figure 17.3: For Parts 2 and 3

3. Support the system by a hanger which is attached to the spring scale and to the metre stick between the suspension point of the 200g mass and the known centre of mass of the stick. Record the metre stick reading, r_s, at the point of suspension. In this case, only m, the metre stick mass, is unknown. It can be determined by either of the conditions

required for static equilibrium:

$$\sum F_{ext} = 0 \tag{17.3}$$

and

$$\sum \tau_{ext} = 0 \tag{17.4}$$

Equating forces up to forces down

$$W = F_a + f + f' \tag{17.5}$$

or

$$M = m_a + m + m' \tag{17.6}$$

where

- M is the *total* suspended mass, read from the spring scale
- m is the mass of the metre stick
- m' is the mass of the clamp at the point of suspension
- m_a is the suspended mass (including the clamp at a)

Thus, m is obtained from

$$m = M - m_a - m' \tag{17.7}$$

From Equation 17.4, the sum of the counterclockwise torques about any point equal the sum of the clockwise torques about that point. Thus, about the point of suspension

$$F_a a = f b \tag{17.8}$$

or

$$m_a a = m b \tag{17.9}$$

and finally

$$m = (a/b)m_a \tag{17.10}$$

4. Calculate r_b and its uncertainty, since, from the diagram, $r_b = r_s + b$.

5. Find r_b directly as in Part 1.

6. Measure the mass of the metre stick using the balance in the lab.

IT2: Fill in the results in Tables 17.2 and 17.3.

Part 3: The Non–Uniform Metre Stick

1. Set up the *non-uniform* metre stick set up exactly as in Part 2. Again we have two unknowns:

   (a) m, the metre stick mass, and

   (b) r_b, the distance from the zero end of the stick to the centre of mass.

2. Repeat the measurements as in Part 2.

3. Find r_b directly as in Part 1.

4. Measure m on the lab scale.

IT3: Fill in the results in Tables 17.4 and 17.5.

IQ2: Does the position of the centre of mass of the non-uniform metre stick make sense given where the holes are?

IQ3: Given what you know about torques, which holes make the biggest contribution to moving the centre of mass away from the 50cm mark?

17.4.3 Analysis (after the lab)

Part 1: Testing the Principle of Moments

1. Calculate to clockwise and counterclockwise torques to see if the Principle of Moments was obeyed.

Part 2: Mass and Center of Mass Determination from a Single Equilibrium Setup

1. Using Equation 17.7, calculate the mass of the metre stick.
2. Using Equation 17.10, and the mass calculated above, calculate the distance b to the centre of mass of the metre stick.
3. Use the metre stick reading for the point of suspension r_s recorded earlier and b to calculate r_b, the *reading on the metre stick at the centre of mass.*
4. Compare your calculated and measured values for r_b and m.

Part 3: The Non–Uniform Metre Stick

1. Perform a similar analysis to that for Part 2 above.

Post-lab Discussion Questions

Q1: Was the Principle of Moments obeyed for the system in Part 1? Explain.

Q2: Did the measured and calculated masses for the metre stick in Part 2 agree?

Q3: Did the the measured position of the centre of mass in Part 2 agree with the value calculated?

Q4: Did the measured and calculated metre stick masses in Part 3 agree?

Q5: Did the the measured position of the centre of mass in Part 3 agree with the value calculated?

17.5 Bonus: Unknown Metre Stick and Unknown Mass

Given

1. a non–uniform metre stick of unknown mass and centre of mass
2. an object of unknown mass,

3. one object of known mass,
4. assorted hangers of known mass,

and *no* spring scale, explain how you could determine the mass of the object. Time permitting, try it and discuss your results.

17.6 Recap

This lab should be a review of:

- calculations with uncertainties
- how to write a lab report

17.7 Summary

Item	Number	Received	weight (%)
In-lab Questions	3	________	50
Post-lab Questions	5		(in report)
Pre-lab Tasks	1	________	10
In-lab Tasks	3	________	40
Post-lab Tasks	0	________	0
Bonus		________	5

17.8 Template

My name:
My student number:
My partner's name:
My other partner's name:
My lab section:
My lab demonstrator:
Today's date:

quantity	symbol	value	effective uncertainty
Part 1			
Instrument			
name			
units			
precision measure			
reading at point of suspension	r_s		
reading at point a	r_a		
reading at point b	r_b		
Instrument			
name			
units			
precision measure			
mass of clamp at a			
mass of clamp at b			
Instrument			
name			
units			
precision measure			
added mass at a			
added mass at b			

Table 17.1: Quantities measured only once (Part 1)

quantity	symbol	value	effective uncertainty
Part 2			
Instrument			
name			
units			
precision measure			
reading at point of suspension	r_s		
reading at centre of mass (direct measurement)	r_b		
reading at point a	r_a		
Instrument			
name			
units			
precision measure			
total mass from spring scale	M		

Table 17.2: Quantities measured only once (Part 2) -to be continued

quantity	symbol	value	effective uncertainty
Part 2 (continued)			
Instrument			
name			
units			
precision measure			
mass of clamp at suspension point s			
mass of clamp at a			
mass of metre stick (direct measurement)	m		
Instrument			
name			
units			
precision measure			
added mass at a			

Table 17.3: Quantities measured only once (Part 2 continued)

quantity	symbol	value	effective uncertainty
Part 3			
Instrument			
name			
units			
precision measure			
reading at point of suspension	r_s		
reading at point a	r_a		
reading at centre of mass (direct measurement)	r_b		
Instrument			
name			
units			
precision measure			
total mass from spring scale	M		

Table 17.4: Quantities measured only once (Part3) -to be continued

quantity	symbol	value	effective uncertainty
Part 3 continued			
Instrument			
name			
units			
precision measure			
mass of clamp at suspension point s			
mass of clamp at a			
mass of metre stick (direct measurement)	m		
Instrument			
name			
units			
precision measure			
added mass at a			

Table 17.5: Quantities measured only once (Part3 continued)

quantity	symbol	value	uncertainty	units
Part 1				

Table 17.6: Given (ie. non-measured) quantities (ie. constants)

quantity	symbol	equation	uncertainty

Table 17.7: Calculated quantities

symbol	factor	bound	units
Sources of *systematic* error			
Sources of *random* error			

Table 17.8: Experimental factors responsible for effective uncertainties

Where to answer		
Question number	Discussion (y/n)	Conclusions (y/n)
In-lab		
Post-lab		
Hints		
	"think" "suggest" "explain" "how" "why" "what"	"agree" "equal" "do (did, does) " "significantly different" "support" "verify"

Table 17.9: Lab Report Organization

Chapter 18

Exercise on Estimation, Bounding, and Order of Magnitude Calculations

18.1 Purpose

The purpose of this exercise is to develop skills in estimating quantities when you can't measure them directly, and to make estimates of some sample quantities.

18.2 Introduction

While we usually think of science as involving *measurement*, there are many times when approximate values for quantities must be produced even when precise measurements are difficult or impossible. This exercise is about how that is done.

18.3 Theory

18.3.1 Estimation

Lots of experiments involve quantities which must be estimated. (For instance, before you *measure* anything, it's good to be able to *estimate* the result you expect, so you can determine what sort of instrument or method you'll need to perform the measurement.) Some estimates may be better than others, but what really matters is that you have a fair idea about how far off your estimate *could* be.

Bounding

Bounding a quantity is forming an estimate of how far off it could be. An *upper bound* is a numerical limit above the expected value, and a *lower bound* is a numerical limit below the expected value.

Picking Realistic Bounds It's often easy to come up with reasonable bounds for a quantity by using similar known quantities which are pretty clearly above or below. For instance, if you are estimating a person's height, then you can compare with known heights of family members or friends. If you have to estimate the mass of an object, you can compare it to objects with which you are familiar.

Range of Possible Values for a Quantity The **range** of values for a quantity is the difference between its upper and lower bounds.

Familiar Comparison

If you're trying to estimate something, and it's similar to something you know, then you can probably make a pretty good estimate through comparison. In other words, if you can establish an upper and a lower bound, then you can estimate something in between.

18.3.2 Order of Magnitude Calculations

Quantities which are too difficult to be estimated directly can be estimated by performing calculations with estimates. For instance, sometimes certain quantities can be measured but others must be estimated. These calculations are called **order of magnitude calculations**[1], since their purpose is to give a result which is within an order of magnitude (i.e. a factor of ten) of the result of the detailed calculation.

[1] or, *"back of the envelope calculations"*

18.3.3 Logarithmic scale

On a logarithmic scale, such as the one in Figure 18.1, the distance of a number from the left end of the scale is proportional to the logarithm of the number. Figure 18.2 and Figure 18.3 show some other possibilities. (Logarithmic scales are often identified by the number of **cycles** they show.) A *cycle* is the space between two numbers which differ by a factor of ten. So, between 1 and 10 is one cycle, between 2 and 20 is one cycle, between 5 and 50 is one cycle, etc. Note that there is *no* zero on a logarithmic scale. All numbers are positive.

18.4 Procedure

18.4.1 Preparation (before the lab)

There is no preparation required for this exercise.

18.4.2 Investigation (in the lab)

Illustration of Comparison

To illustrate, we'll try to make a few simple estimates. The first question we want to answer is: *How tall am I?*

1. Find someone who thinks they are shorter than me. Record that person's height. If that someone is shorter than I am, that person's height is definitely a lower bound for my height.

2. Find someone who thinks they are taller than me. Record that person's height. If that someone is taller than I am, that person's height is definitely an upper bound for my height.

3. Estimate my height according to the two known heights. If, for example, you are closer in height to the person who has the lower bound height than to the person who has the higher bound height, you would estimate your height closer to that of the lower bound person.

4. Estimate the bounds you would place on my height. (For instance, if you saw someone about my height commit a crime, what range of

heights would you give to investigators so that it would be of use in identifying suspects?)

IQ1: Give the heights of the people who you thought were taller and shorter than me. From these heights estimate my height and explain how you came up with my height. What bounds would you place on my height? Are they as far apart as the two heights of the people you knew?

> The goal when making estimates is to try and make them "safe" but "useful"; i.e. you are *pretty sure* about lower and upper bounds on your estimate, but the bounds are close enough together to make the estimate usable.

IT1: Form a group of 3 or 4, and use items in the class to make comparisons. Fill in Table 18.1 with a reasonable estimate and bounds for each of the following:

- my height (from above)
- mass of a block of wood (comparing to known masses)
- volume of liquid

Less Familiar Comparison

Often it's not easy to make a clear comparison with something very similar, and so the bounds and thus the estimate have to be a bit more fuzzy.

IT2: Based on your experience, fill in Table 18.2 with suggested bounds and a reasonable estimate for each of the following:

- height of this building (in metres)
- length of this building (in metres)
- mass of lab table (in kilograms)

IQ2: For one of those quantities, explain how you came up with the bounds and the estimate. Was the range of values for this comparison proportionally larger than for the familiar comparisons above? Explain.

Order of Magnitude Calculations

In labs, order of magnitude calculations are often done with preliminary data to see if the results seem to be as expected. Using the wrong units, a common mistake in labs, shows up easily in order of magnitude calculations. In these calculations, the formula may the one that will ultimately be used with the complete data, but with incomplete or estimated values it is an order of magnitude calculation. (In these cases, if you're using a calculator, there's no need to type in the known values to more than one or two significant figures since the result will be approximate anyway.)

> Since an order of magnitude calculation is supposed to be within one order of magnitude, you should be fairly confident that 10 times the value is *clearly* too big, and 1/10 of the value is *clearly* too small.

Illustration of Order of Magnitude Calculation

The next question we want to answer is: *How many coffees are sold in the Tim Horton's in the Science building every day?*

How can we figure this out?

There are several things we know about this problem. We know:

- They have lineups at certain times of the day.
- They have to sell enough coffee to pay their staff.
- It takes a certain amount of time to serve a coffee.
- There are a fixed number of coffeemakers and pots that process all of the coffee each day.

Each of these facts can be used with a bit of reasoning to produce a calculation of the number of coffees sold in a day, provided we can estimate the quantities involved. An example with a related problem can be used to illustrate.

How many gold parking spaces are there at Laurier?

1. Gold parking lots are for faculty and staff. So, there are probably less gold spaces than the total number of faculty and staff at Laurier, since some will walk, bus, carpool, etc.

2. There are several lots of various sizes, so an estimate of the number of lots and the average size will allow estimating the total number of spots.

In the first case, the lot in front of the science building has maybe 50 spots; (about 25 on each side). I think there are around 10 or 15 gold lots around campus, so let's say ten. If we call the average number of spots in a lot S, and the number of lots L, then the number of spaces, N, is given by

$$N = S \times L$$

or

$$N \approx 50 \times 10 = 500$$

So I'd estimate around 500 gold spots on campus. Does this make sense?

- *Are there more than 50 spots?* Since there are probably about 50 in front of the science building alone, then yes, there are more than 50.

- *Are there less than 5000 spots?* Since Laurier has around 10000 students, there would have to be one faculty or staff member for every two students to be able to use 5000 spots, which doesn't fit with what I observe. (I'd guess that there are more like one faculty or staff person for every 10 students or so.) So, yes, there are less than 5000.

Putting these together, my order-of-magnitude estimate of 500 spots is reasonable.

One of the important things to understand about order of magnitude calculations is that they are somewhat self-correcting; i.e. you're likely to overestimate some quantities but underestimate others, so the effects will cancel somewhat.

IQ3: Using the facts you know about Tim's that I listed earlier, produce two different formulas for the number of coffees in a day, with definitions of each of the quantities in the equation. Using estimates for each of the quantities involved, calculate results for each method. Discuss whether the two results are within an order of magnitude of each other. (If they aren't, suggest which particular estimates are the most suspect.)

When is a calculation an Order of Magnitude Calculation?

Any time you have to do a calculation using an estimated quantity, you are performing an **order of magnitude calculation**. The *order of magnitude* of a quantity refers specifically to the power of ten in its measurement. For instance, the height of the building would be in metres, while the length would be in tens of metres. In more general terms, the order of magnitude of a quantity refers to the cycle of a logarithmic scale to which the quantity belongs. Thus we could say that the order of magnitude value for the length of the science building is

- around 100 metres
- between 50 and 200 metres

Both of these are order of magnitude estimates.

When are Order of Magnitude Calculations used?

Order of magnitude calculations are quite commonly done in science *before* an experiment is performed. This is so that the range of expected data can be determined. They are also often done *as* the data are being collected to see if the experimental results appear to be in the correct ballpark.

> An order of magnitude calculation is *any* kind of calculation which will produce an answer which should be close to the *"real"* answer. Any calculation involving at least one estimated quantity is an order of magnitude calculation. Generally, the more estimated quantities involved in an order of magnitude calculation, the wider the distance between the upper and lower bounds produced.

Logarithmic scale

If several people make estimates, they will no doubt vary. However, they will probably still be in a common ballpark. This can be more easily observed by plotting the values on a *logarithmic scale*, such as the one in Figure 18.1.

Where would zero be, if you wanted to show it?

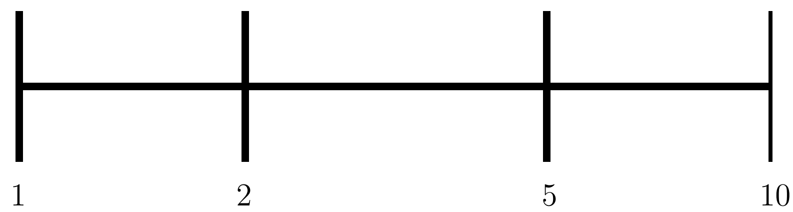


Figure 18.1: Logarithmic scale

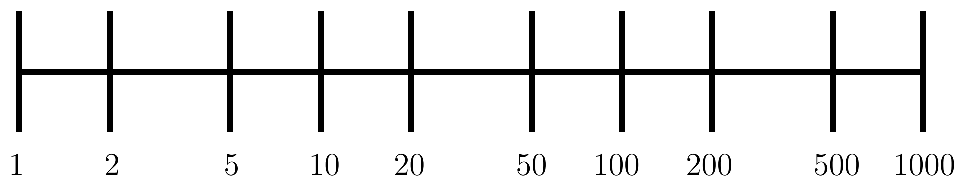


Figure 18.2: Three cycle logarithmic scale

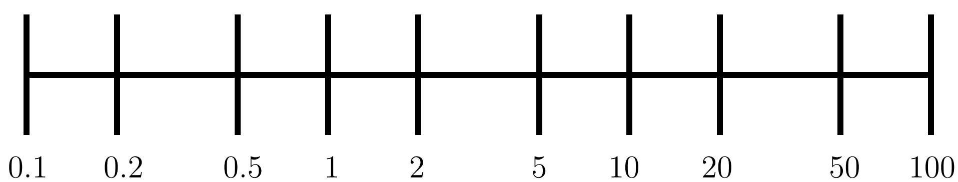


Figure 18.3: Logarithmic scale with numbers less than one

There are many things which we *perceive* on a logarithmic scale (such as the volume of music).

How big is an order of magnitude on a logarithmic scale?

> If two numbers are within the same cycle of a logarithmic scale, they are within an order of magnitude of each other.

IT3: For the quantity estimated in **IQ3**, use the logarithmic scale of Figure 18.4 and mark 3 or 4 of the class estimates on it.

IQ4: Do all of the estimates for that quantity fall within a single cycle of the scale? (In other words, between 2 and 20, 5 and 50, 10 and 100, etc.) Explain.

Uncertainties

A quantity that is bounded can be expressed as an estimate with an uncertainty. (This is a little less cumbersome than giving the estimate, the lower bound, and the upper bound.) Usually it's easiest to express uncertainties in linear (i.e. non-logarithmic) terms, so that an estimate can be given which is *"plus or minus"* some amount. In order to do this, it may require adjusting one of the bounds so that the uncertainty can be the same in both directions. For instance, the length of the building was estimated to be between 50 and 200 metres. If I think it's probably around 100 metres I could modify my estimate of *"between 50 and 200 metres"* to be *"between 50 and 150 metres"* which I could state as "100 ± 50 *metres*".

> If you have upper and lower bounds for a quantity, then the uncertainty can be *estimated* as one half of the range; i.e.
>
> $$uncertainty \approx 1/2(upper\ bound - lower\ bound)$$

IT4: In the same group of 3 or 4, using the upper and lower bounds for the list of quantities in Task 1 above, and give final estimates with an uncertainty using your upper and lower bounds in Table 18.3.

IQ5: Did all of your estimates fall midway between your upper and lower bounds? If not, how did you choose your uncertainty? If so, how would you choose your uncertainty if that happened?

Comparing Quantities with Uncertainties

IQ6: Compare your answers to the answers from one other group. Do the answers from the two groups for one item in Task 4 *agree* with each other? Give values of both groups to help explain your answer.

18.4.3 Analysis (after the lab)

There are no post-lab requirements since next week final marks will be calculated.

18.5 Recap

By the end of this exercise, you should understand the following terms:

- estimate
- bound
- logarithmic scale
- order of magnitude calculation

All of these concepts are important in research.

18.6 Summary

Item	Number	Received	weight (%)
Pre-lab Questions	0	________	0
In-lab Questions	6	________	50
Post-lab Questions	0	________	0
Pre-lab Tasks	0	________	0
In-lab Tasks	4	________	50
Post-lab Tasks	0	________	0

18.7 Template

My name:
My student number:
My partner's name:
My other partner's name:
My lab section:
My lab demonstrator:
Today's date:

quantity	estimate	units	upper bound	lower bound
my height				
mass of block of wood				
volume of liquid				

Table 18.1: Estimates for Task 1

quantity	estimate	units	upper bound	lower bound
height of building		m		
length of building		m		
mass of lab table		kg		

Table 18.2: Estimates for Task 2

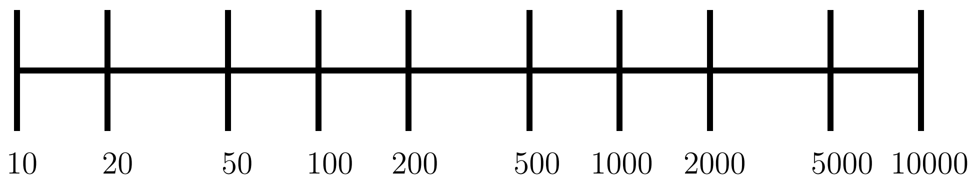


Figure 18.4: For Task 3

quantity	estimate	units	upper bound	lower bound	uncertainty
my height					
mass of block of wood					
volume of liquid					

Table 18.3: Determining Uncertainties for Task 4

Appendix A

Common Uncertainty Results

Following are some common results about uncertainties which you may find useful. If there are others which you feel should be here, inform the lab supervisor so that they may included in future versions of the lab manual.

$$\Delta(x^n) \approx n|x|^{n-1}\Delta x$$

$$\Delta(\sin x_R) \approx |\cos x_R|\,(\Delta x)_R$$

$$\Delta(\tan x_R) \approx (\sec x_R)^2(\Delta x)_R$$

where x_R denotes x in *radians*.

$$\Delta \ln x \approx \frac{1}{x}\Delta x = \frac{\Delta x}{x}$$

$$\Delta x^y \approx \left|x^{y-1}y\right|\Delta x + \left|x^y \quad \ln x\right|\Delta y$$

$$\Delta \sqrt[y]{x} \approx \left|x^{\left(\frac{1}{y}-1\right)} \quad \frac{1}{y}\right|\Delta x + \left|x^{\frac{1}{y}} \quad \ln x\right|\frac{\Delta y}{y^2}$$

$$\Delta f(x,y,z) \approx \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \left|\frac{\partial f}{\partial z}\right|\Delta z$$

Appendix B

Common Approximations

Following are some common approximations which you may find useful. If there are others which you feel should be here, inform the lab supervisor so that they may included in future versions of the lab manual.

Taylor Series Expansion

$$f(x+h) = \sum \frac{f^n(x)}{n!} \approx f(x) + hf'(x)$$

The following derive from the Taylor series expansions, where $x \ll 1$. In cases where an approximation is given with more than one term, the first term alone may be sufficient in some cases.

$$(1 + x^n) \approx 1 + nx$$

$$\ln(1 + x) \approx x - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2!}$$

The following also assume x is in *radians*.

$$\sin(x) \approx x - \frac{x^3}{3!}$$

$$\cos(x) \approx 1 - \frac{x^2}{2!}$$

Appendix C

Use of the Standard Form for Numbers

C.1 Introduction

Learning how to use the standard form is easy; learning *when* to use it is a bit harder. Following are some examples which should help you decide when to use it. It is perhaps easiest to illustrate by comparing results using the standard form and not using it. Remember the point is to present things more concisely.

1. Voltage V of 2.941 Volts; uncertainty of 0.4517 Volts

    - First round the uncertainty to one significant figure; thus

      $\Delta V \approx 0.4$ V

    - Next round the quantity so the last digit displayed is in the same decimal place as the uncertainty; thus

      $V \approx 2.9$ V (when rounded to the same decimal place as the uncertainty)

    - So

      $V = 2.9 \pm 0.4$ V

      In this case, scientific notation is not needed.

2. Mass m of 140.6 grams; uncertainty of 531.7 grams

   This is an example of a situation where the uncertainty is larger than the quantity itself. The process involved is the same.

   - First round the uncertainty to one significant figure; thus

     $\Delta m \approx 500$ g
   - Next round the quantity so the last digit displayed is in the same decimal place as the uncertainty; thus

     $m \approx 100$ g (when rounded to the hundreds place)
   - So

     $m = 100 \pm 500$ g

     In this case, scientific notation *is* needed, because we need to get rid of the placeholder zeroes. One option would be to write

     $m = (1 \pm 5) \times 10^2$ g

     Another option would be to write

     $m = 0.1 \pm 0.5$ kg

     Note that this last option is more concise.

3. Diameter d of 0.727 cm; uncertainty of 0.015 cm

   - First round the uncertainty to one significant figure; thus

     $\Delta d \approx 0.02$ cm
   - Next round the quantity so the last digit displayed is in the same decimal place as the uncertainty; thus

     $d \approx 0.73$ cm (when rounded to the hundredths place)
   - So

     $d = 0.73 \pm 0.02$ cm

     In this case, scientific notation is not needed, because there are no placeholder zeroes.

     Another option would be to write

     $d = 7.3 \pm 0.2$ mm

Note that this last option is slightly more concise, since it gets rid of two zeroes, and puts d in proper form for scientific notation (even though in this case the exponent would be zero).

4. Time t of 943 s; uncertainty of 29 s

   - First round the uncertainty to one significant figure; thus

     $\Delta t \approx 30$ s

   - Next round the quantity so the last digit displayed is in the same decimal place as the uncertainty; thus

     $t \approx 940$ s (when rounded to the tens place)

   - So

     $t = 940 \pm 30$ s

     In this case, scientific notation is needed, because there are place-holder zeroes.

     One option would be to write

     $t = (9.4 \pm 0.3) \times 10^2$ s

     This is in the standard form.

     Another option would be to write

     $t = 9.4 \pm 0.3$ hs

     However, since “hectoseconds” are not commonly used I would avoid this (although it is also correct).

     Similarly you could write

     $t = 94 \pm 3$ das

     Again, since “dekaseconds” are not commonly used I would avoid this (although it is also correct).[1]

[1] I actually had to check on the spelling and notation for dekaseconds and hectoseconds, so that illustrates how (un)familiar they are.

Appendix D

Order of Magnitude Calculations

To check your conversions, (among other things), do a rough calculation of your results carrying every value to just 1 significant[1] figure. (This is easy to do quickly on a piece of paper without a calculator, so you can be sure that calculated results are in the right ballpark.)
For instance, for the *"Measuring 'g' "* experiment,

$$h \approx 5m$$

$$t \approx 1s$$

Thus

$$g \approx \frac{2 \times 5}{1^2} \approx 10m/s^2$$

Be sure to write out these calculations; that way you'll be clear on the units you used, etc. If you make an error and have to correct it, you'll want a record of it so you don't make it when you do the "real" calculations.
Since this result is about what you'd expect, then you know any values in that range should be reasonable.

D.1 Why use just one or two digits?

There are a couple of reasons:

[1] If the digit is a 1 or a two, then you may carry 2 figures. If you do this then your answer should be within about 10% of the value you'd get with a detailed calculation. This is easily close enough to spot any major errors.

1. Since we're only using one data point, instead of all of our values, the result will be approximate, so the extra digits aren't needed.

2. When people use calculators, they tend to just automatically write down any answer without thinking. If they made a typing mistake, they often don't notice. So by doing it by hand using only one or two digits, they keep their brains engaged and are more likely to notice an error.

D.2 Using the median instead of the average

If you have several measurements of a quantity, do the calculation with one value instead of averaging all of them. The median is easy to find, and should be close to the average.

D.3 Order of Magnitude Calculations for Uncertainties

In a similar way, you can check to see if your uncertainties are reasonable. In the above example, if

$$\Delta h \approx 5 \text{ cm}$$

and

$$\Delta t \approx 0.1 \text{ s}$$

then

$$\begin{aligned}
\Delta g &\approx g\left(\frac{\Delta h}{h} + 2\frac{\Delta t}{t}\right) \\
&\approx 10\left(\frac{0.05}{5} + 2\frac{0.1}{1}\right) \\
&\approx 10\,(0.01 + 0.2) \\
&\approx 10\,(0.21) \\
&\approx 0.2 \text{ m/s}^2
\end{aligned}$$

This makes sense, and so your detailed uncertainty calculations should produce something in this ballpark.

Appendix E

Simple Statistical Approximations

Statistical calculations can be tedious and time consuming in the lab. There are some ways to get approximate results quickly.

E.1 Simple Method; The Method of Quartiles

There is a way [1] to get values very close to those given by calculating the mean and standard deviation of the mean with very little calculation. (This will be true if the data have a Gaussian[1] distribution.) The method involves dividing the data into *quartiles*. The first quartile is the value which is above 1/4 of the data values; the second quartile is the value which is above 1/2 of the data values[2] and so on. The second quartile gives a good estimate for the average, and the third quartile minus the first quartile gives a good estimate[3] for the standard deviation. Thus,

$$\bar{x} \pm \alpha \approx Q_2 \pm \frac{(Q_3 - Q_1)}{\sqrt{n}}$$

If you use a number of data values which is a perfect square, such as 16, then the only calculation is *one* division!

[1] or "normal"

[2] which is also the median

[3] Actually the inter-quartile distance or $IQR \approx 1.35\ \sigma$ for normally distributed data.

E.1.1 Example with an even number of points

Previously, the data in Table E.1 were used for an example:

i	x_i
1	1.1
2	1.4
3	1.3
4	1.2

Table E.1: Sample Data

From this,

$$\overline{x} = 1.25$$

and

$$\sigma = 0.129$$

and finally

$$\alpha = 0.0645$$

Using the method of quartiles, we first rearrange the data, as in Table E.2:

i	x_i
1	1.1
2	1.2
3	1.3
4	1.4

Table E.2: Ordered Data

Now,

$$Q_1 = \frac{1.1 + 1.2}{2} = 1.15$$

$$Q_2 = \frac{1.2 + 1.3}{2} = 1.25$$

$$Q_3 = \frac{1.3 + 1.4}{2} = 1.35$$

Thus

$$\overline{x} \approx Q_2 = 1.25$$

and

$$\sigma \approx \frac{Q_3 - Q_1}{1.35} = \frac{1.35 - 1.15}{1.35} = 0.15$$

and finally

$$\alpha \approx \frac{0.15}{2} = 0.075$$

The estimates of σ and α are within about 15% of their correct values.

E.1.2 Example with an odd number of points

The data in Table E.3 will be used for this example:

i	x_i
1	1.01
2	0.97
3	1.03
4	0.99
5	0.95

Table E.3: Sample Data

From this,

$$\overline{x} = 0.99$$

and

$$\sigma = 0.032$$

and finally

$$\alpha = 0.014$$

Again, we first rearrange the data, as in Table E.4:
Now, the quartiles overlap

$$Q_1 = median\ of\ 0.95, 0.97, 0.99 = 0.97$$

$$Q_2 = median\ of\ all\ values = 0.99$$

$$Q_3 = median\ of\ 0.99, 1.01, 1.03 = 1.01$$

i	x_i
1	0.95
2	0.97
3	0.99
4	1.01
5	1.03

Table E.4: Ordered Data

Thus

$$\overline{x} \approx Q_2 = 0.99$$

and

$$\sigma \approx \frac{Q_3 - Q_1}{1.35} = \frac{1.01 - 0.97}{1.35} = 0.03$$

and finally

$$\alpha \approx \frac{0.03}{\sqrt{5}} = 0.013$$

The estimates of σ and α are within about 10% of their correct values.

E.2 Simple Method; Using the K factor

For small numbers of measurements, it is possible [2] to get a value close to the standard deviation by using the *range* of values to estimate the standard deviation.

To estimate the standard deviation for a sample of n data points,

$$\sigma \approx K_n \times w$$

where w is the *width* of the range of data (i.e. the highest minus the lowest value). The *efficiency* indicates how well this estimate compares to the correctly calculated value. For example, with 10 data points, the efficiency of 0.85 indicates that the K factor estimation of the standard deviation is as statistically reliable as a standard deviation calculated with $0.85 \times 10 = 8.5$ data points.

For convenience, I have defined $K_{\alpha n}$, which is used to estimate α, the standard deviation of the mean, from the range. Since

$$\alpha = \frac{\sigma}{\sqrt{n}}$$

n	K_n	Efficiency	$K_{\alpha n}$
2	0.89	1.00	0.63
3	0.59	0.99	0.34
4	0.49	0.98	0.25
5	0.43	0.96	0.19
6	0.40	0.93	0.16
7	0.37	0.91	0.14
8	0.35	0.89	0.12
9	0.34	0.87	0.11
10	0.33	0.85	0.10

Table E.5: K factors

then

$$\alpha \approx \frac{K_n \times w}{\sqrt{n}}$$

so

$$K_{\alpha n} = \frac{K_n}{\sqrt{n}}$$

and thus

$$\alpha \approx K_{\alpha n} \times w$$

E.2.1 example

Again we are using the data from Table E.1. $n = 4$, so

$$K_n = 0.49$$

and the width of the range is

$$w = 1.4 - 1.1 = 0.3$$

and so

$$\sigma \approx 0.49 \times 0.3 = 0.15$$

and finally

$$\alpha \approx \frac{0.15}{2} = 0.075$$

which are the same values (to two decimal places) as given by the method of quartiles.

Appendix F

Lab Checklist

This marking checklist will be used for lab reports this term. You need to print one off and attach it to each lab report you hand in. Lab reports will be marked as follows:

- Start with 90

  For items *not* in italics

- Subtract 1 for each ˜.
- Subtract 2 for each − .

  For items *in italics*

- Subtract 3 for each ˜.
- Subtract 6 for each − .

Note the importance of items in italics. These are very important in a report, and so are weighted accordingly.
The other 10 marks will be based on how well the post-lab discussion questions were answered within the text of the report. *Remember that the answers to these questions should be an integral part of the report, not merely an afterthought.*

Lab Format Checklist (V3.0ng)

A. General

1.	Your own work	________
2.	Complete	________
3.	Clear and appropriate "Purpose"	________
4.	Flows	________
5.	Did not require help on or after due date	________
6.	Correct grammar	________
7.	Correct spelling	________
8.	Complete sentences where required	________
9.	Legible	________
10.	Professionally presented	________
11.	Properly identified (e.g. name, partner)	________
12.	On time	________
13.	Checklist included	________
14.	Template included	________

B. *Plagiarism Avoidance*

1.	*Data only shared with partner(s)*	________
2.	*Individual choice of sample data*	________
3.	*Individual formatting*	________
4.	*Individual structure of text*	________

C. Data

1.	Neat	________
2.	Table column headings informative	________
3.	Units given	________
4.	Uncertainties given	________
5.	Reasonable values	________
6.	Reasonable uncertainties	________
7.	Correct number of significant figures	________
8.	Tables labeled (e.g. "Sample 1 Data")	________
9.	Tables given numbers (e.g. "Table #2")	________

D. Calculations and Results

1.	Any required derivations done correctly	________
2.	Analysis explained where needed	________
3.	Correct formulas used	________
4.	Sample calculations shown where needed	________
5.	All required values calculated	________
6.	Uncertainties included	________
7.	Units included	________
8.	Correct number of significant figures	________
9.	Appropriate use of standard form	________
10.	Theoretical or reasonable value	________
11.	Agreement of experiment with theory	________

E. *Error Discussion*

1.	*Sources listed are significant*	________
2.	*Sources are prioritized*	________
3.	*Systematic error consequences*	________
4.	*Evidence: i.e. test or bound*	________
5.	*Reasonable suggestions for improvement*	________

F. *Conclusions*

1.	*Relate to purpose*	________
2.	*Major results stated*	________
3.	*Comparisons made where appropriate*	________
4.	*Agreement noted when found*	________
5.	*% difference only when no agreement*	________

G. References

1.	Source(s) of constants listed	________

Appendix G

Marking Scheme

PC131Lab and Exercise Weighting Fall 2016(Best adding to 100 will count)

A. Required

1.	MyLearningSpace Quizzes	10
2.	Repeated Measurements Exercise	10
3.	Processing Uncertainties Exercise	10
4.	Estimation, etc. Exercise	10
5.	Measuring "g"*[1]	10
6.	Torque and the Principle of Moments*[1]	10
7.	**Measuring "g" report**	**40**

B. Optional (Can replace other marks if higher)

1.	*Additional* weight of Measuring "g" lab report, if over 50	10
2.	*Additional* weight of lab report, if over 80	10
3.	Lab test	20
4.	Torque lab report	20

[1]except post-lab questions, which will be in the report

Fall 2016

Bibliography

[1] L.M. Brown. The use of quartiles in teaching error estimation. *Physics Education*, 12(4):261–263, 1977.

[2] R.B. Dean and W.J. Dixon. Simplified statistics for small numbers of observations. *Analytical Chemistry*, 23(4):636–638, 1951.

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