Graphical Uncertainites
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Overview

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  and when to use each one

When graphs are used for data analysis, the *parameters* of the graph, (such as slope and y-intercept for a linear graph), will have uncertainties based on the data.

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- For data shown on a graph, **error bars** are used to show the range of possible values for the *points*.
- Parameters of an equation determined from the data, such as the slope and y-intercept of a straight line, will have uncertainties which represent the range of realistic values needed to make all of the data points fit the curve.

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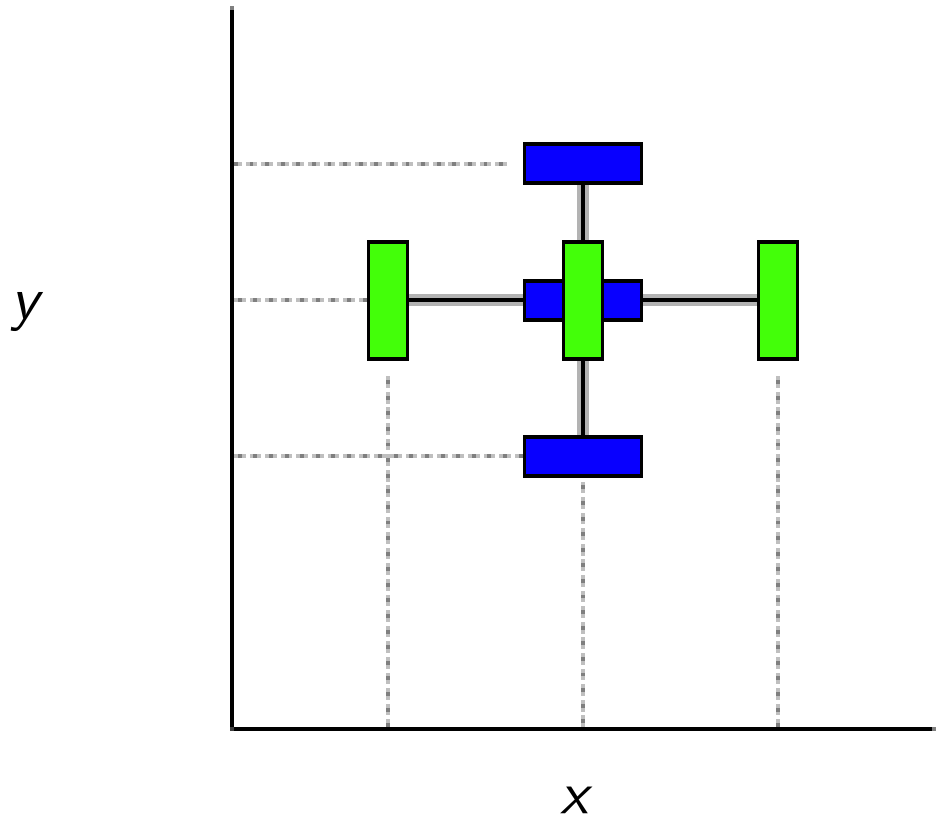
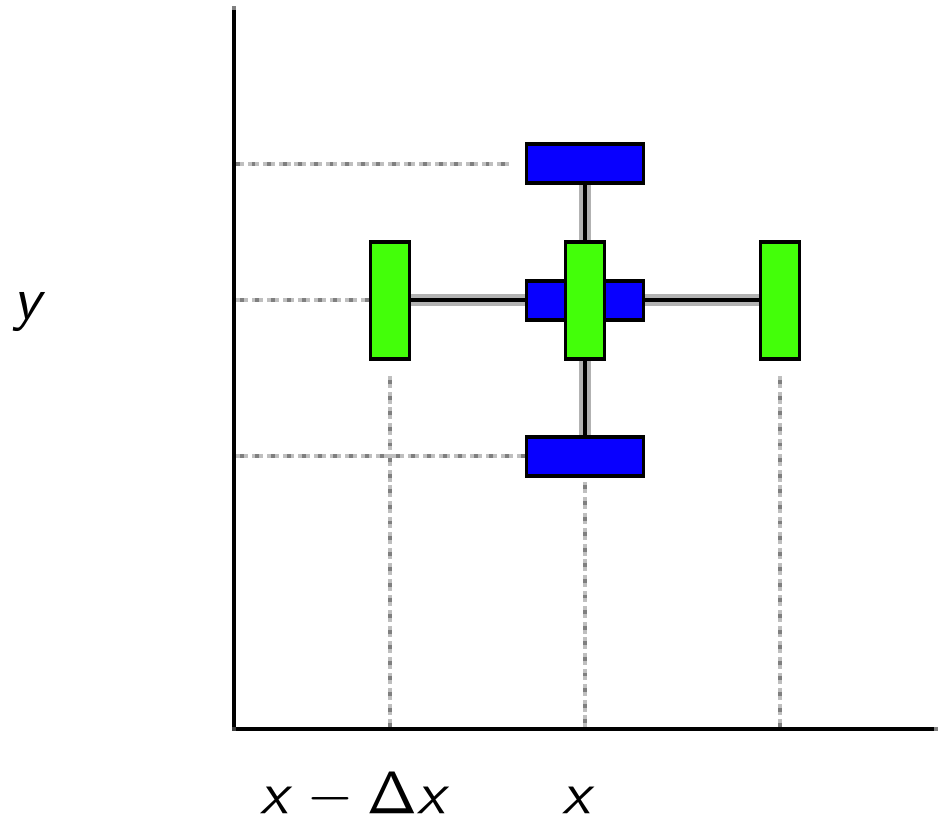


Point with error bars



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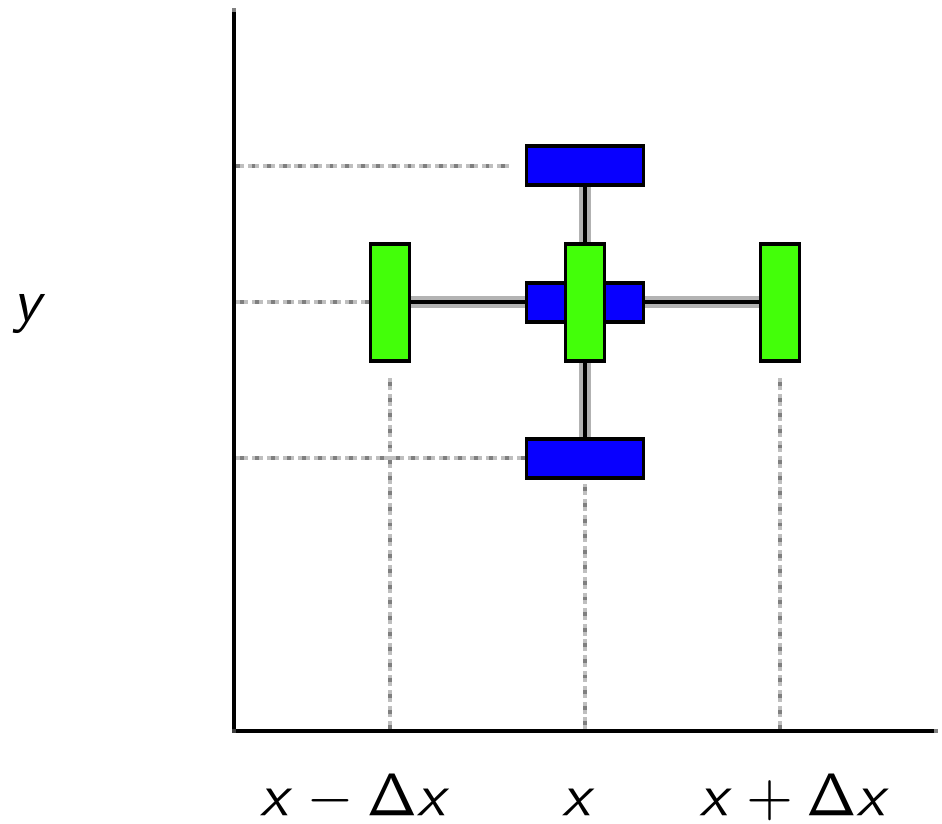

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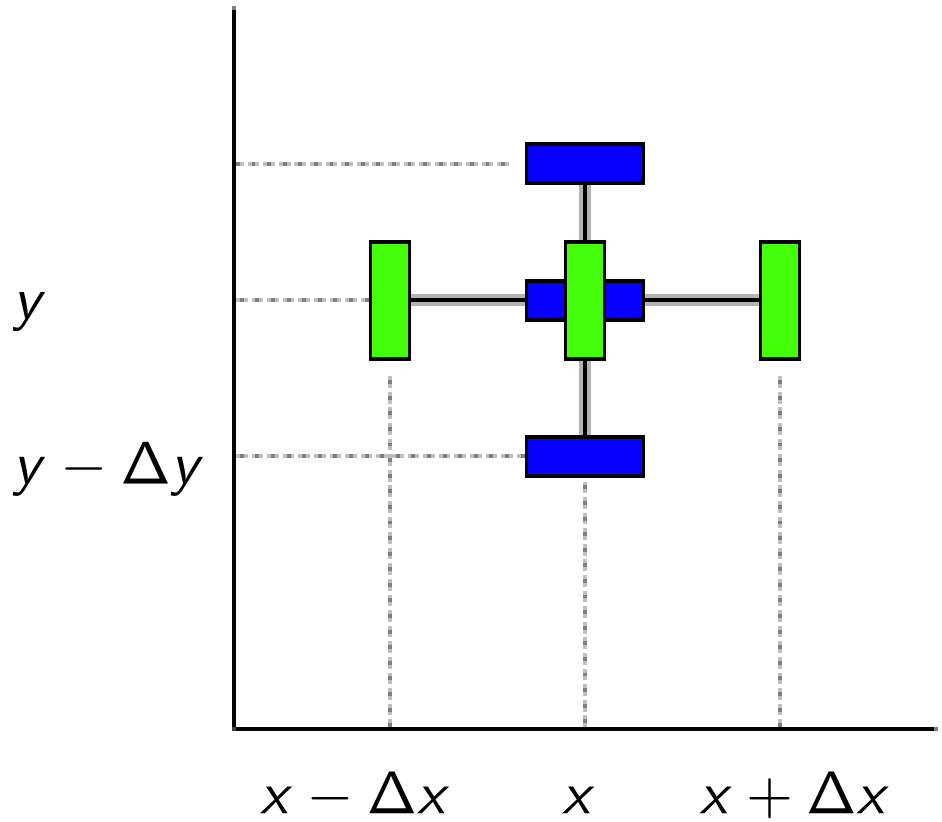

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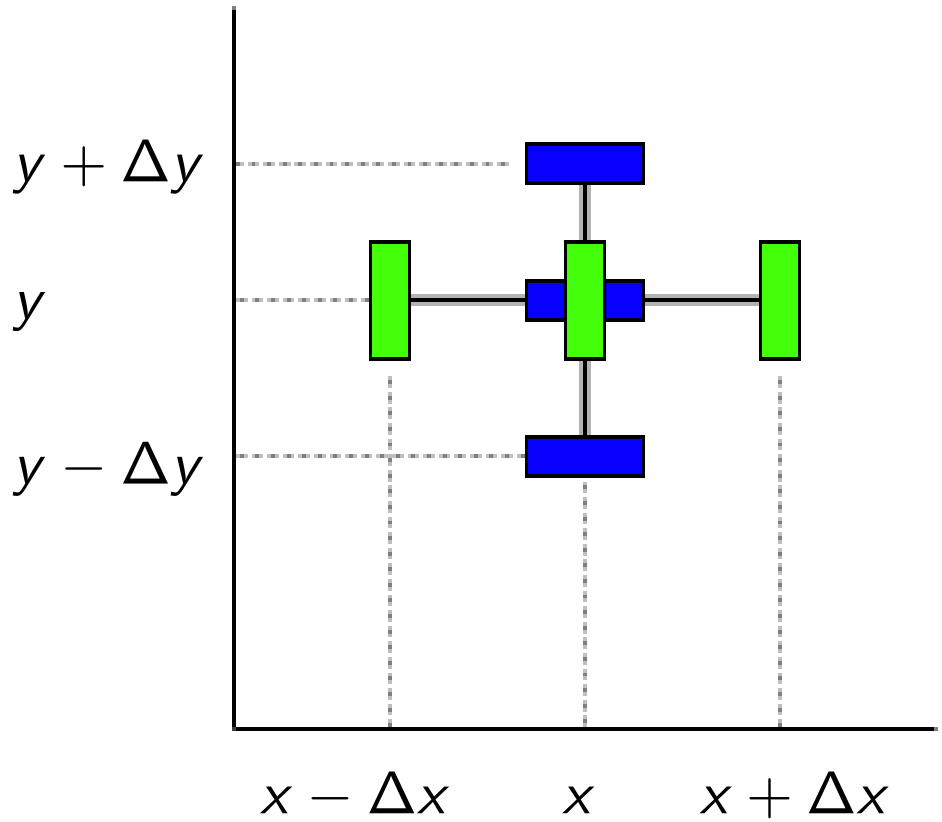

Point with error bars

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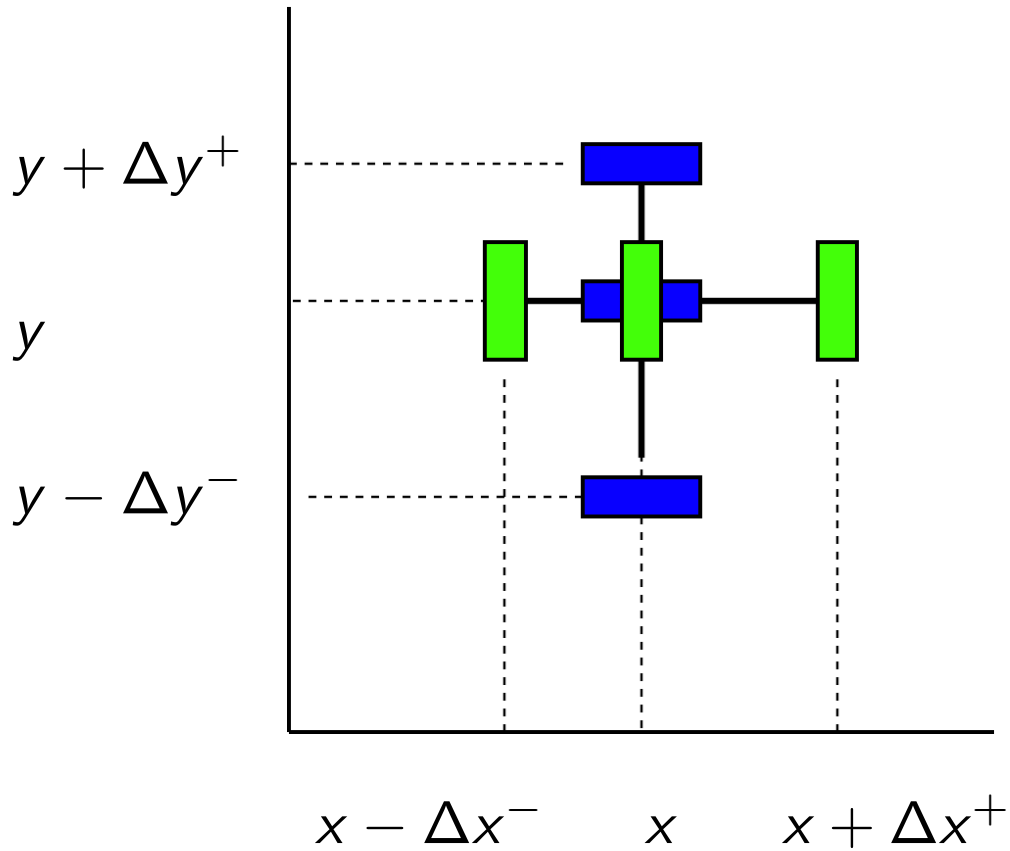
- They may even be different in the positive and negative directions.



Graph with unequal error bars in positive and negative directions

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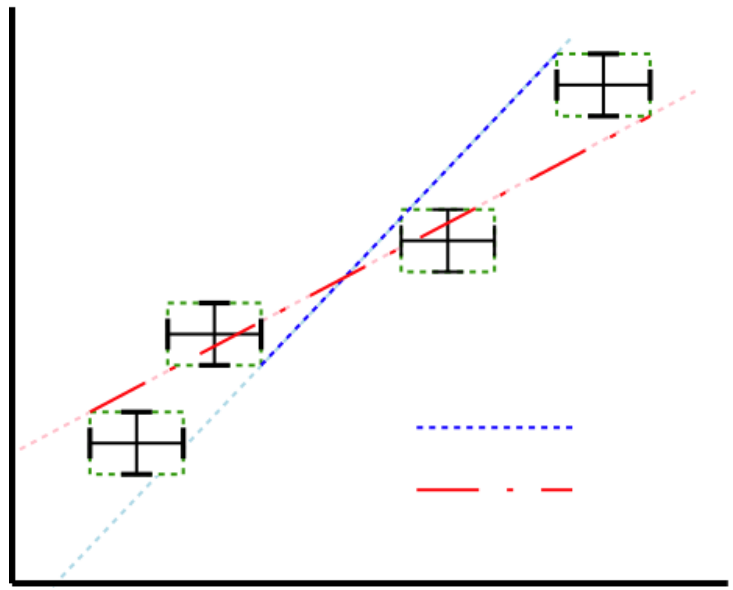
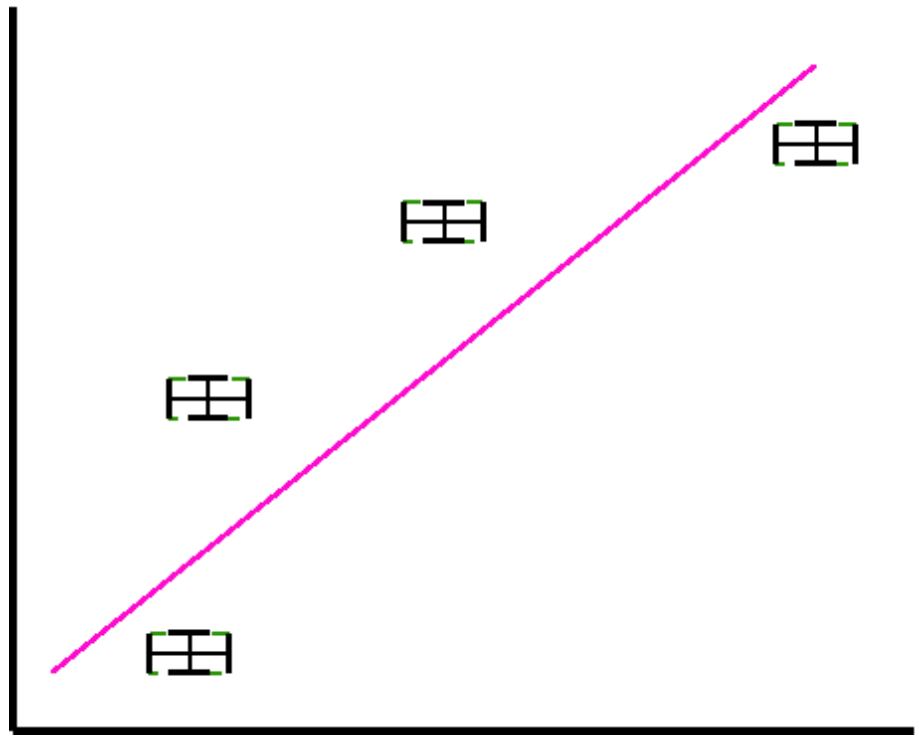
The slope and intercept can be found graphically, by eye or using the least squares fit method.



Case I: Small Scatter of Data Points; line of best fit crosses all error bars



Case II: Large Scatter of Data Points; *no* line crosses all error bars

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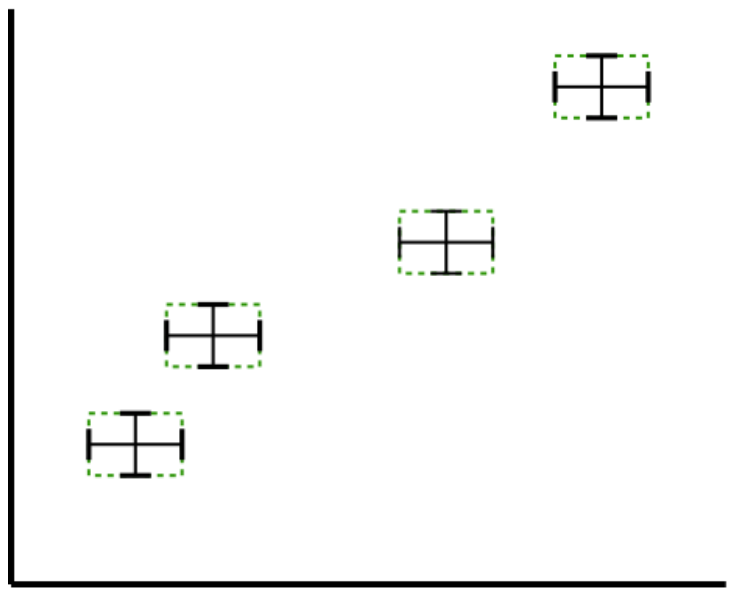

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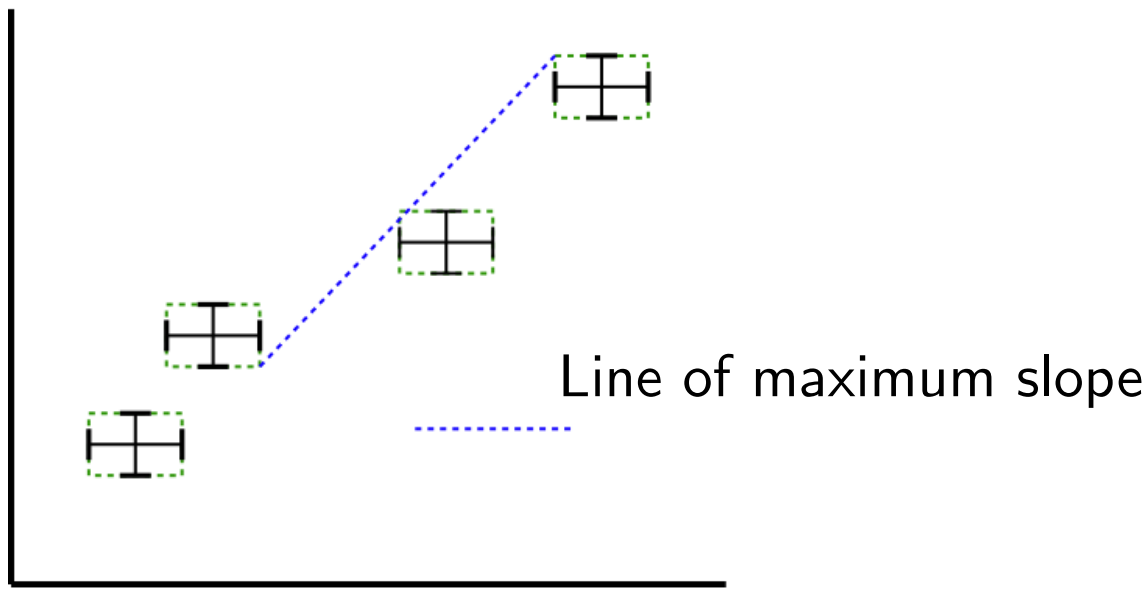

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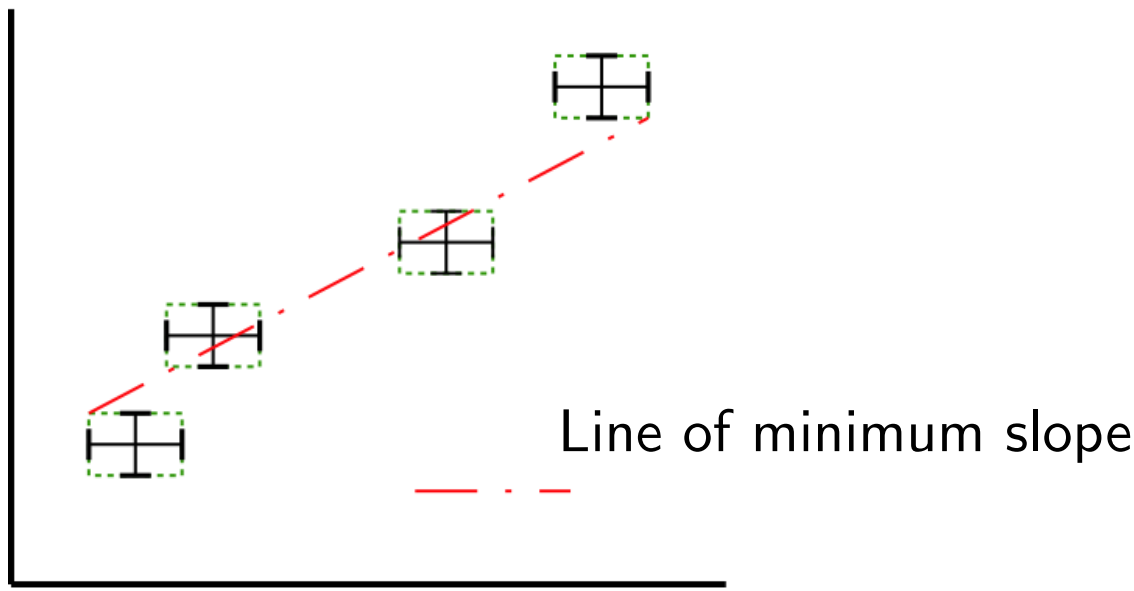

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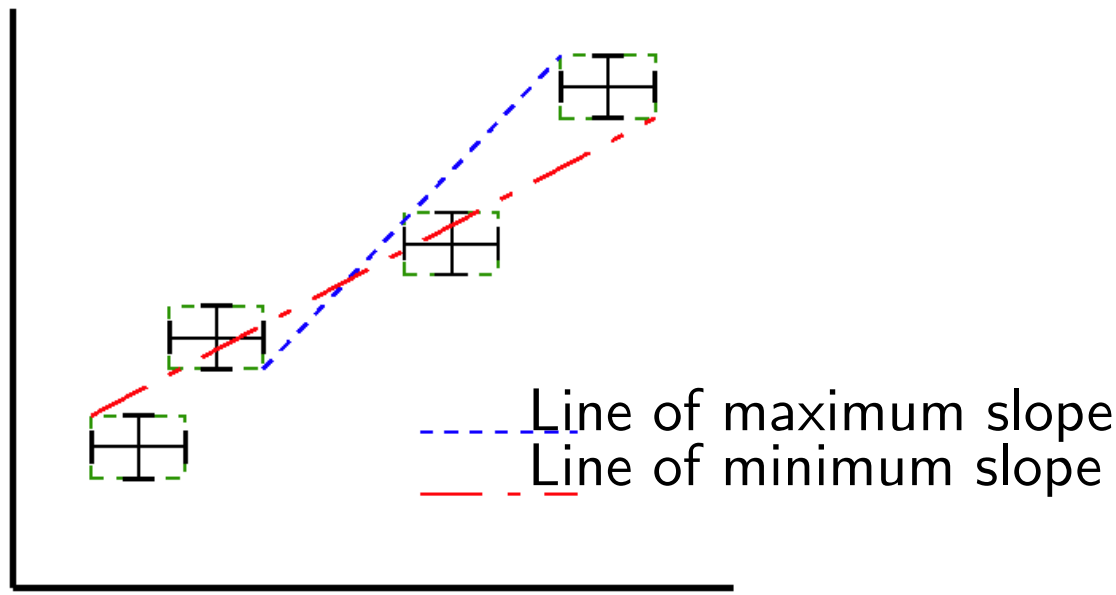
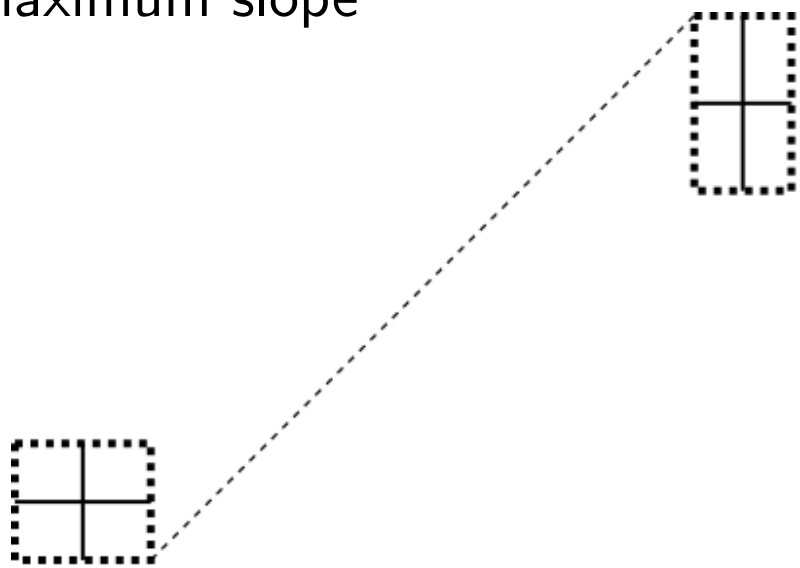


For positive slope

line of maximum slope



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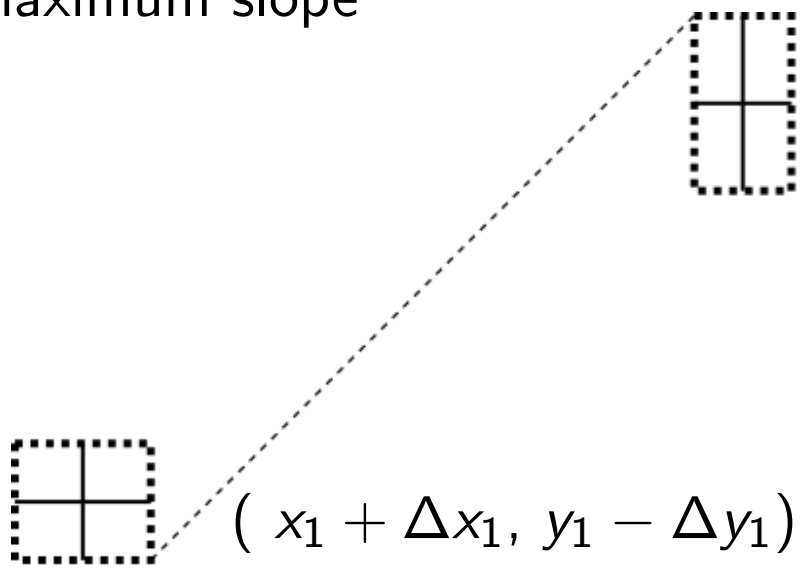

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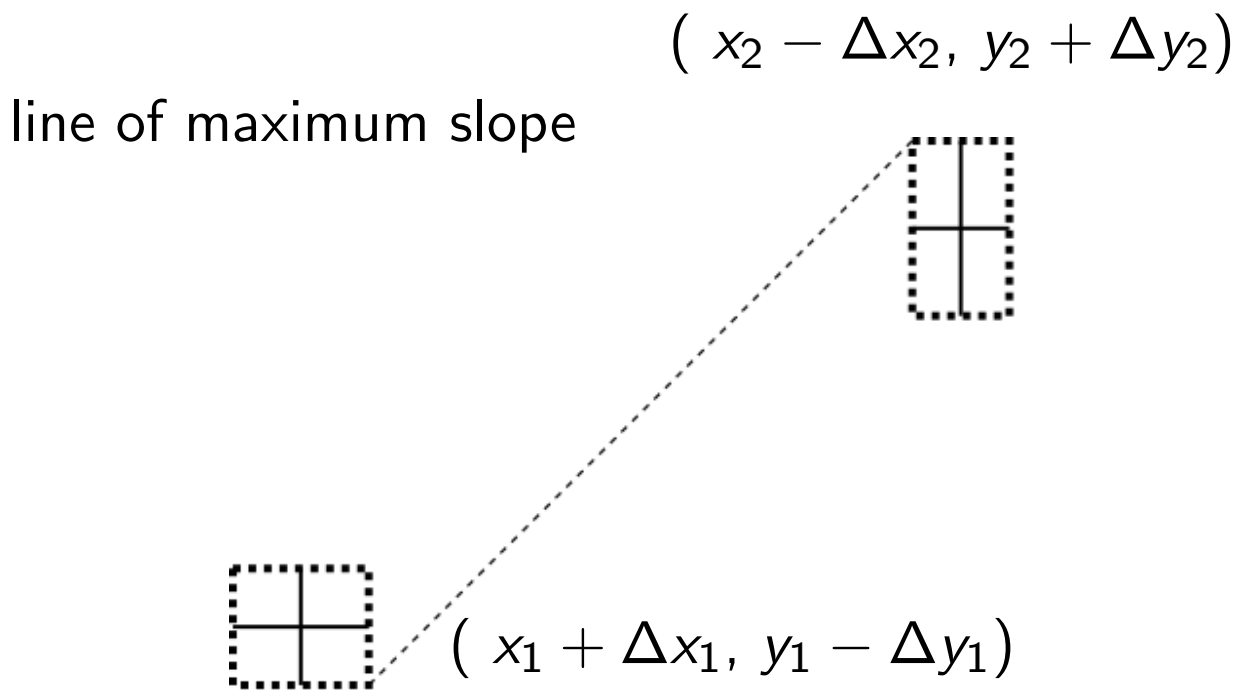

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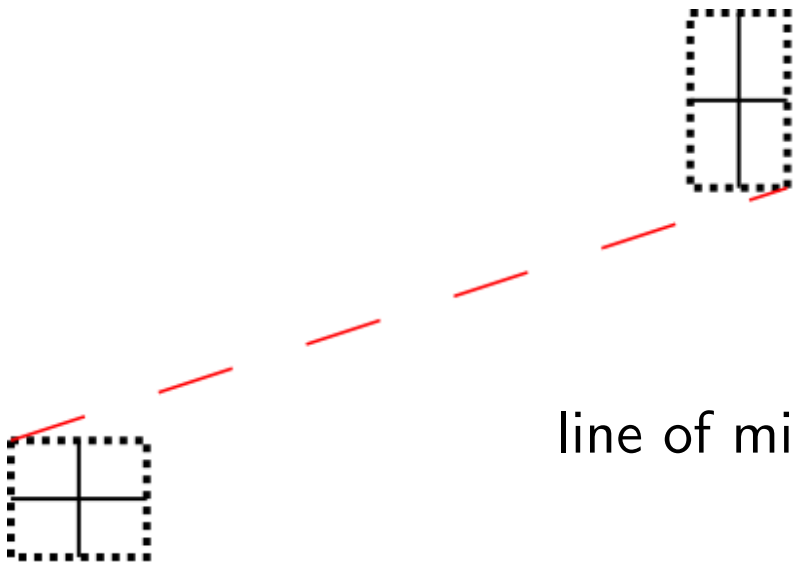
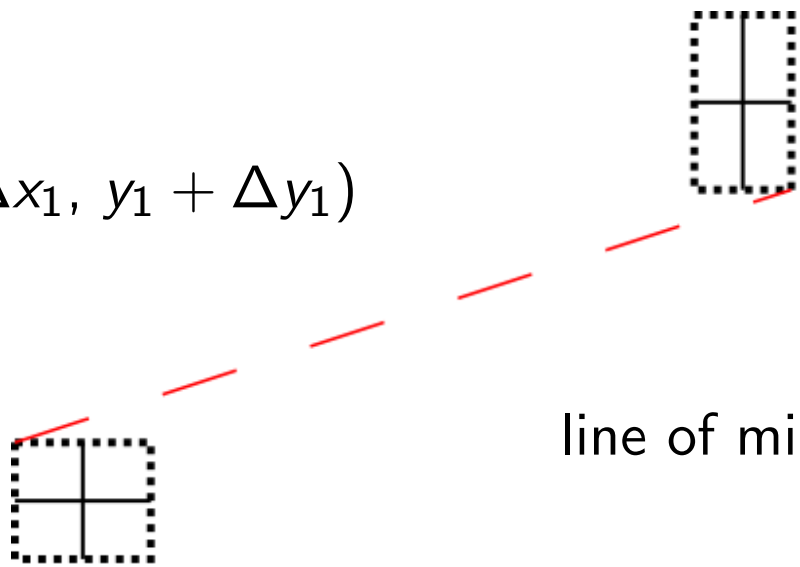


For positive slope

$(x_1 - \Delta x_1, y_1 + \Delta y_1)$



line of minimum slope

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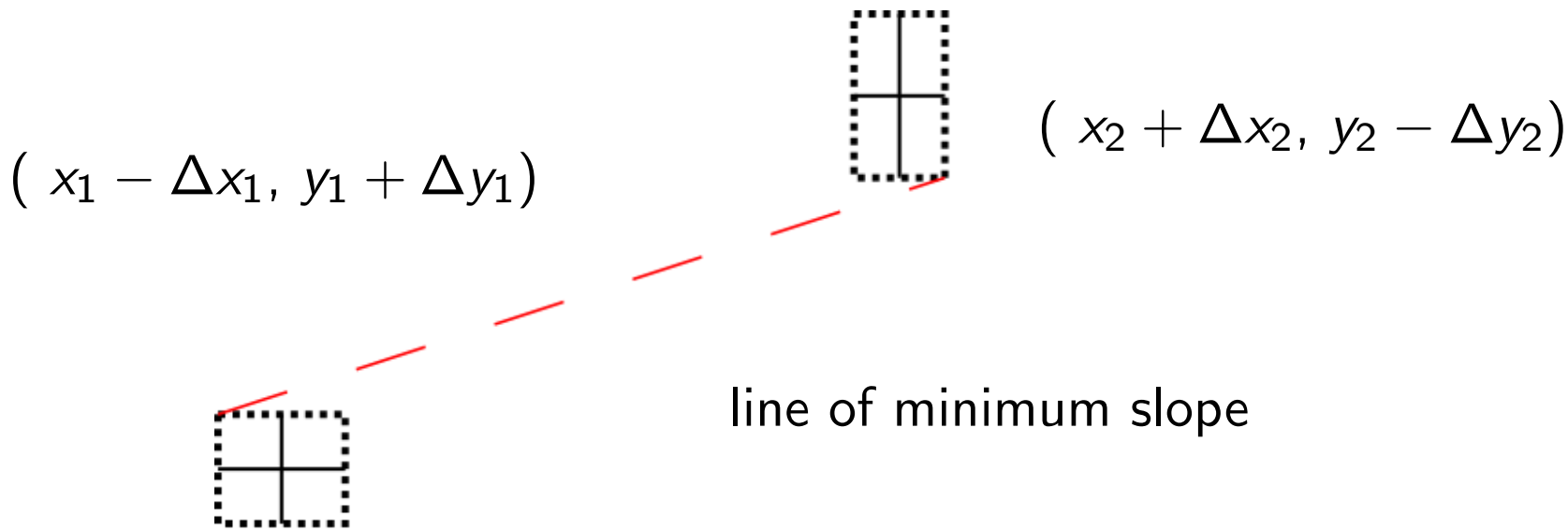
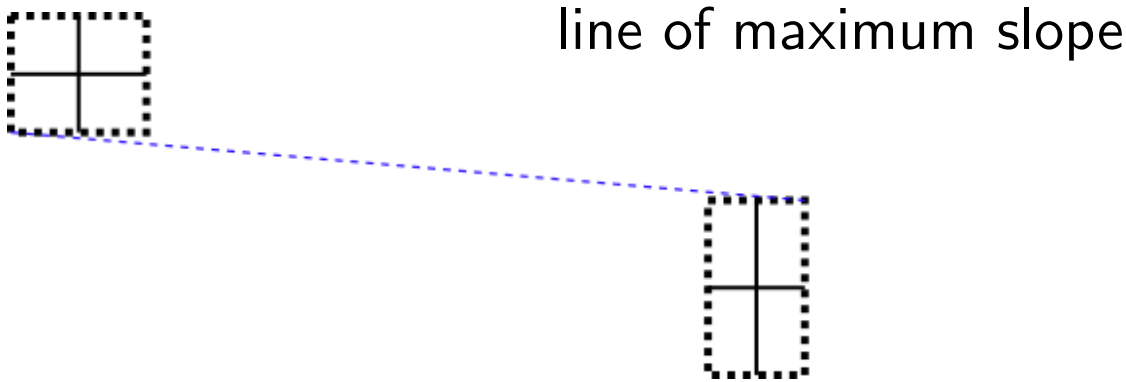


For negative slope



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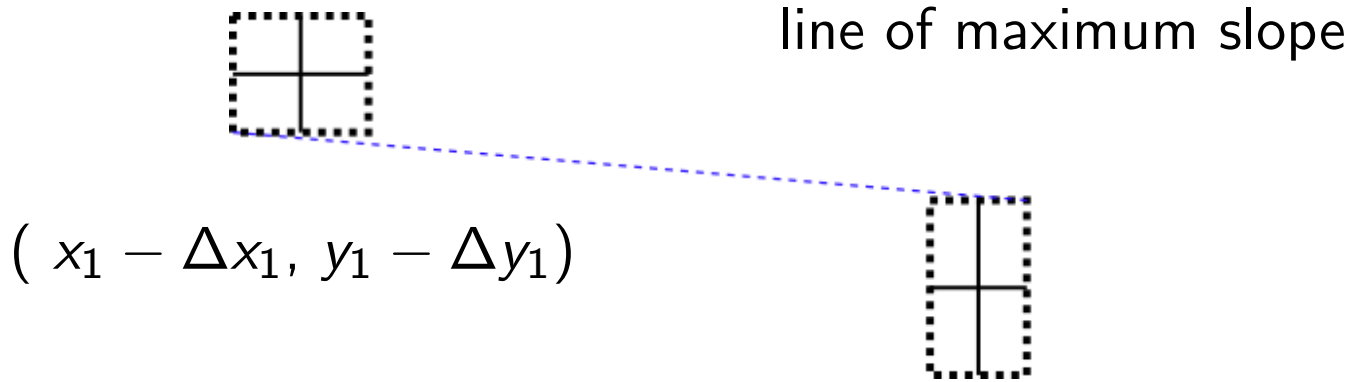

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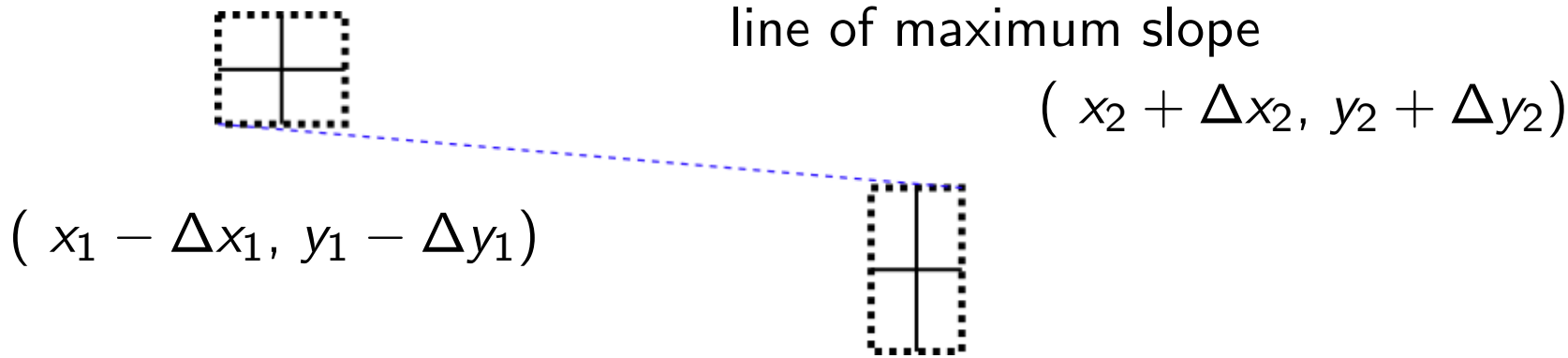

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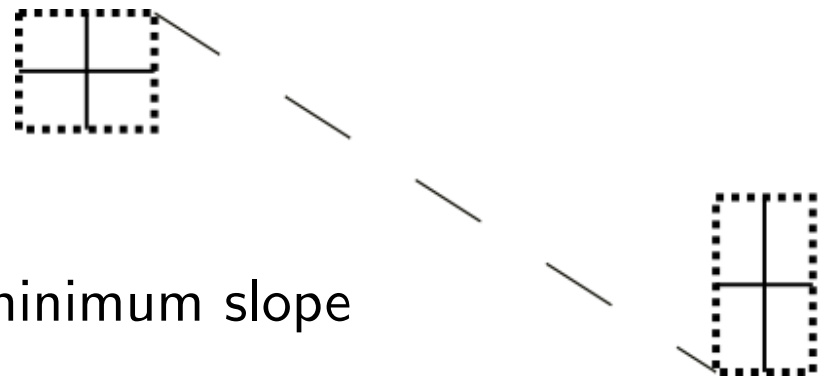

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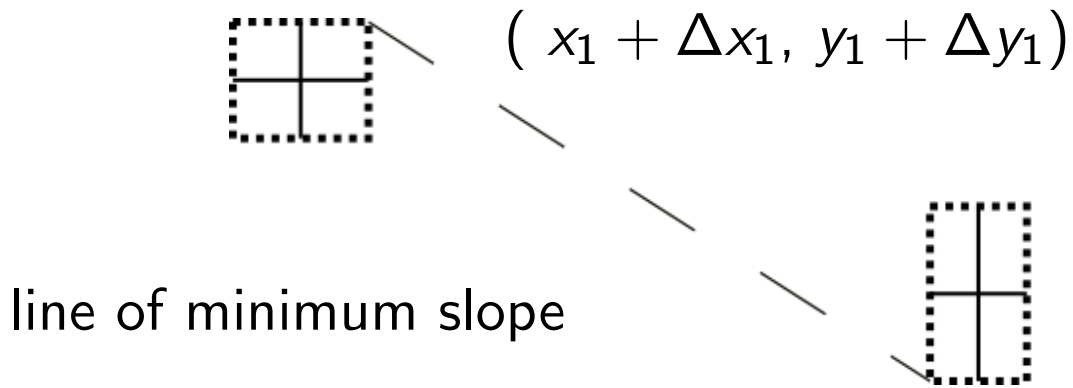

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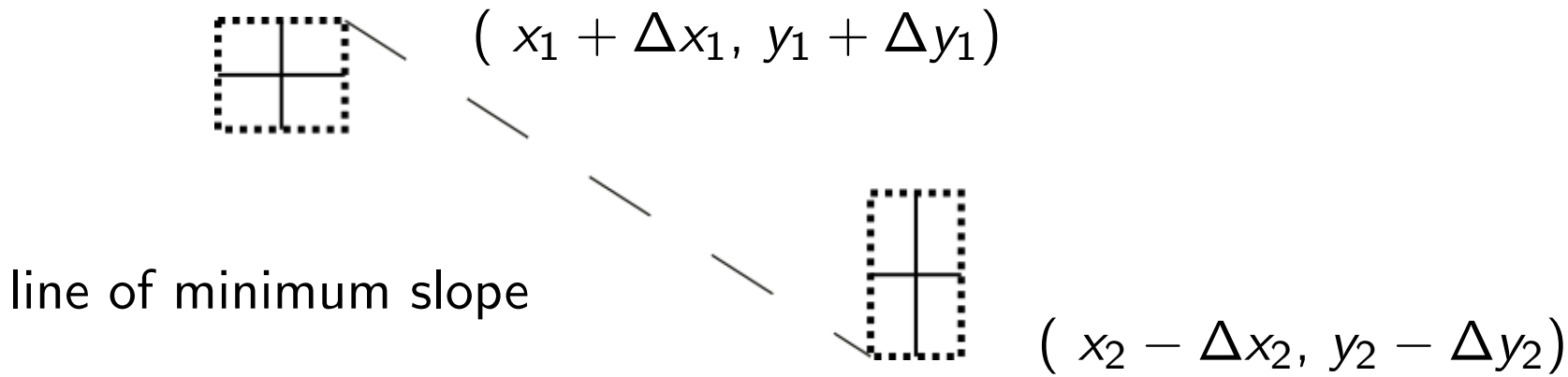

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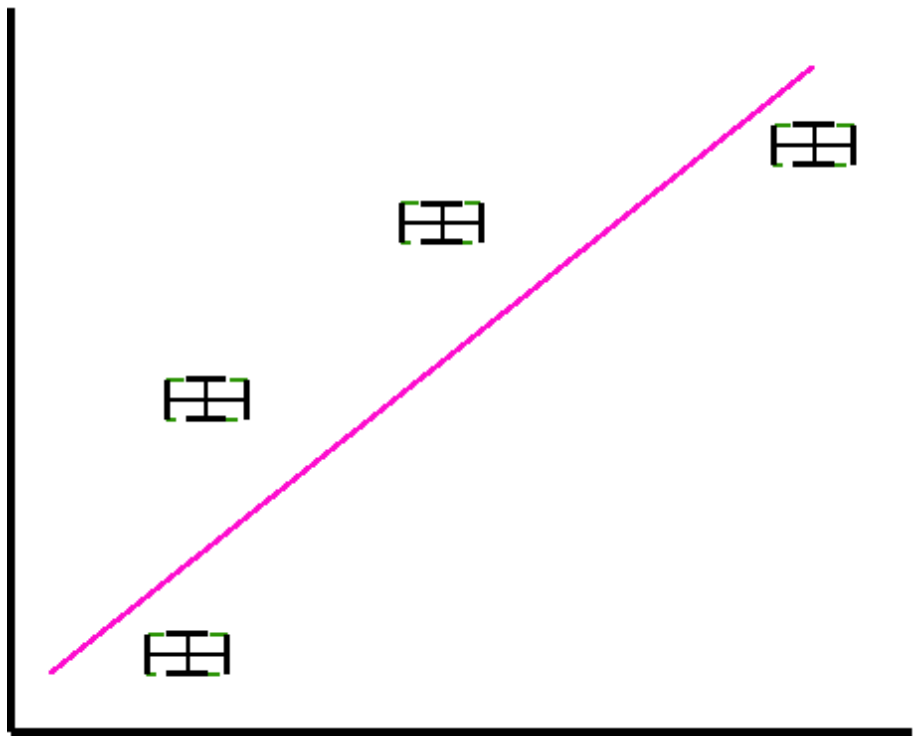

Large Scatter of Data; *no* line crosses all error bars

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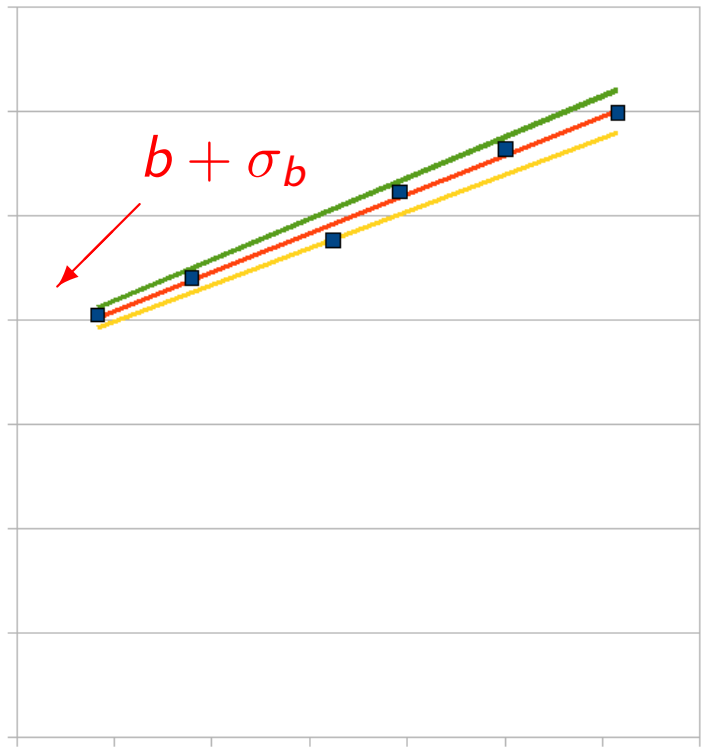
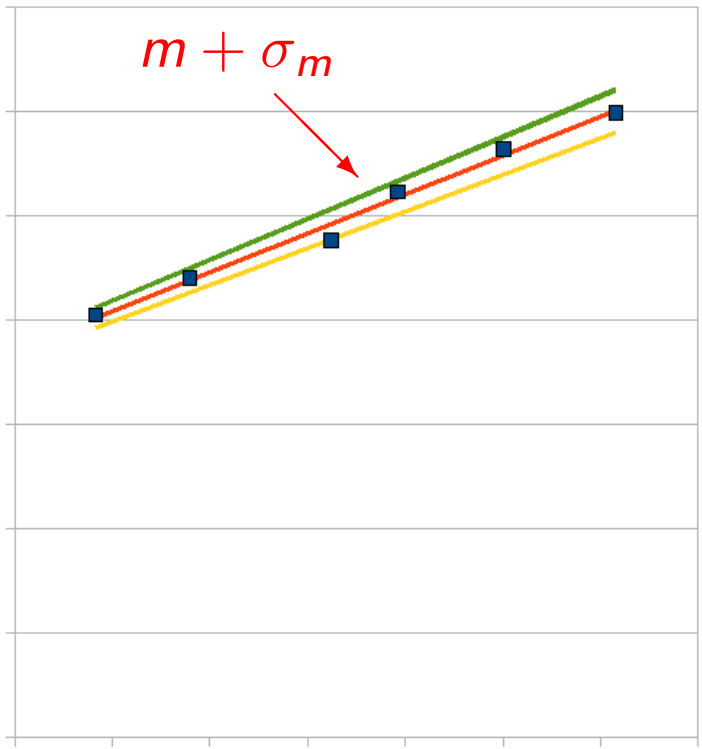
$$\sigma_m = \sigma \sqrt{\frac{N}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2}}$$

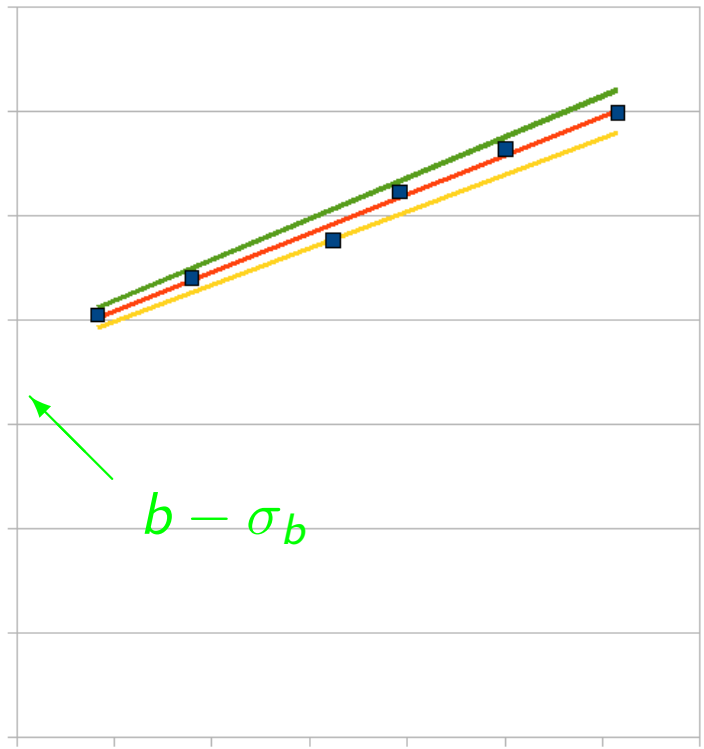
High and low values for the slope and y-intercept can be calculated using the standard errors as illustrated in the following figure.



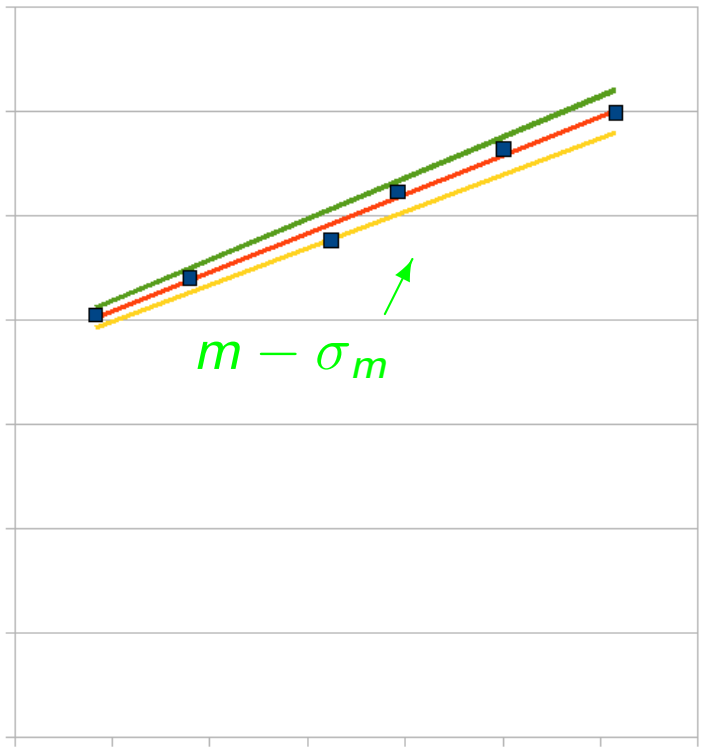
High value for y-intercept given by standard error in b



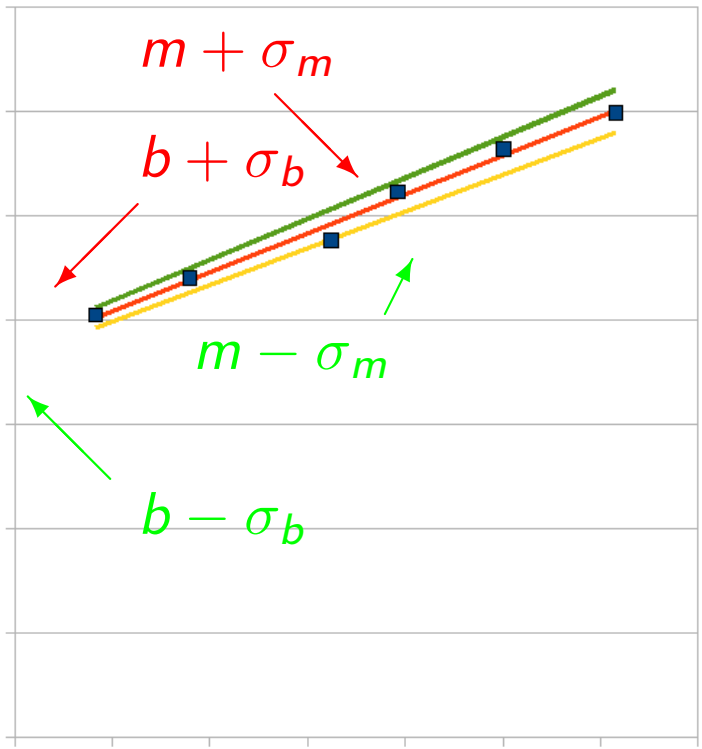
High value for slope given by standard error in m



Low value for y-intercept given by standard error in b



Low value for slope given by standard error in m



Possible fits given by standard error between the lines shown

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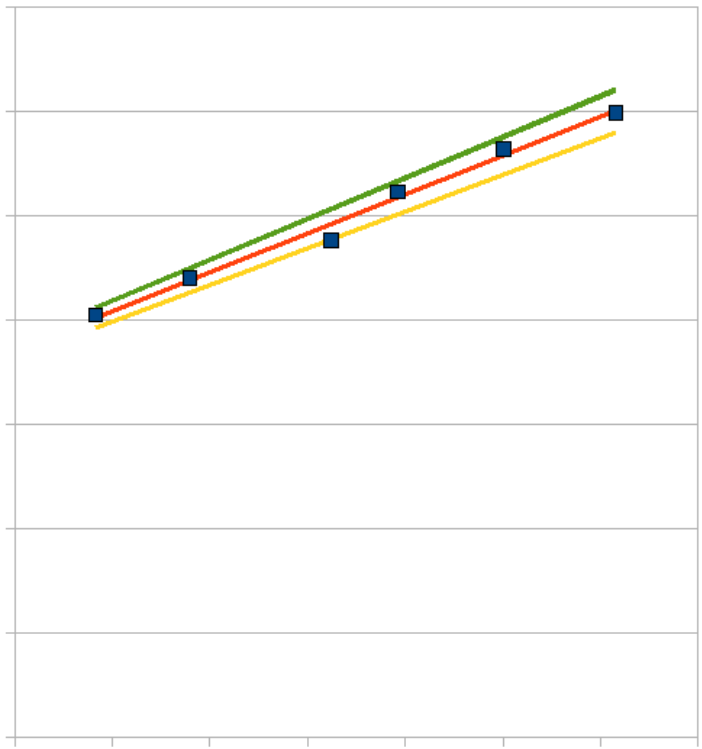
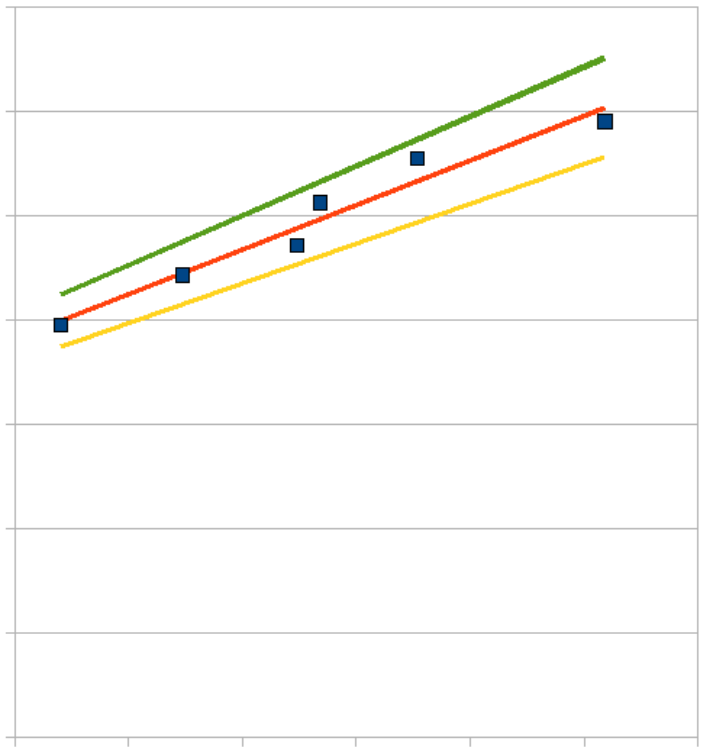
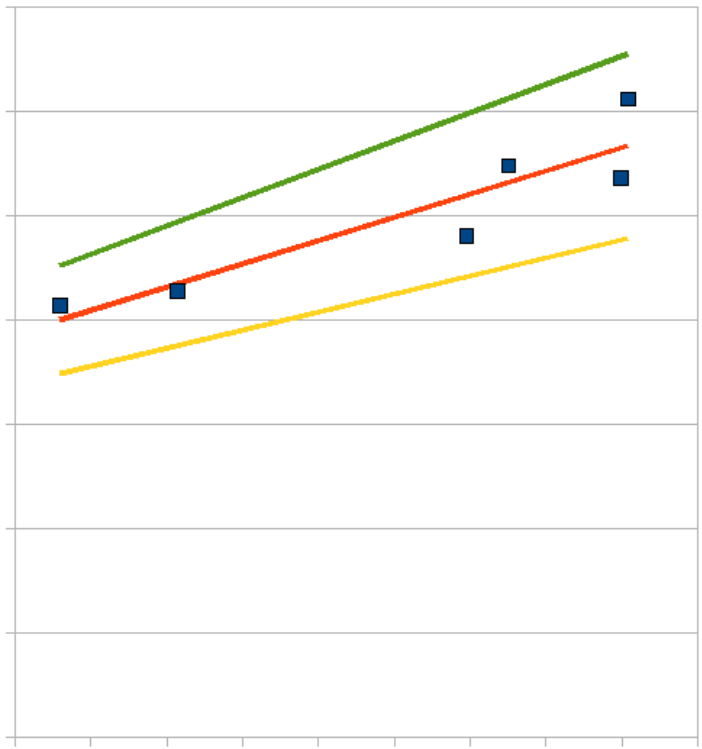


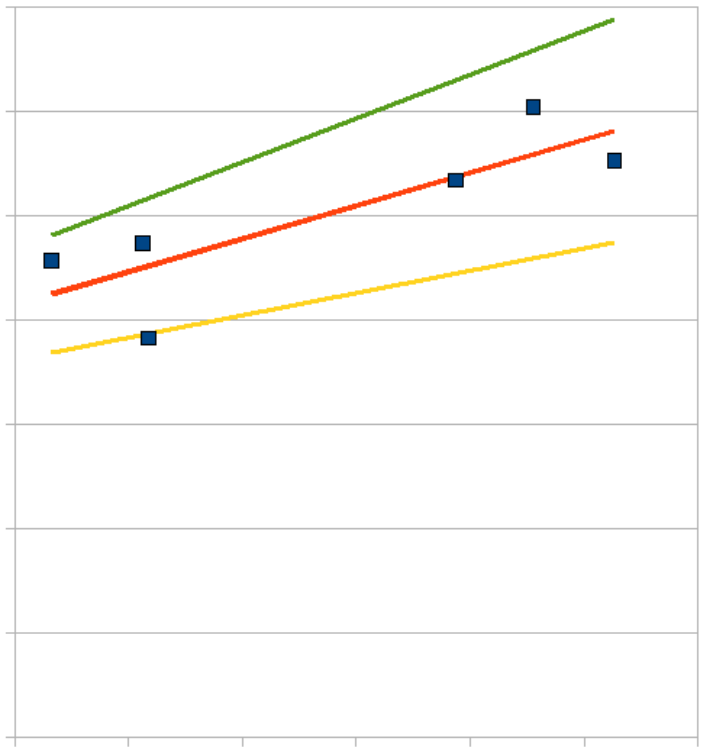
Small scatter of points



Moderate scatter of points



Fairly large scatter of points



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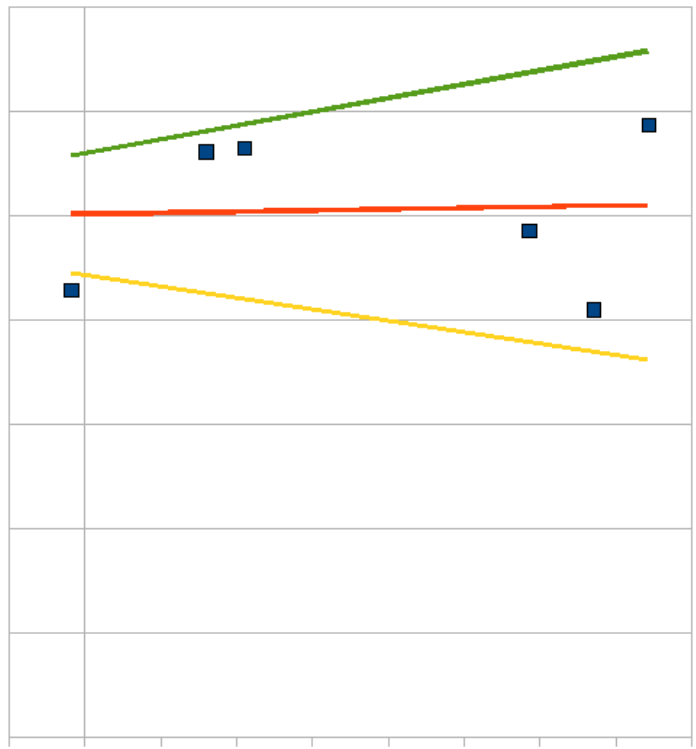

Extreme scatter of points

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Recap

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   If error bars are *big* compared to the scatter of the data points, use *maximum and minimum slopes* to determine the uncertainties in the slope and y-intercept;

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   If error bars are *big* compared to the scatter of the data points, use *maximum and minimum slopes* to determine the uncertainties in the slope and y-intercept;

   If error bars are *small* compared to the scatter of the data points, use *standard errors* to determine the uncertainties in the slope and y-intercept.