

Large and Small Scatter

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When does a line cross an error bar?

Introduction

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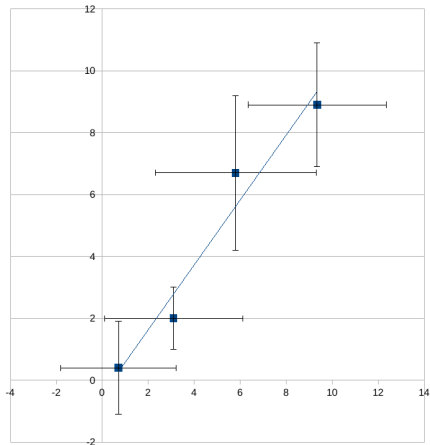
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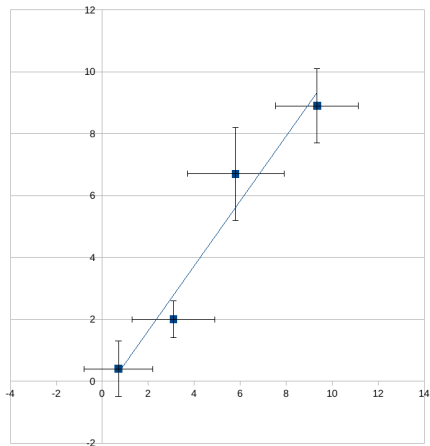
In this case, the *uncertainties* in the slope and y -intercept will be determined by the *error bars* on the graph.

When does a line cross an error bar?



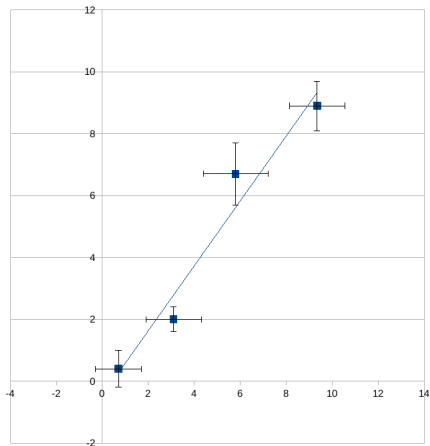
Least squares fit *clearly* crosses all error bars.

When does a line cross an error bar?



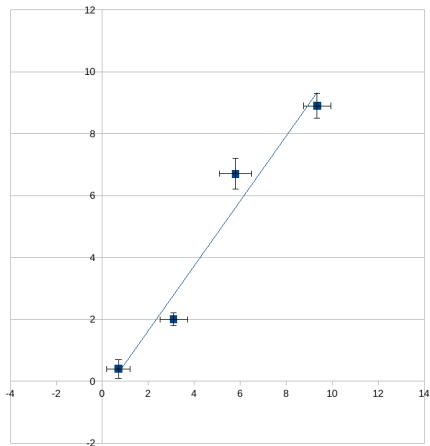
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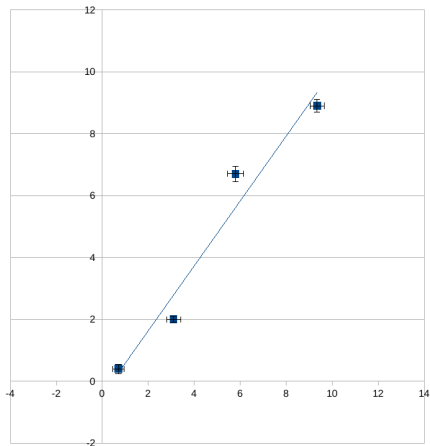
Least squares fit *clearly* crosses all error bars.

When does a line cross an error bar?



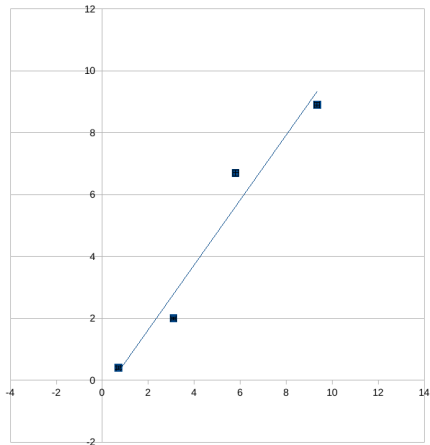
Least squares fit *maybe* crosses all error bars.

When does a line cross an error bar?



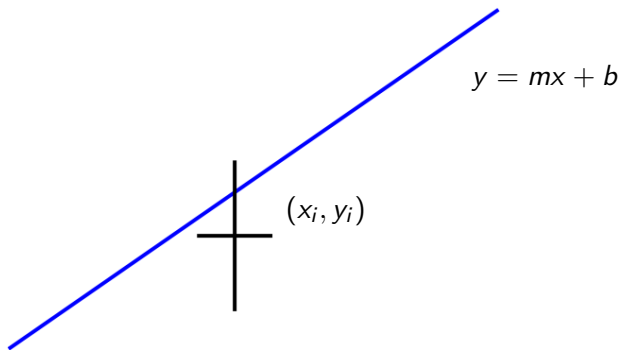
Least squares fit doesn't cross all error bars.

When does a line cross an error bar?

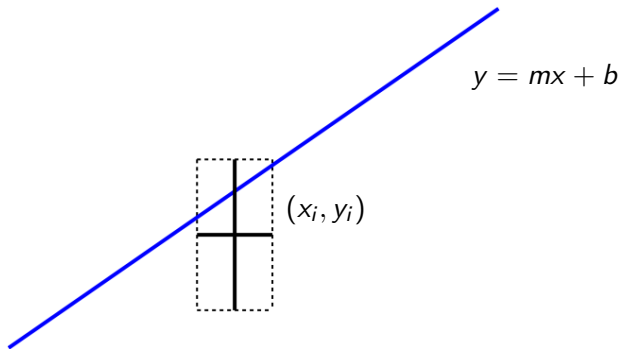


Least squares fit definitely doesn't cross all error bars.

When does a line cross an error bar?



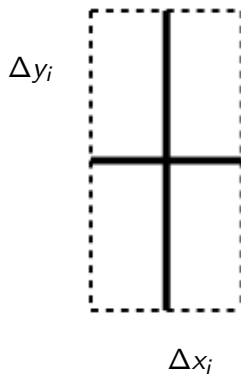
The line clearly crosses the vertical error bar.



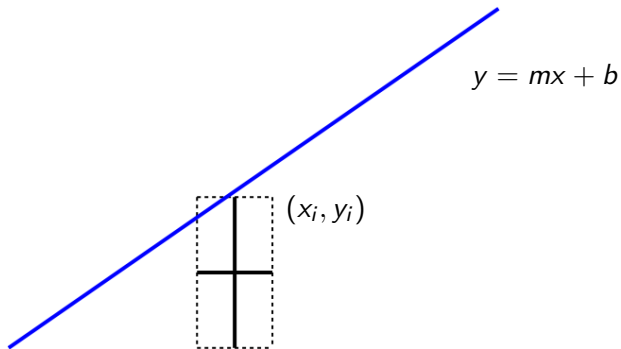
Actually, the error bars form a box.

When does a line cross an error bar?

Another case where this doesn't work



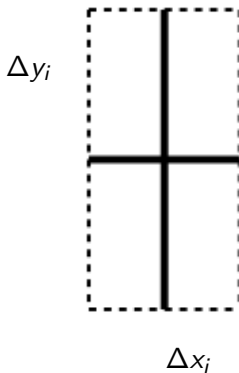
This is the box close up.



The line goes through the box, even though it doesn't cross either bar.

When does a line cross an error bar?

Another case where this doesn't work

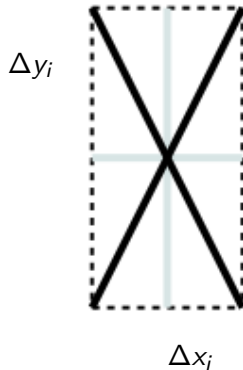


Here's the box again.

When does a line cross an error bar?

When this doesn't work

Another case where this doesn't work

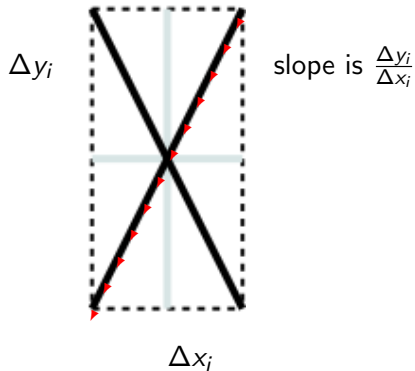


We can define it by *diagonal* lines.

When does a line cross an error bar?

When this doesn't work

Another case where this doesn't work

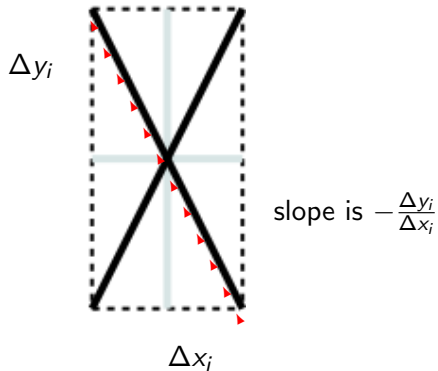


Here's one.

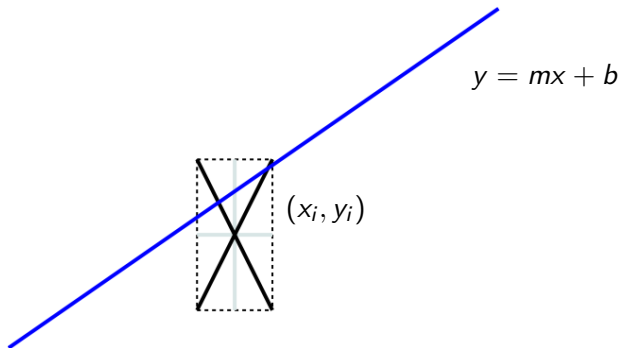
When does a line cross an error bar?

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Here's the other.



A line only needs to cross either diagonal.

The slope of the diagonal with the positive slope is

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since, by definition, (x_i, y_i) must be on the line.

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For two lines $y = m_1x + b_1$ and $y = m_2x + b_2$, they cross when there is a single point (x_c, y_c) that satisfies both equations.

When does a line cross an error bar?

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and so

$$x_c = \frac{b_2 - b_1}{m_1 - m_2} \quad (5)$$

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and

$$x_{ci-} = \frac{b - b_{-i}}{m_{-i} - m} \quad (7)$$

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When this doesn't work

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Another case where this doesn't work

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If the x co-ordinate has no uncertainty, (i.e. there are only error bars in the y direction), then this won't work either.

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If the x co-ordinate has no uncertainty, (i.e. there are only error bars in the y direction), then this won't work either. However, in that case, it is easy to apply a similar test to see if the line crosses all of the (vertical) error bars.

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$$y_i - \Delta y_i \leq mx_i + b \leq y_i + \Delta y_i$$

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or

$$|y_i - (mx_i + b)| \leq \Delta y_i$$