Max/Min Slopes Example
Wilfrid Laurier University

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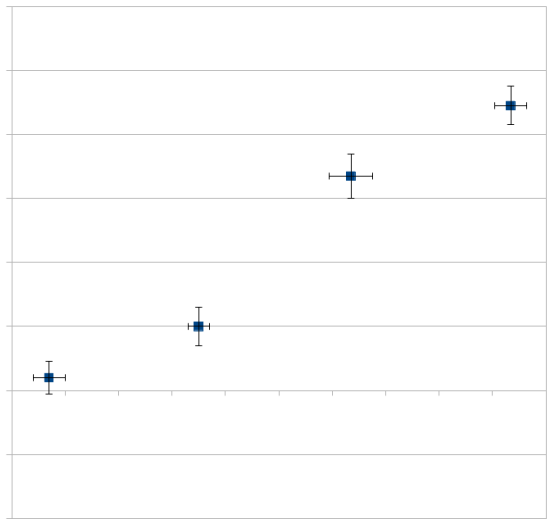
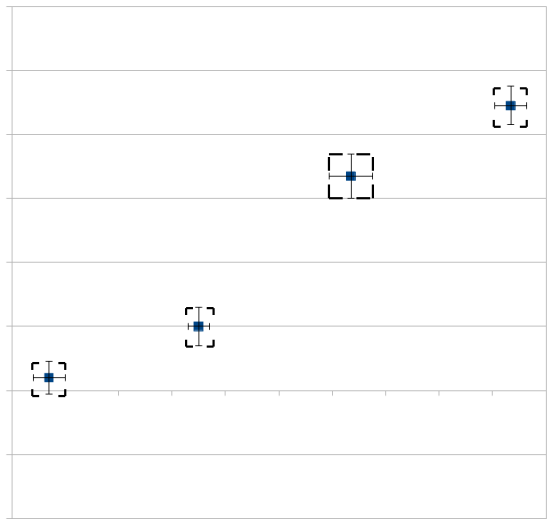
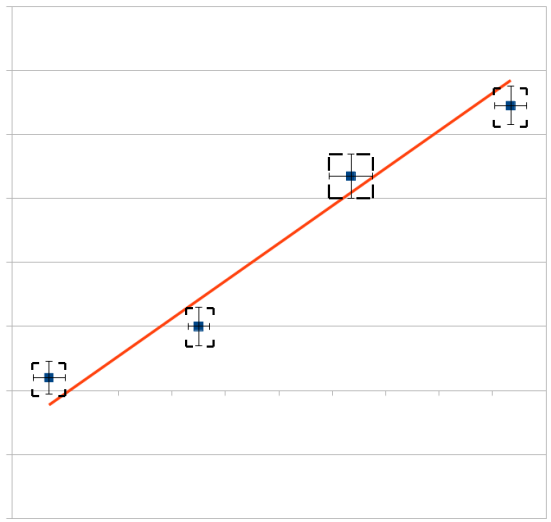
An $x - y$ graph with error bars is shown on the following page.

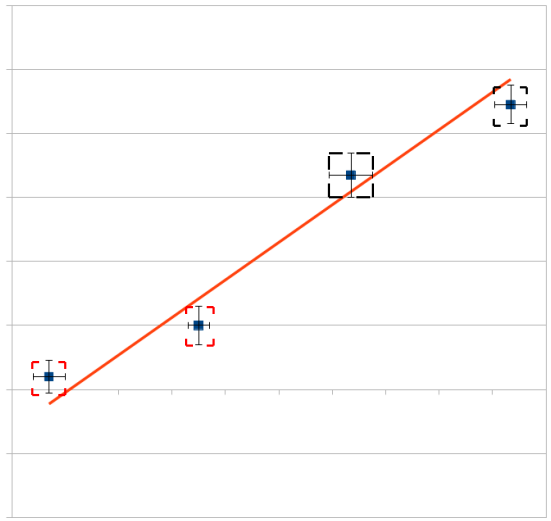




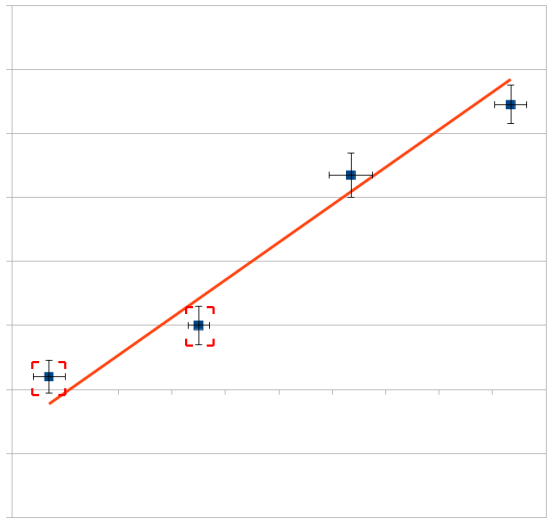
The error bars for a point mean the actual value could be anywhere inside the rectangle.



A line "crossing the error bars" actually means one touching each of the rectangles.



This line is made between opposite corners of the first two points and crosses the others.



$m = 1.17$

$b = -1.27$

The slope and y-intercept can be calculated in the normal way.

For a line to be valid as a line of maximum (or minimum) slope, it must cross *all* of the error bars.

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If it's a maximum or a minimum, then it must *just touch* the corners of two rectangles.

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If it's a maximum or a minimum, then it must *just touch* the corners of two rectangles.

That means the number of possible lines to check is $N(N-1)/2$, if N is the number of data points.

If the slope of a graph is *positive*:

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The line of maximum slope will be from the *lower right corner of the left point* to the *upper left corner of the right point*.

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The line of maximum slope will be from the *lower right corner of the left point* to the *upper left corner of the right point*.

The line of minimum slope will be from the *upper left corner of the left point* to the *lower right corner of the right point*.

If the slope of a graph is *positive*:

The line of maximum slope will be from the *lower right corner of the left point* to the *upper left corner of the right point*.

The line of minimum slope will be from the *upper left corner of the left point* to the *lower right corner of the right point*.

If the slope of a graph is *negative*:

If the slope of a graph is *positive*:

The line of maximum slope will be from the *lower right corner of the left point* to the *upper left corner of the right point*.

The line of minimum slope will be from the *upper left corner of the left point* to the *lower right corner of the right point*.

If the slope of a graph is *negative*:

The line of maximum slope will be from the *lower left corner of the left point* to the *upper right corner of the right point*.

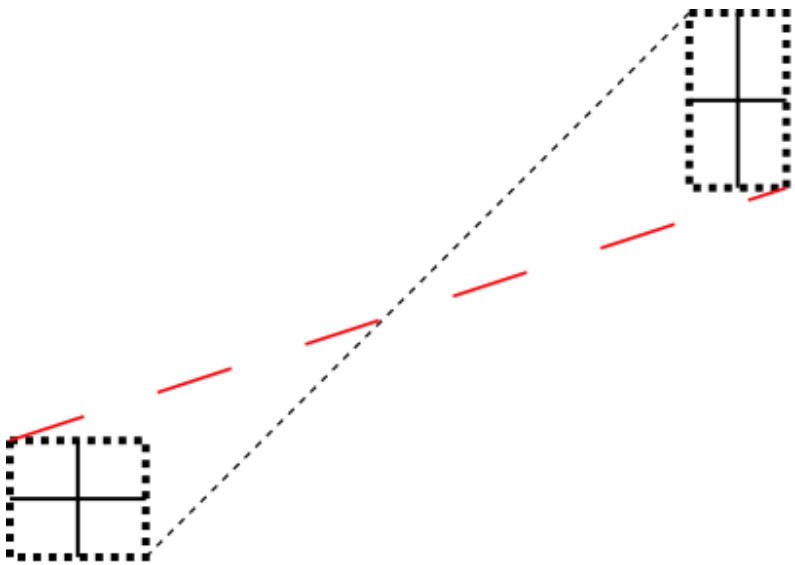
If the slope of a graph is *positive*:

- The line of maximum slope will be from the *lower right corner of the left point* to the *upper left corner of the right point*.
- The line of minimum slope will be from the *upper left corner of the left point* to the *lower right corner of the right point*.

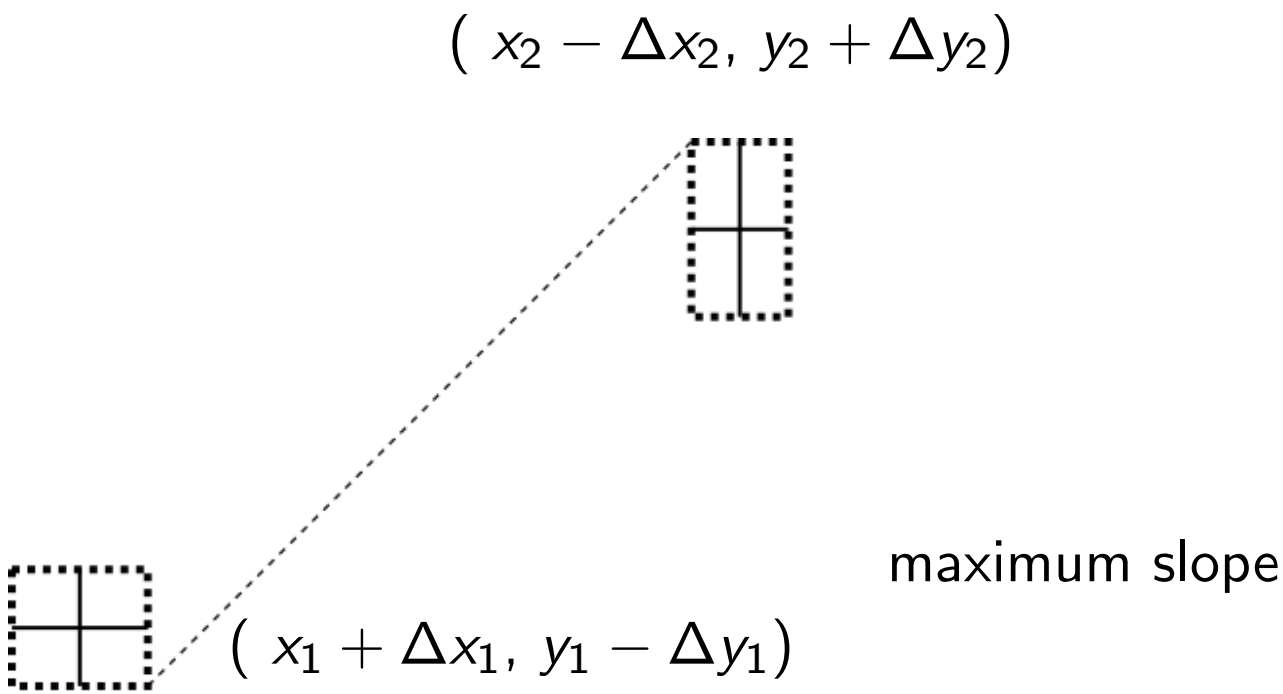
If the slope of a graph is *negative*:

- The line of maximum slope will be from the *lower left corner of the left point* to the *upper right corner of the right point*.
- The line of minimum slope will be from the *upper right corner of the left point* to the *lower left corner of the right point*.

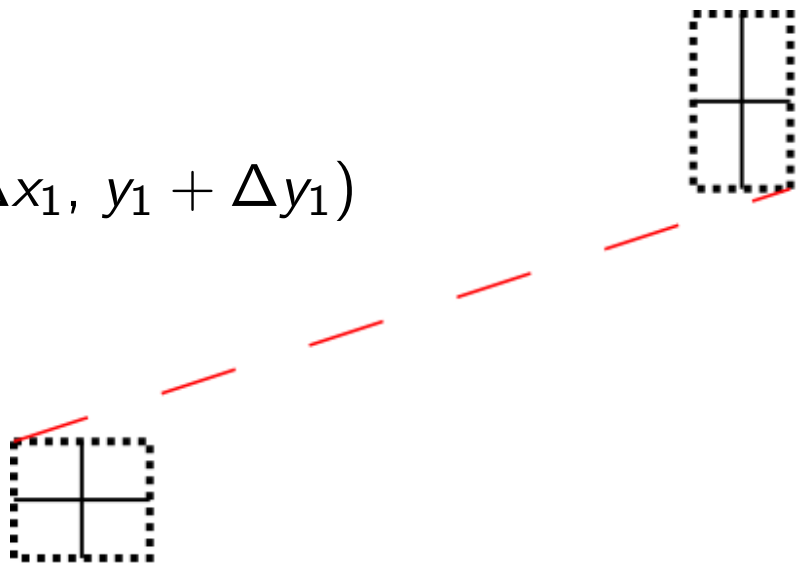
Maximum and minimum slope coordinates for positive slope:



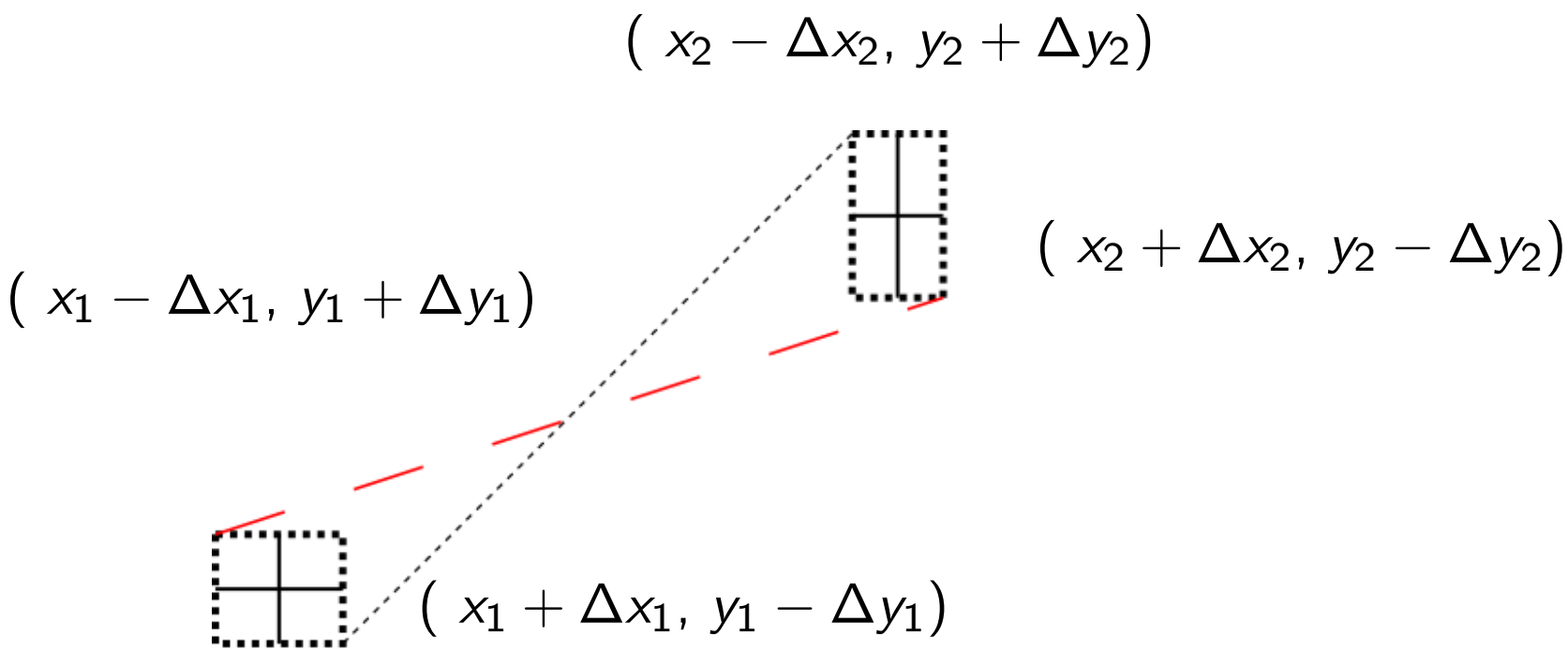
Maximum and minimum slope coordinates for positive slope:



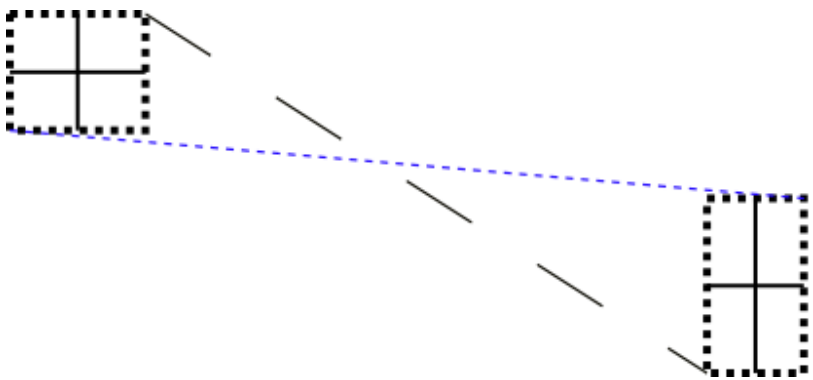
Maximum and minimum slope coordinates for positive slope:



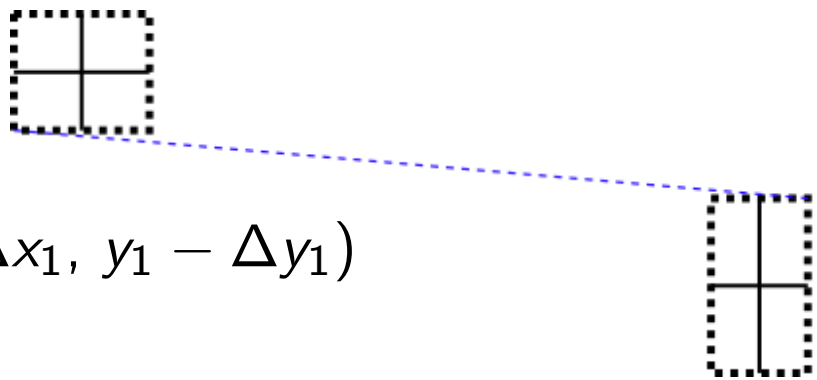
Maximum and minimum slope coordinates for positive slope:



Maximum and minimum slope coordinates for negative slope:



Maximum and minimum slope coordinates for negative slope:

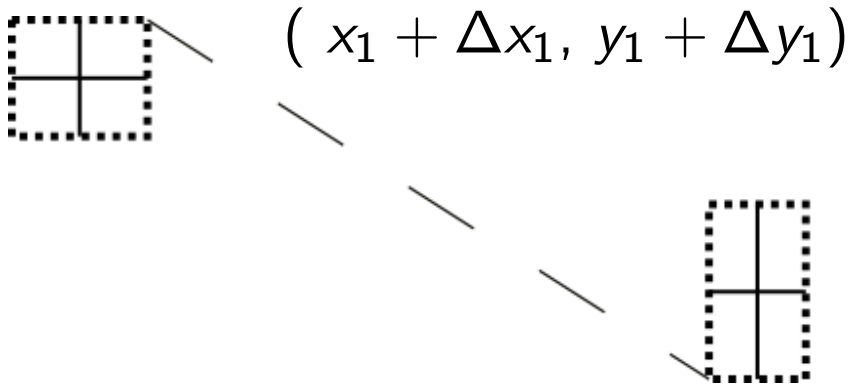


$(x_2 + \Delta x_2, y_2 + \Delta y_2)$

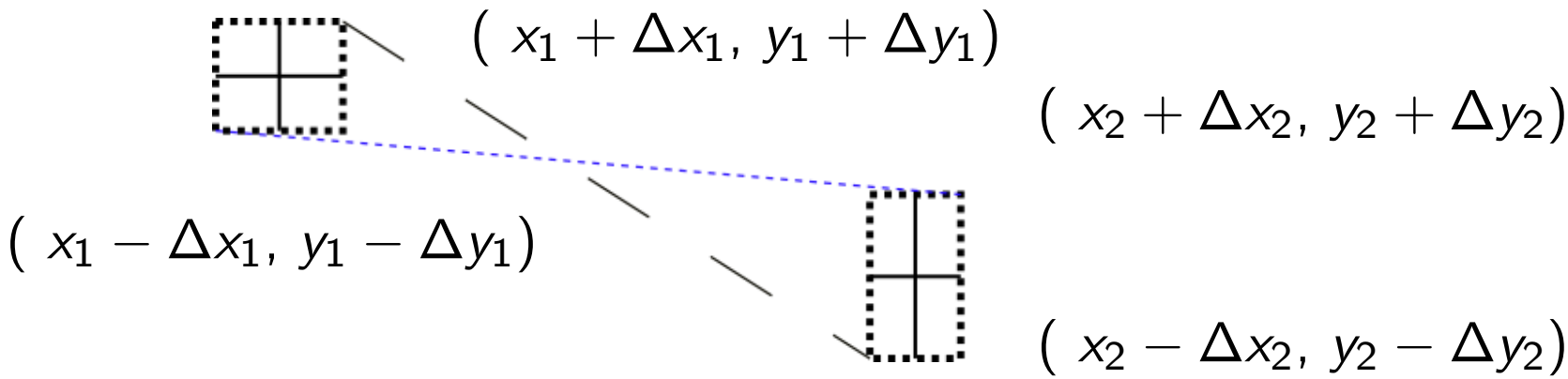
$(x_1 - \Delta x_1, y_1 - \Delta y_1)$

maximum slope

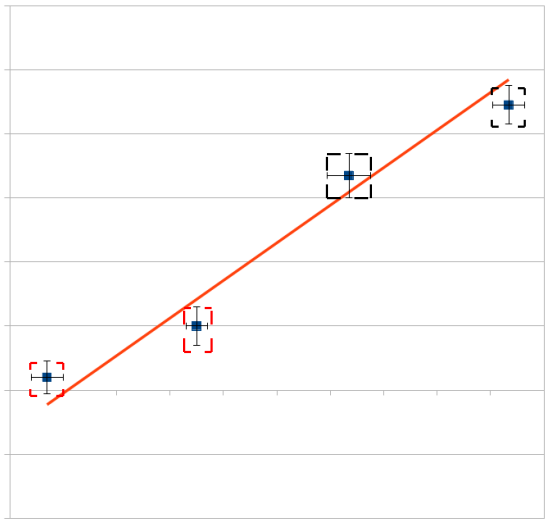
Maximum and minimum slope coordinates for negative slope:



Maximum and minimum slope coordinates for negative slope:



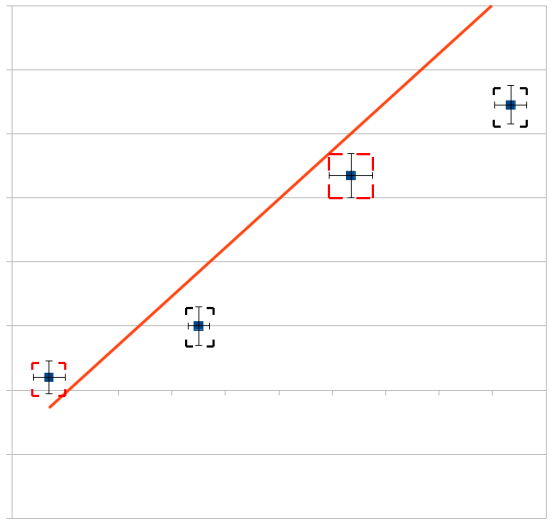
Here are the candidates for the line of maximum slope for four points with a positive slope.



$m = 1.52$

$b = -1.62$

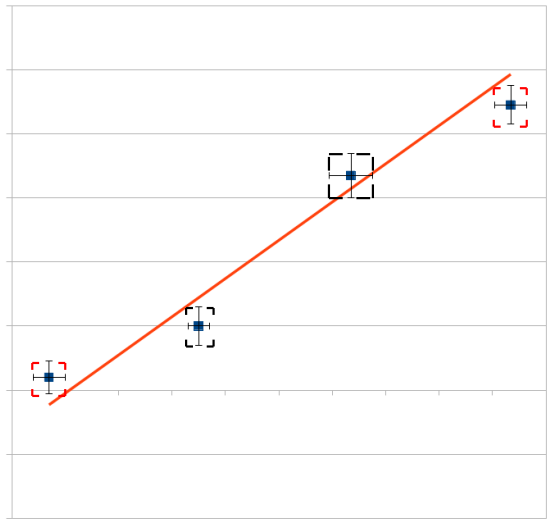
The line connecting the corners for points 1 and 2 looks like this.



$m = 1.52$

$b = -1.62$

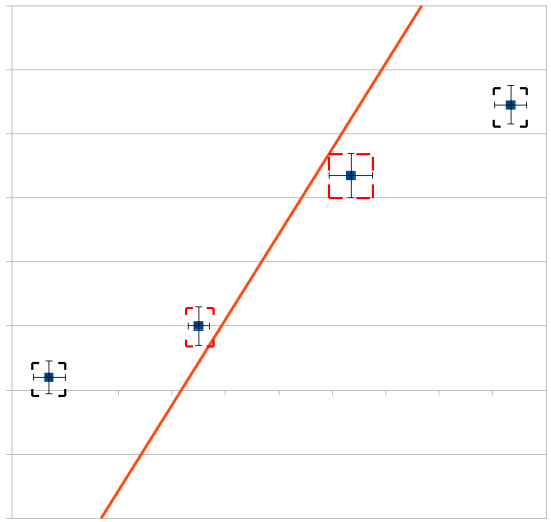
Here it is for points 1 and 3.



$m = 1.19$

$b = -1.19$

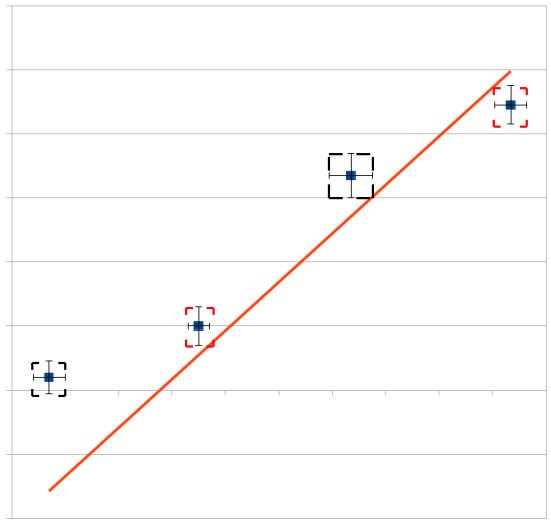
Here it is for points 1 and 4.



$m = 2.67$

$b = -8.47$

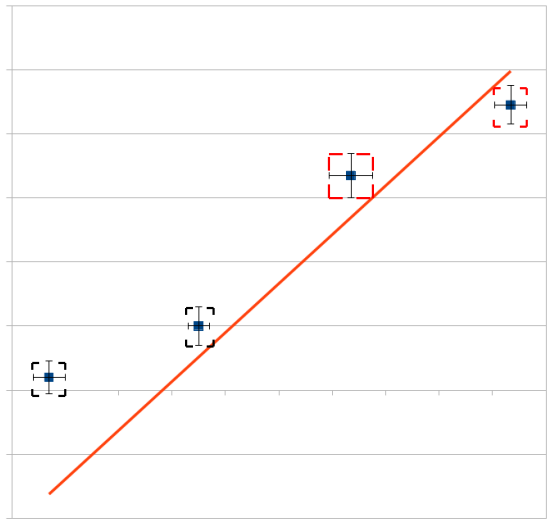
This is for points 2 and 3.



$m = 1.52$

$b = -4.21$

This is for points 2 and 4.

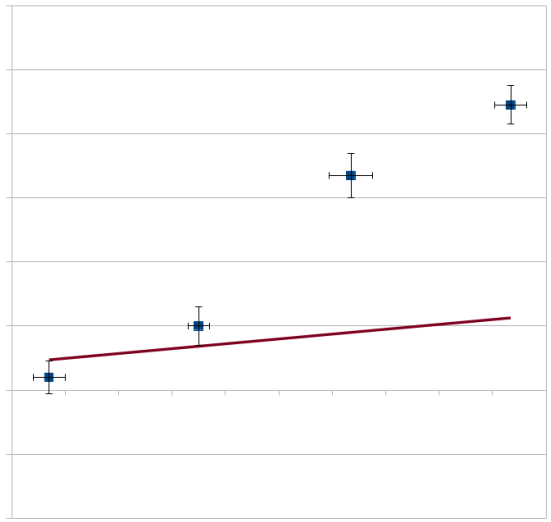


$m = 1.53$

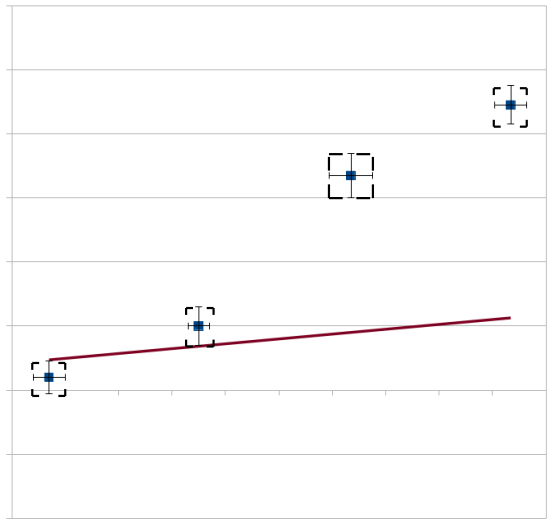
$b = -4.32$

Points 3 and 4 finishes all of the possibilities.

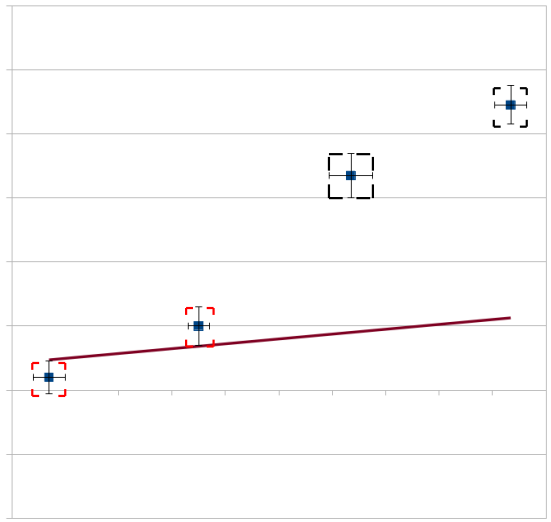
For the same graph, here are the choices for the line of minimum slope.



For a line of *minimum* slope, it also has to cross all of the error bars.



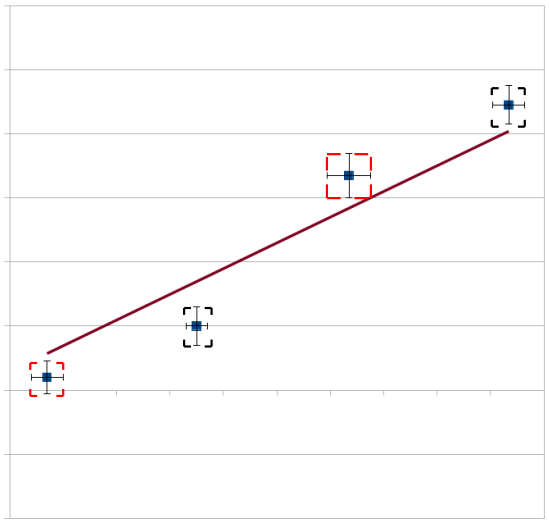
As before, that really means touching each of the rectangles.



$m = 0.15$

$b = 0.84$

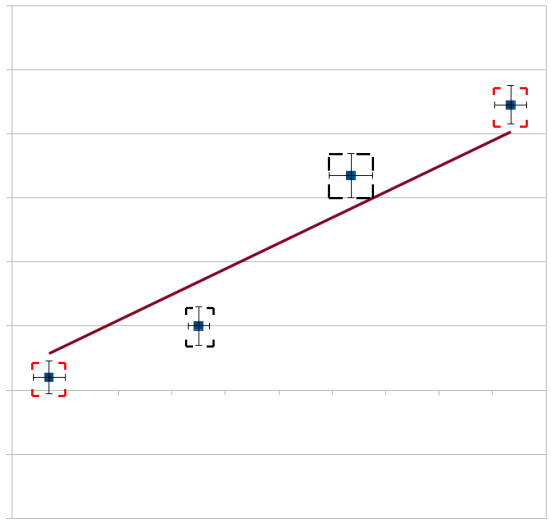
Note the corners used, as shown here for points 1 and 2.



$m = 0.80$

$b = 0.58$

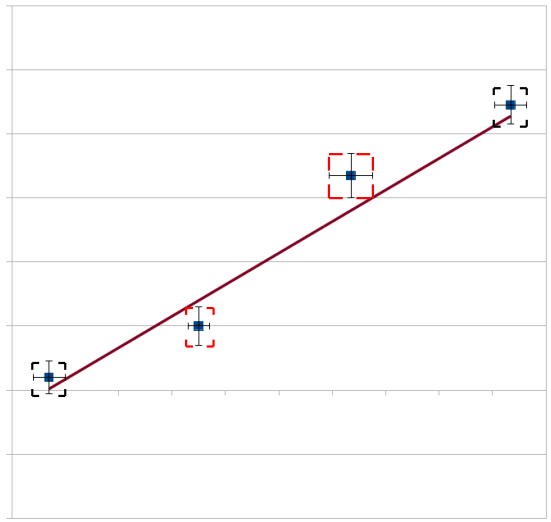
Here it is for points 1 and 3.



$m = 0.80$

$b = 0.58$

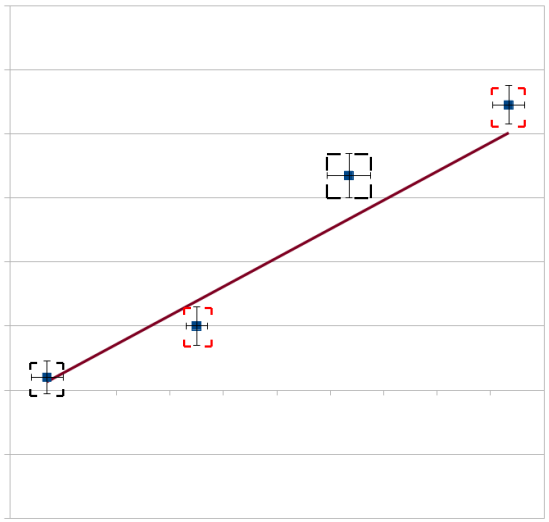
Here it is for points 1 and 4.



$m = 0.99$

$b = -0.65$

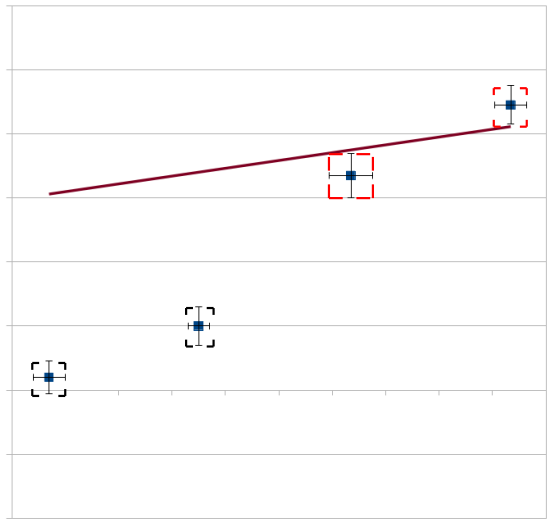
This is for points 2 and 3.



$m = 0.90$

$b = -0.37$

This is for points 2 and 4.



$m = 0.24$

$b = 5.95$

Here's the final choice of points 3 and 4.

Let's look at the candidates for line of maximum slope again, and eliminate any that don't cross all of the rectangles.

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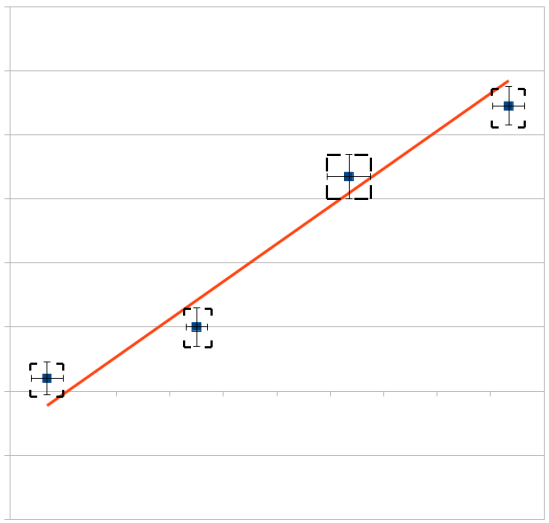
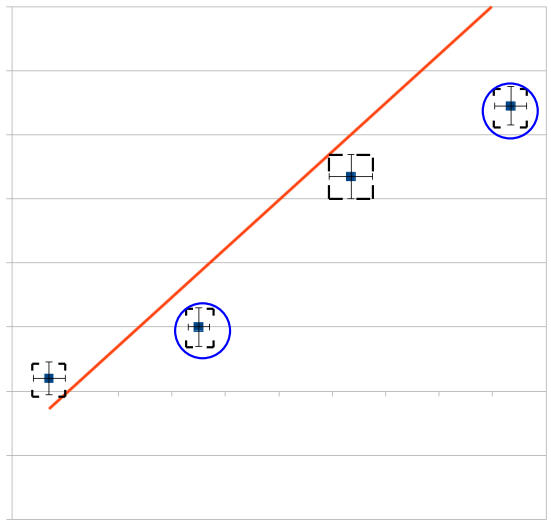
In the following figures, rectangles that are missed are circled.



$m = 1.17$

$b = -1.27$

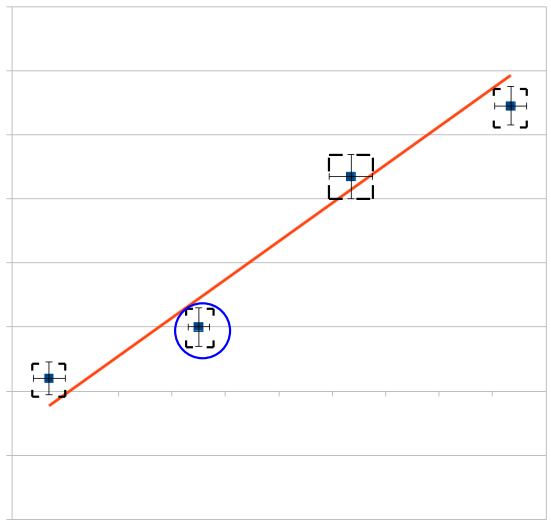
This one doesn't miss any rectangles, so it's a possibility. (Points used are 1 and 2.)



$m = 1.52$

$b = -1.62$

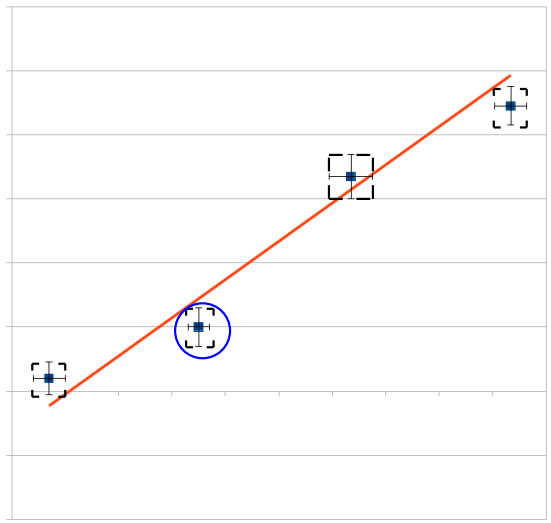
This one misses point 2 and point 4.



$m = 1.19$

$b = -1.19$

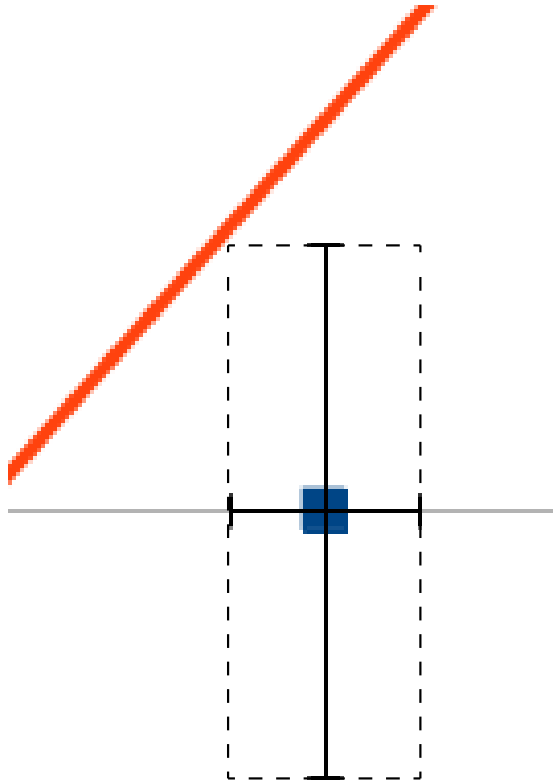
This one misses point 2.



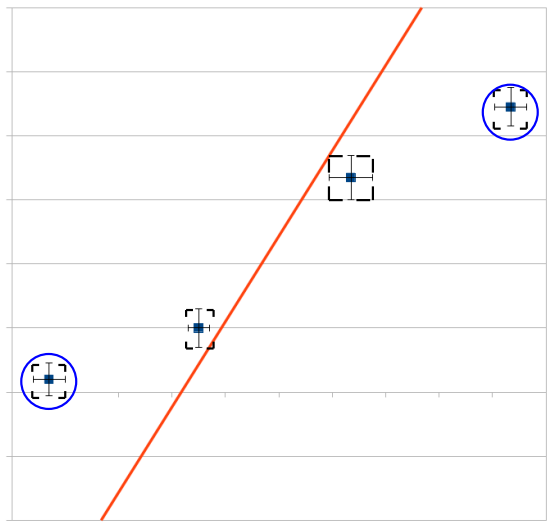
$m = 1.19$

$b = -1.19$

This one misses point 2. On this scale it's hard to see.



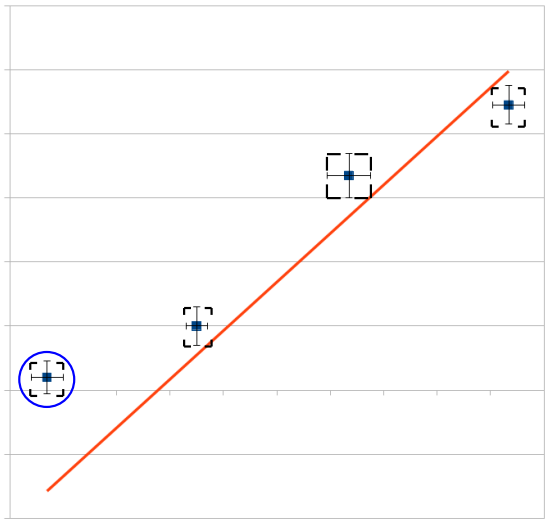
It's *really* close; it would not be very wrong to include this line as a possibility.



$m = 2.67$

$b = -8.47$

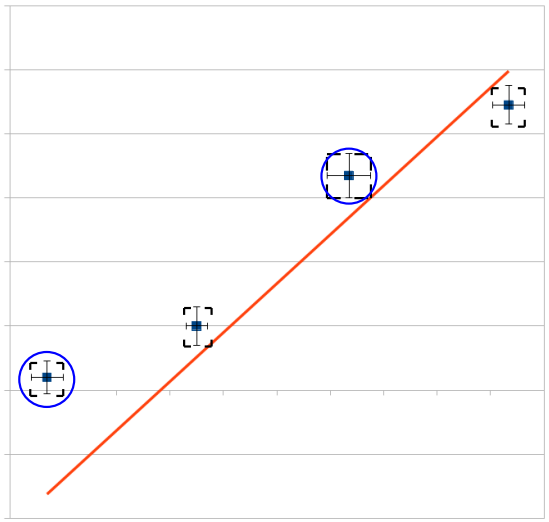
This one misses points 1 and 4.



$m = 1.52$

$b = -4.21$

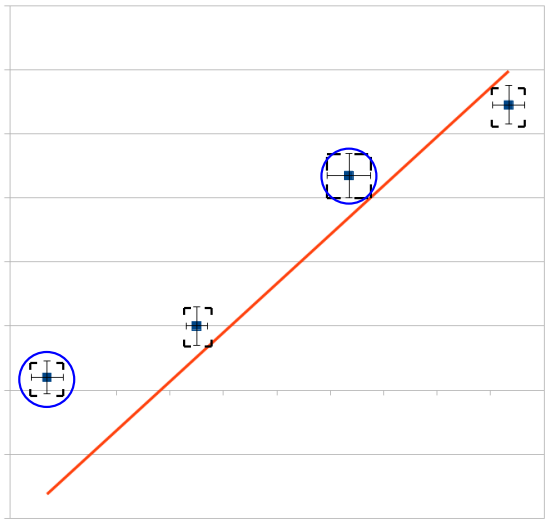
This one misses point 1.



$m = 1.53$

$b = -4.32$

This one misses points 1 and 3.



$m = 1.53$

$b = -4.32$

This one misses points 1 and 3. It just misses 3 by a tiny amount.

The only line that didn't miss any points was the one connecting points 1 and 2, so that's our line of maximum slope.

Now we'll look at the candidates for line of *minimum* slope again, and do the same thing.

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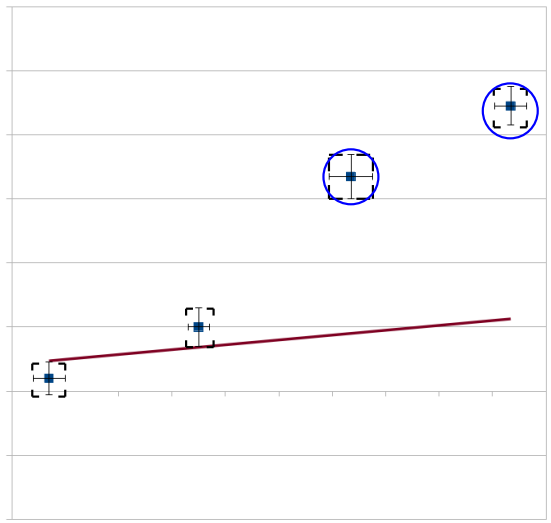
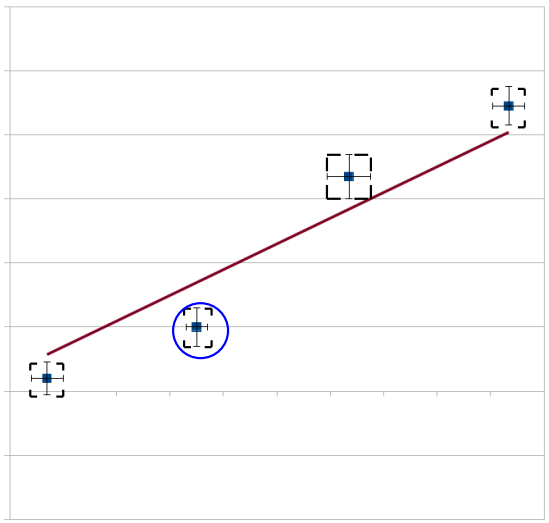
As before, rectangles that are missed are circled.



$m = 0.15$

$b = 0.84$

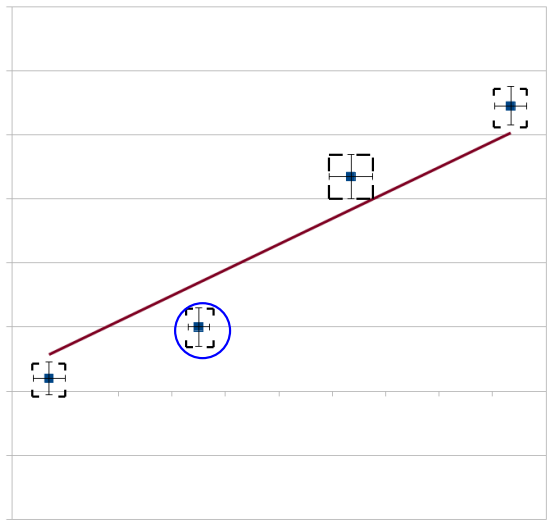
Points 3 and 4 were missed.



$m = 0.80$

$b = 0.58$

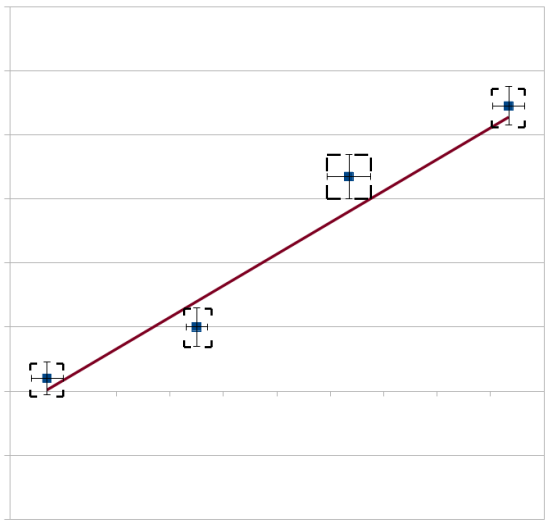
Point 2 was missed.



$m = 0.80$

$b = 0.58$

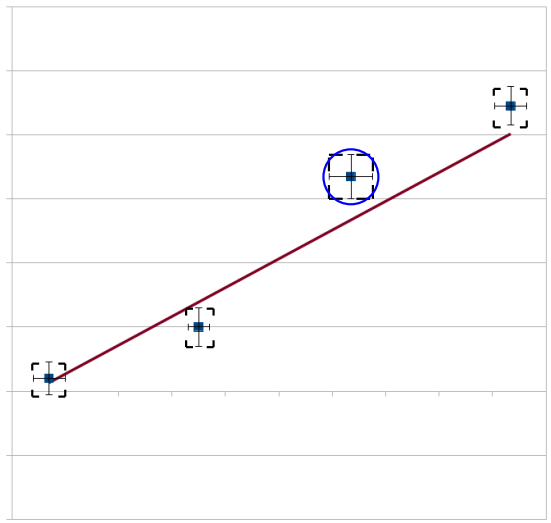
This also misses point 2. (It's *very* similar to the one just before.)



$m = 0.99$

$b = -0.65$

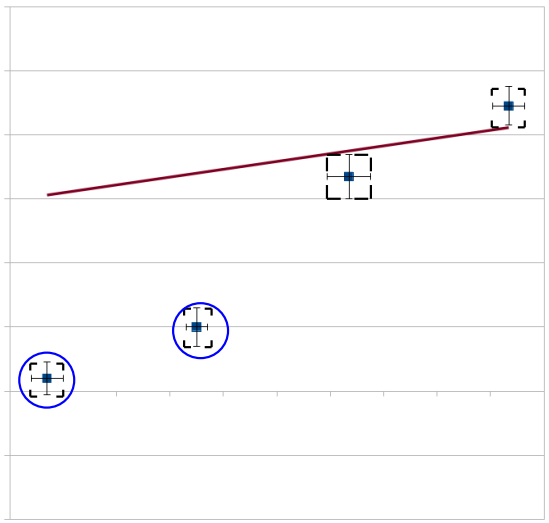
No points were missed, so this one is possible.



$m = 0.90$

$b = -0.37$

Point 3 was missed.

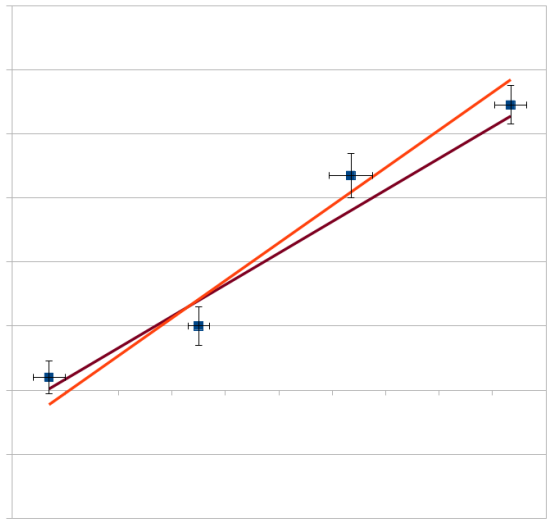


$m = 0.24$

$b = 5.95$

Points 1 and 2 were missed.

The only line that didn't miss any points was the one connecting points 2 and 3, so that's our line of minimum slope.



Here's the graph showing both lines.

The resulting line of maximum slope gives us:

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$m_{max} = 1.17$

$b_{min} = -1.27$

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$m_{max} = 1.17$

$b_{min} = -1.27$

The resulting line of minimum slope gives us:

The resulting line of maximum slope gives us:

$m_{max} = 1.17$

$b_{min} = -1.27$

The resulting line of minimum slope gives us:

$m_{min} = 0.99$

$b_{max} = -0.65$

The resulting line of maximum slope gives us:

$m_{max} = 1.17$

$b_{min} = -1.27$

The resulting line of minimum slope gives us:

$m_{min} = 0.99$

$b_{max} = -0.65$

for an uncertainty in the slope of:

The resulting line of maximum slope gives us:

$m_{max} = 1.17$

$b_{min} = -1.27$

The resulting line of minimum slope gives us:

$m_{min} = 0.99$

$b_{max} = -0.65$

for an uncertainty in the slope of:

$\Delta m = \frac{m_{max} - m_{min}}{2} = \frac{1.17 - 0.99}{2} = 0.09$

The resulting line of maximum slope gives us:

$m_{max} = 1.17$

$b_{min} = -1.27$

The resulting line of minimum slope gives us:

$m_{min} = 0.99$

$b_{max} = -0.65$

for an uncertainty in the slope of:

$\Delta m = \frac{m_{max} - m_{min}}{2} = \frac{1.17 - 0.99}{2} = 0.09$

and an uncertainty in the y-intercept of:

The resulting line of maximum slope gives us:

$m_{max} = 1.17$

$b_{min} = -1.27$

The resulting line of minimum slope gives us:

$m_{min} = 0.99$

$b_{max} = -0.65$

for an uncertainty in the slope of:

$\Delta m = \frac{m_{max} - m_{min}}{2} = \frac{1.17 - 0.99}{2} = 0.09$

and an uncertainty in the y-intercept of:

$\Delta b = \frac{b_{max} - b_{min}}{2} = \frac{-0.65 - (-1.27)}{2} = 0.31$