Linearizing Equations
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December 12, 2014

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- why it's useful
- how to handle uncertainties when linearizing

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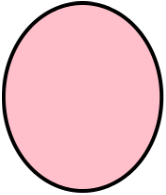

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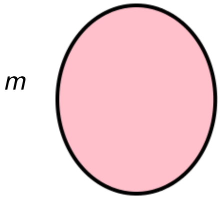

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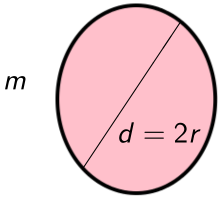

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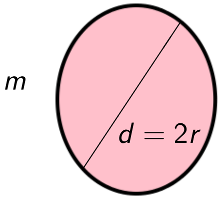

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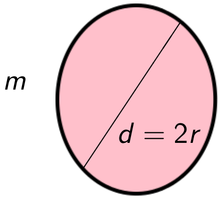

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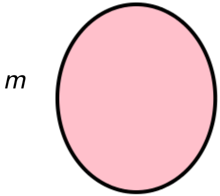
  Thus $\rho = \frac{m}{\frac{4}{3}\pi r^3} = \frac{3m}{4\pi r^3}$

  $= \frac{3\times 0.05}{4\pi(0.7\times 10^{-2})^3}$ (in Si units)

Since radius changes, we could take several values of r:



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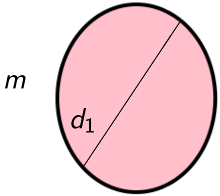

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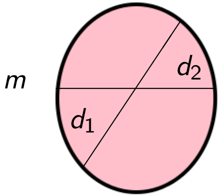

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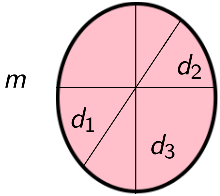

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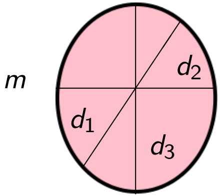

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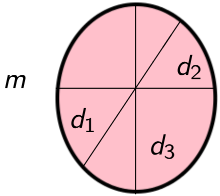

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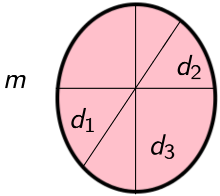

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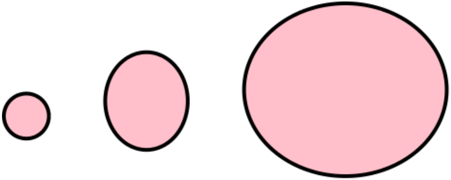
What if we had several marbles of the same material?



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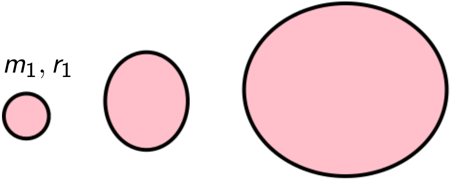

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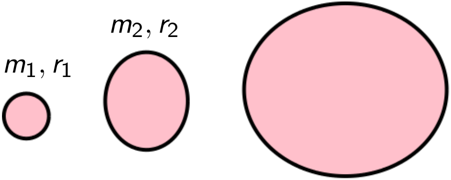

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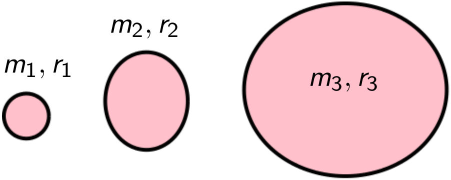

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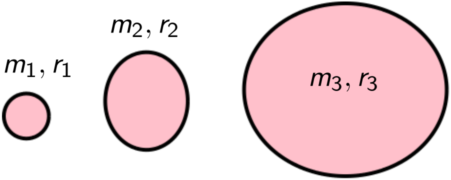

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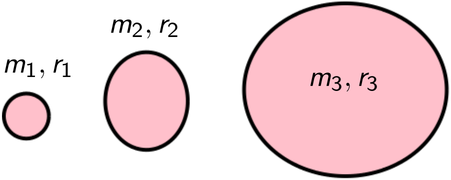

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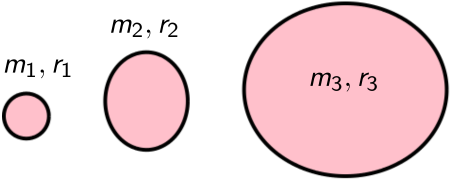

- We could average the m values and average the r values so that
- $\rho = \frac{3\bar{m}}{4\pi\bar{r}^3}$
- This is *not* the best solution. (The small marble will have almost no effect on the calculation.)

Q:Why not just calculate the density for each of the marbles and average the results?

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Is there some way to prevent one data point from having a disproportionate effect on the result?

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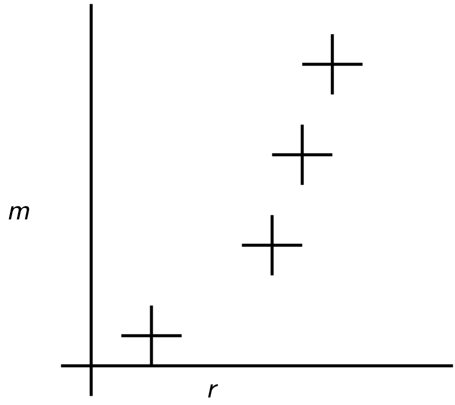
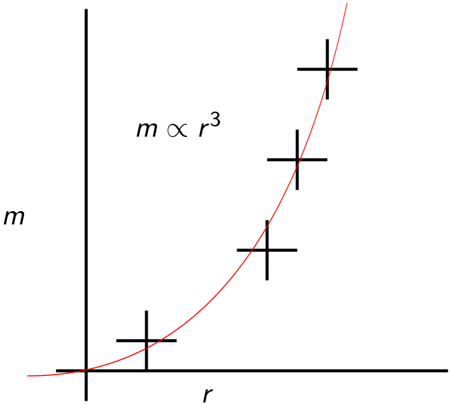
- We can show a mathematical relationship between variables by a graph.



The graph of m vs. r is not linear.



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- Knowing the two constants, M and B, tells us how to calculate one variable given the other.

Linearizing equations is the process of modifying an equation to produce new variables which can be plotted to produce a straight line graph.

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- A fit equation replaces a bunch of data with a few parameters: $M, \Delta M, B$ and ΔB.

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- The slope, $M = \frac{4\pi\rho}{3}$
- The y-intercept, B, should be zero.

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- **Note that after linearization, the original variables, m and r, are gone. The density only depends on the slope.**

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$\ln(m) = 3\ln(r) + \ln\left(\frac{4\pi\rho}{3}\right)$

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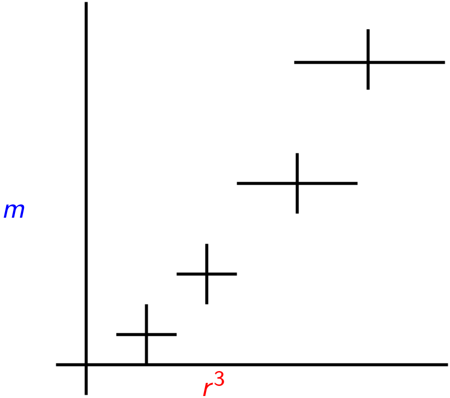
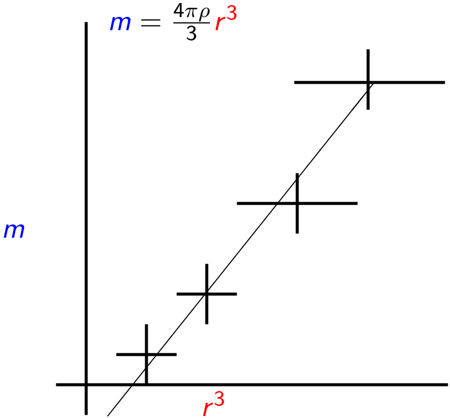
- **Note that after linearization, the original variables, m and r, are gone. The density only depends on the y-intercept.**

Here's how the graphs look for both linearizations.


m
r3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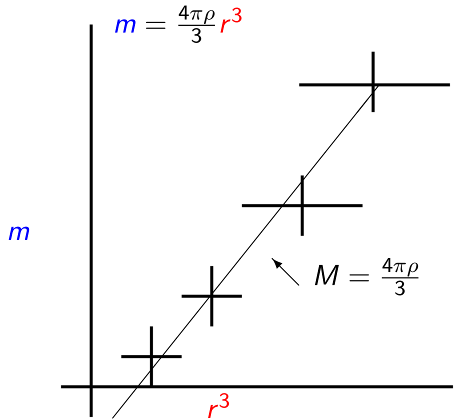
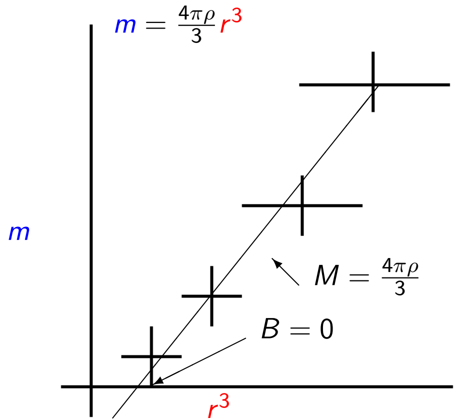
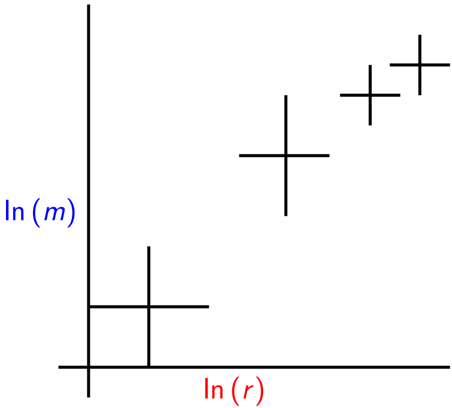


The density can be calculated from the *slope*.



The density can be calculated from the *slope*. The y-intercept *isn't* zero, so there may be a systematic error.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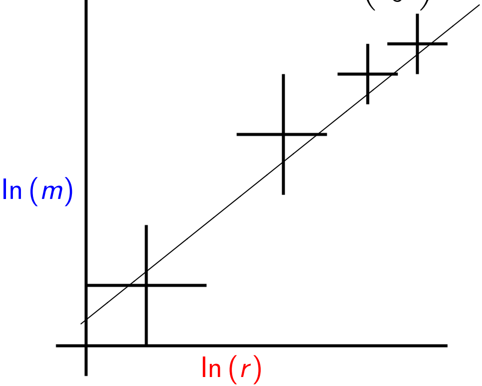
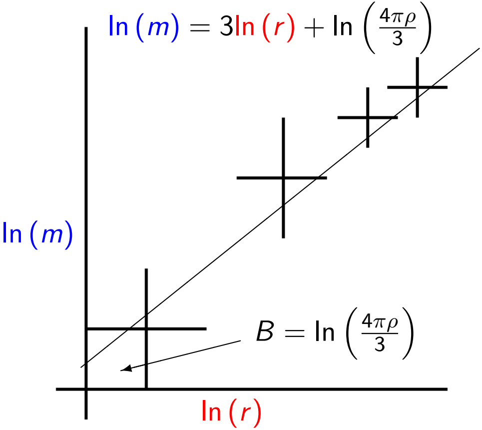
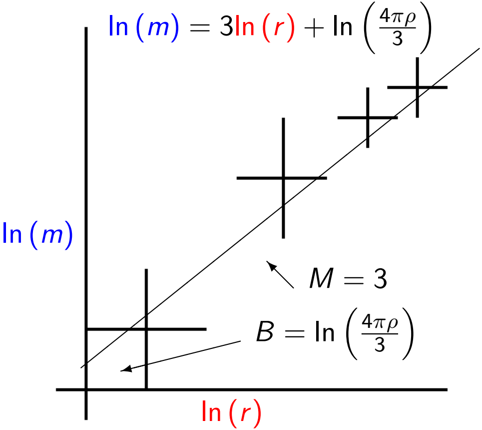




The density can be calculated from the *y-intercept*.



The density can be calculated from the *y-intercept*. The slope *should be* 3, so this can be checked to look for systematic error.

All of the calculations follow the same rules as for any other calculations. There are two specific types of calculations:

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   We'll look at the earlier examples

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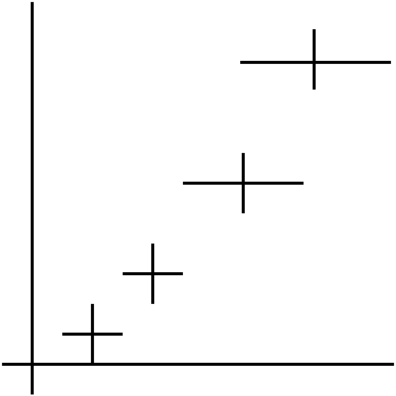
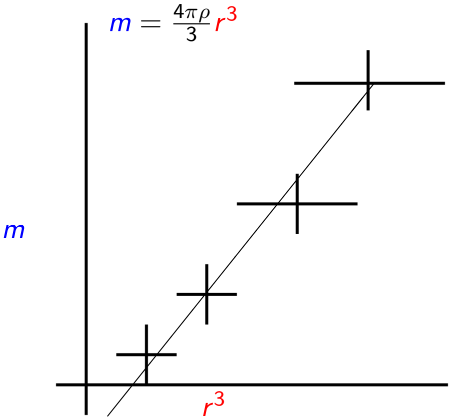
  (If Δr is the same for all of the values it's easiest to see the result.)

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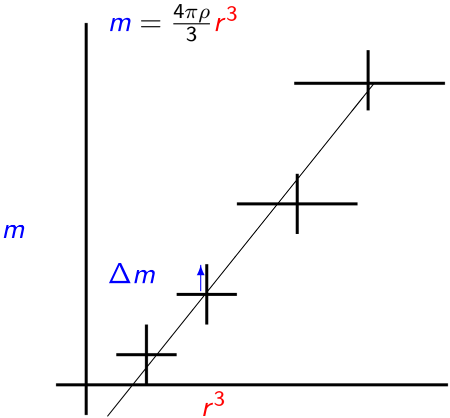
r^3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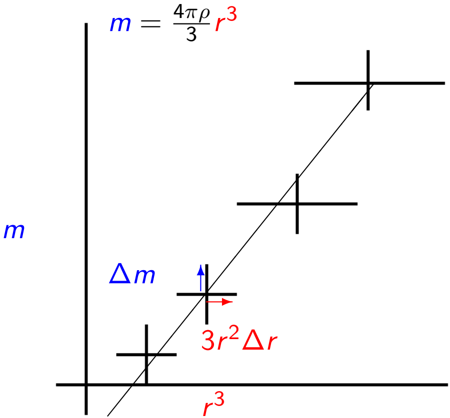
Δm
r³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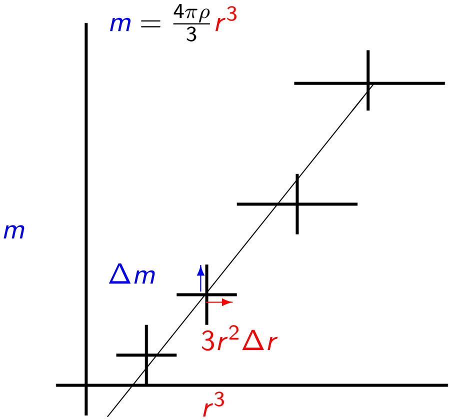
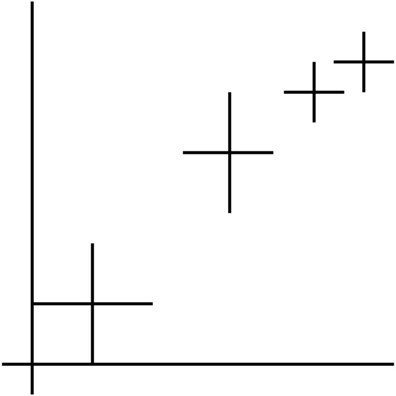
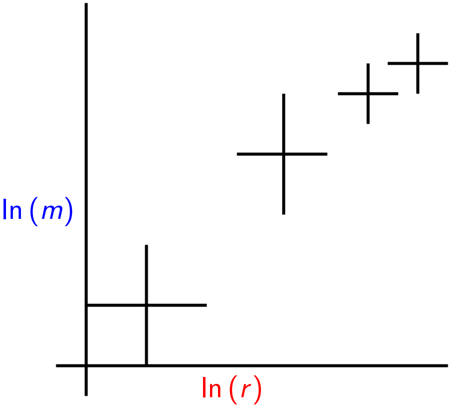
$3r^2\Delta r$
r^3



Error bars get *bigger* as r gets bigger, since $\Delta r^3 \approx 3r^2\Delta r$

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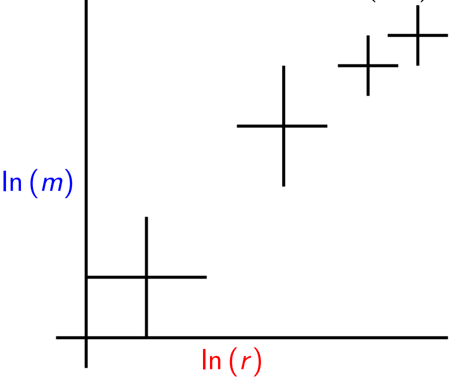

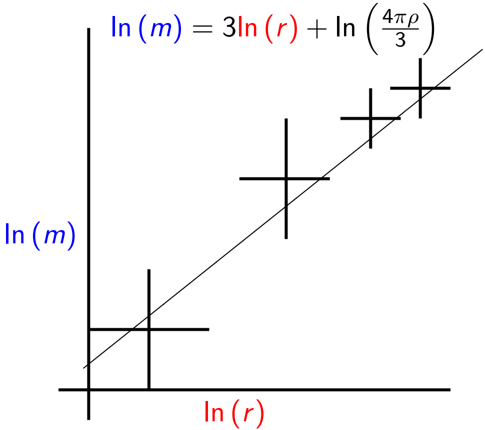
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ln (m)
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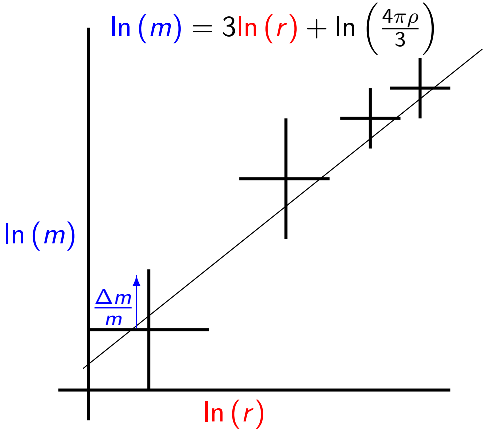
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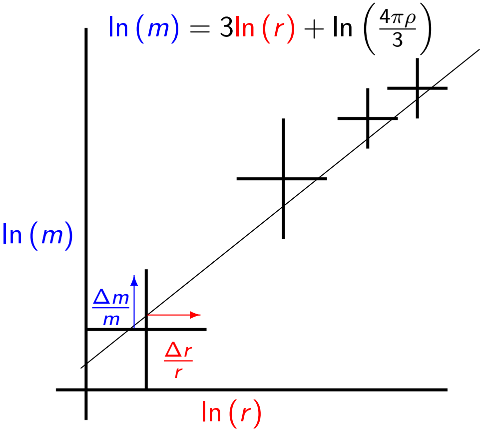
ln (m)
Δm/m
ln (r)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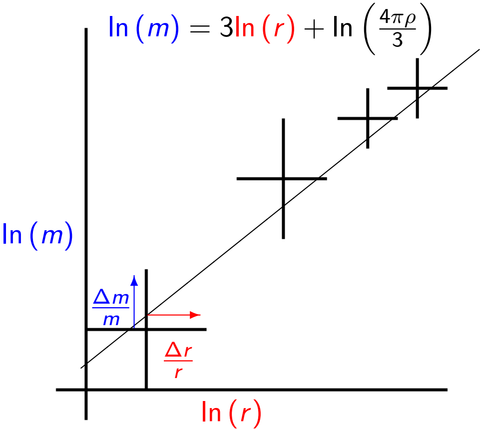

Error bars get *smaller* as m and r get bigger, since $\Delta \ln(r) \approx \frac{\Delta r}{r}$ and $\Delta \ln(m) \approx \frac{\Delta m}{m}$

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1. Linearizing an equation means rearranging it so that you can make it into a straight line graph.
2. A linearized graph is convenient to use, and makes it easier to spot possible errors.
3. After linearization, the only parameters left are the slope, y-intercept, and their uncertainties.
4. There are often several different linearizations for a single equation.