Plotting Curves with a Spreadsheet
Wilfrid Laurier University

Terry Sturtevant

Wilfrid Laurier University

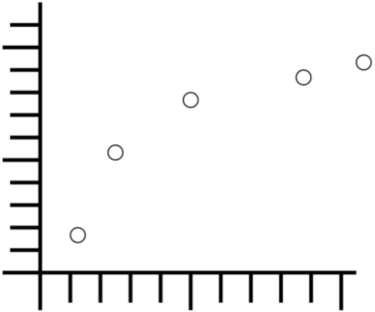
January 30, 2014

Using a spreadsheet to graph data is convenient once you know what you're doing, and even plotting error bars and straight lines is relatively simple.

Using a spreadsheet to graph data is convenient once you know what you're doing, and even plotting error bars and straight lines is relatively simple.
However, once you need to plot data which is not linear, things become a bit more complicated.

Consider the data here:

Consider the data here:



First of all, the graph does not have error bars.

First of all, the graph does not have error bars.
However it it also not smooth.

First of all, the graph does not have error bars.
However it it also not smooth.
Error bars have not been shown so that the smoothness of the curve can be seen easily.

First of all, the graph does not have error bars.

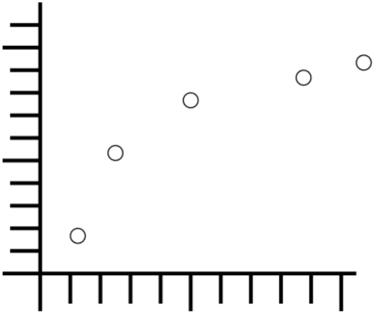
However it it also not smooth.

Error bars have not been shown so that the smoothness of the curve can be seen easily.

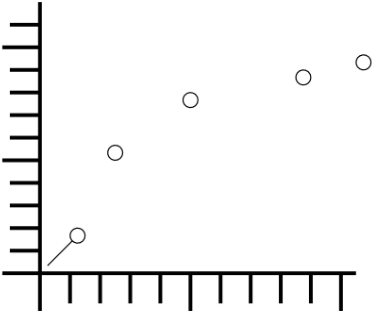
As always, error bars should be included unless you are specifically told to omit them.

You might be tempted to do this:

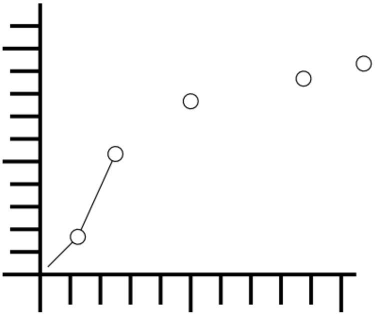
You might be tempted to do this:



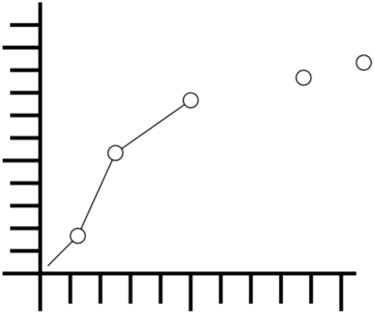
You might be tempted to do this:



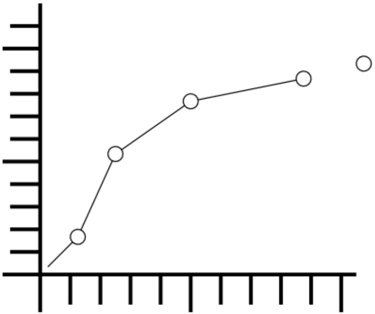
You might be tempted to do this:



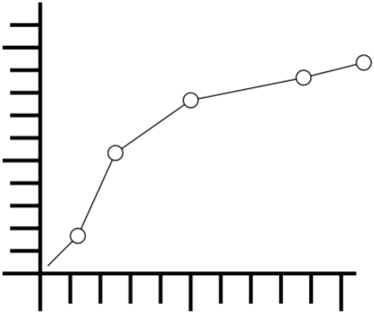
You might be tempted to do this:



You might be tempted to do this:



You might be tempted to do this:



Dot-to-dot drawings do not usually belong in scientific reports!

Dot-to-dot drawings do not usually belong in scientific reports!

Smooth curves are usually more appropriate.

Dot-to-dot drawings do not usually belong in scientific reports!

Smooth curves are usually more appropriate.

The following discussion should help you to use a spreadsheet to produce non–linear plots.

Displaying Curves

Displaying Curves

A *line* can be drawn by simply connecting its endpoints.

Displaying Curves

A *line* can be drawn by simply connecting its endpoints.

A *curve* can be *approximated* by a series of very short line segments between points along the curve.

Displaying Curves

A *line* can be drawn by simply connecting its endpoints.

A *curve* can be *approximated* by a series of very short line segments between points along the curve.

If the points are close enough, the line will look smooth.

If we wish to plot a curve in a spreadsheet, proceed as follows:

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \ldots N$ where

$$x_i = x_{min} + i\left(\frac{x_{max} - x_{min}}{N}\right)$$

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \ldots N$ where

$$x_i = x_{min} + i\left(\frac{x_{max} - x_{min}}{N}\right)$$

so that

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \dots N$ where

$$x_i = x_{min} + i \left(\frac{x_{max} - x_{min}}{N} \right)$$

so that

$$x_0 = x_{min}$$

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \ldots N$ where

$$x_i = x_{min} + i \left(\frac{x_{max} - x_{min}}{N} \right)$$

so that

$$x_0 = x_{min}$$

and

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \ldots N$ where

$$x_i = x_{min} + i\left(\frac{x_{max} - x_{min}}{N}\right)$$

so that

$$x_0 = x_{min}$$

and

$$x_N = x_{max}$$

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \dots N$ where

$$x_i = x_{min} + i\left(\frac{x_{max} - x_{min}}{N}\right)$$

so that

$$x_0 = x_{min}$$

and

$$x_N = x_{max}$$

and for each point

If we wish to plot a curve in a spreadsheet, proceed as follows: Instead of just using two values, x_{min} and x_{max} , for an x–series, create a series of values $x_i, i = 0 \ldots N$ where

$$x_i = x_{min} + i\left(\frac{x_{max} - x_{min}}{N}\right)$$

so that

$$x_0 = x_{min}$$

and

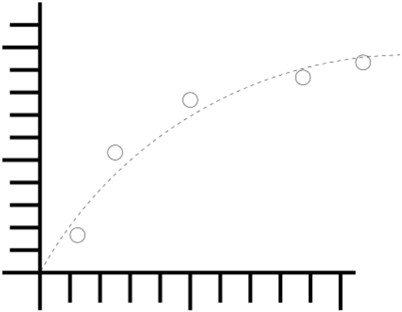
$$x_N = x_{max}$$

and for each point

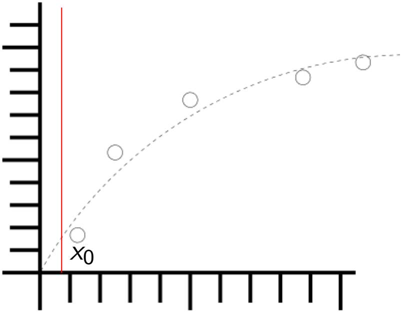
$$y_i = f(x_i)$$

Consider the example, with $N = 5$:

Consider the example, with $N = 5$:

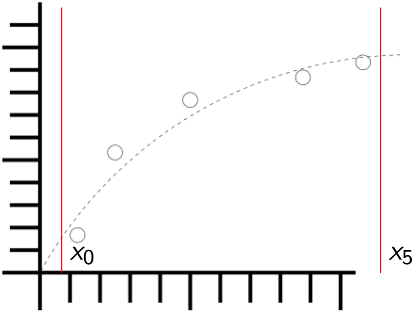


Consider the example, with $N = 5$:



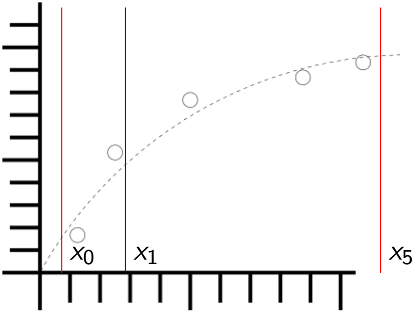
$x_0 = x_{min}$

Consider the example, with $N = 5$:



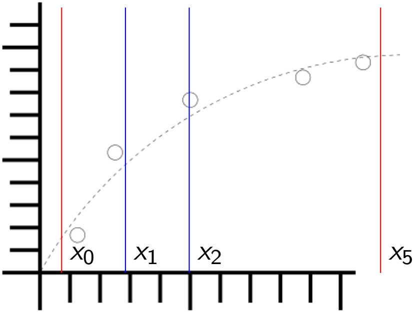
$x_5 = x_{max}$

Consider the example, with $N = 5$:



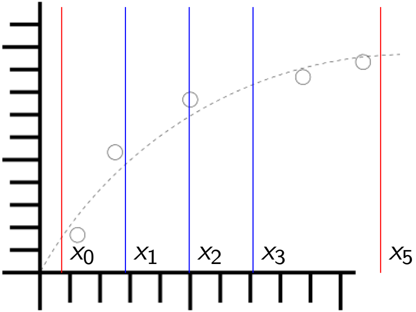
$x_1 = x_0 + 1\left(\frac{x_{max} - x_{min}}{5}\right)$

Consider the example, with $N = 5$:



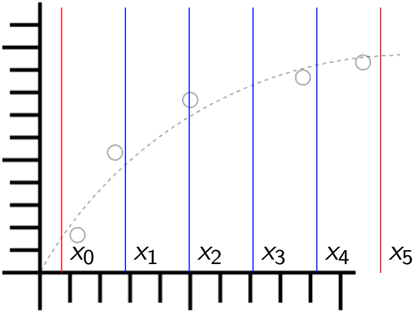
$$x_2 = x_0 + 2\left(\frac{x_{max} - x_{min}}{5}\right)$$

Consider the example, with $N = 5$:



$$x_3 = x_0 + 3\left(\frac{x_{max} - x_{min}}{5}\right)$$

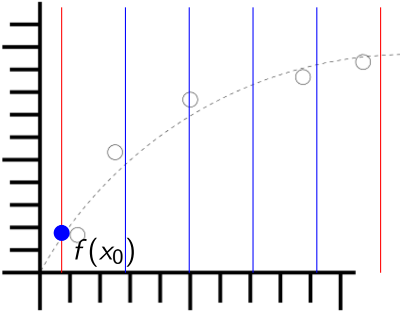
Consider the example, with $N = 5$:



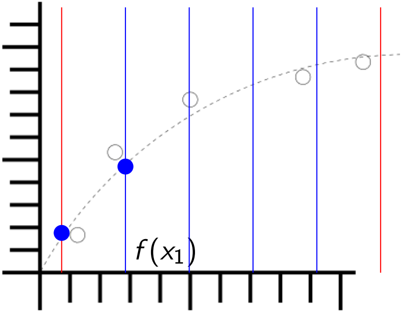
$$x_4 = x_0 + 4\left(\frac{x_{max} - x_{min}}{5}\right)$$

Calculate the points on the curve:

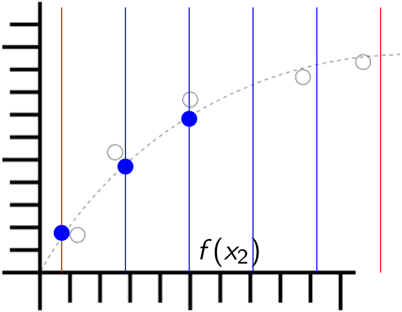
Calculate the points on the curve:



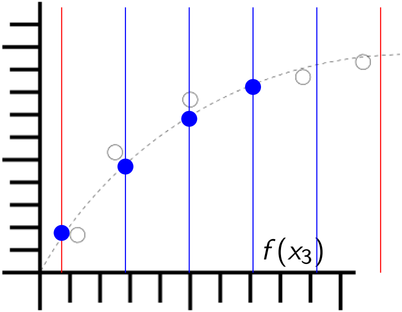
Calculate the points on the curve:



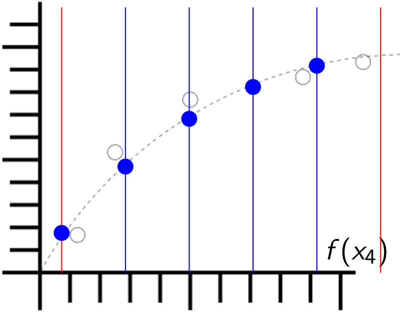
Calculate the points on the curve:



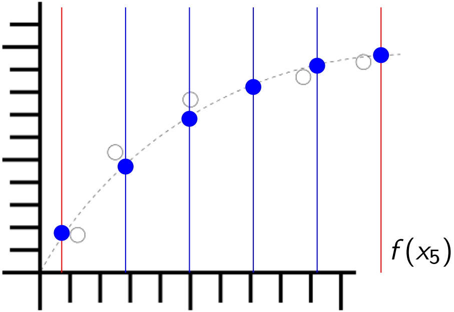
Calculate the points on the curve:



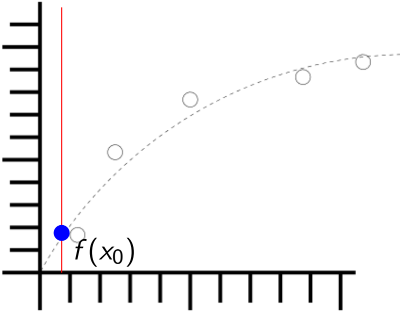
Calculate the points on the curve:



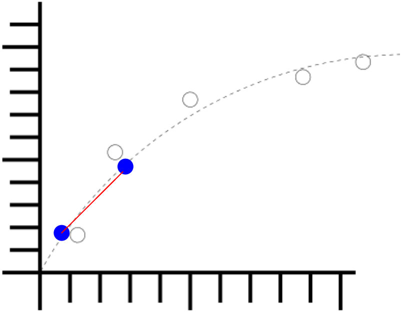
Calculate the points on the curve:



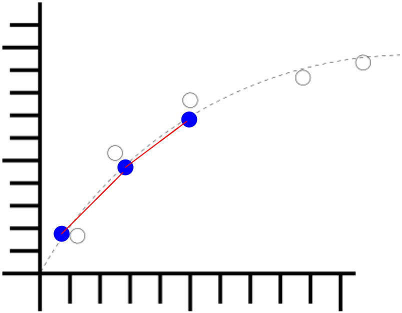
Joint the points on the curve with line segments:



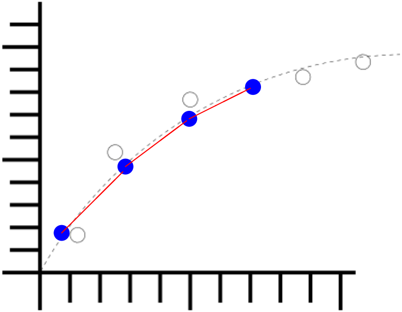
Joint the points on the curve with line segments:



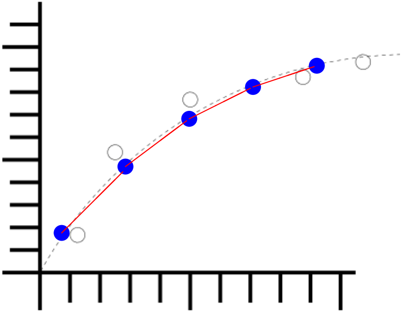
Joint the points on the curve with line segments:



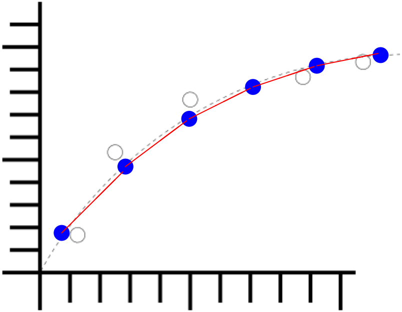
Joint the points on the curve with line segments:



Joint the points on the curve with line segments:

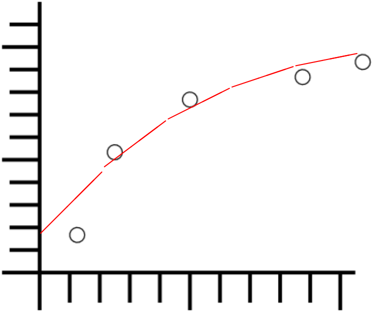


Joint the points on the curve with line segments:

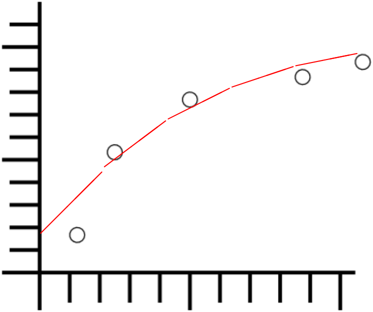


The final product:

The final product:

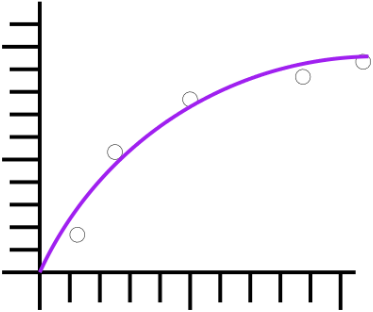


The final product:



Obviously if we used more points we could make it much smoother.

The final product:



Obviously if we used more points we could make it much smoother. (Like this.)

Note that even though the x values for the first and last point are taken from the original data, the y values are not, since the purpose is to find points on the *curve*.

Note that even though the x values for the first and last point are taken from the original data, the y values are not, since the purpose is to find points on the *curve*.
A value of 100 for N should be sufficient to make a smooth curve. If not, make it bigger. You can try making it smaller if that many points aren't needed.

Note that even though the x values for the first and last point are taken from the original data, the y values are not, since the purpose is to find points on the *curve*.
A value of 100 for N should be sufficient to make a smooth curve. If not, make it bigger. You can try making it smaller if that many points aren't needed.

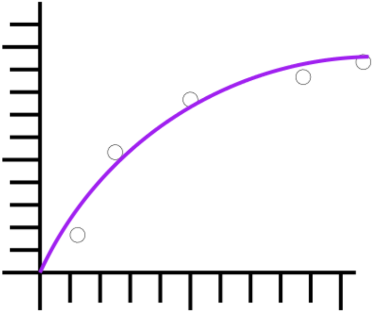
1. For each of the x_i, calculate the corresponding y_i value from the curve equation and add this series to the graph.

Note that even though the x values for the first and last point are taken from the original data, the y values are not, since the purpose is to find points on the *curve*.
A value of 100 for N should be sufficient to make a smooth curve. If not, make it bigger. You can try making it smaller if that many points aren't needed.

1. For each of the x_i, calculate the corresponding y_i value from the curve equation and add this series to the graph.
2. Remove the markers and add lines for this series. All points in this series will thereby be joined with line segments. You should be able to produce a graph such as the following. (Error bars have been left off for simplicity. They can be produced in the usual way.)

Note that in the graph shown, none of the data points actually fall on the curve. This is often the case.

Note that in the graph shown, none of the data points actually fall on the curve. This is often the case.



Piecewise Defined Functions

Piecewise Defined Functions

In some cases, a curve must be made from data which must be fit to different equations in different regions.

Piecewise Defined Functions

In some cases, a curve must be made from data which must be fit to different equations in different regions.

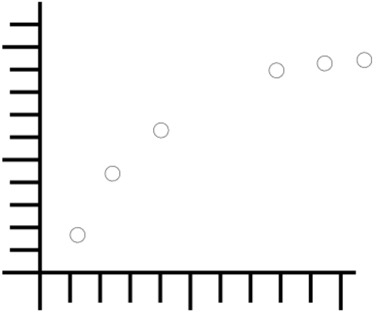
In this case the trick comes in trying to join the two fit equations smoothly.

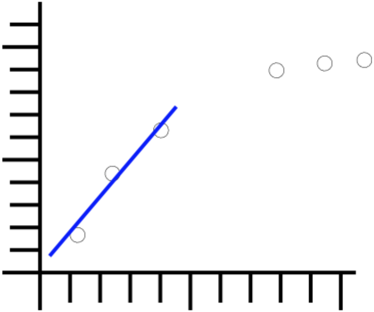
Piecewise Defined Functions

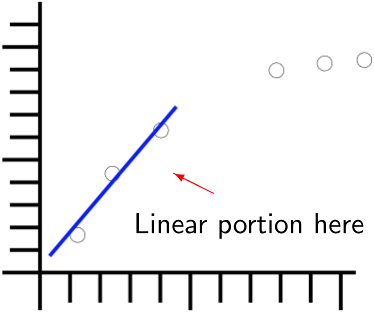
In some cases, a curve must be made from data which must be fit to different equations in different regions.

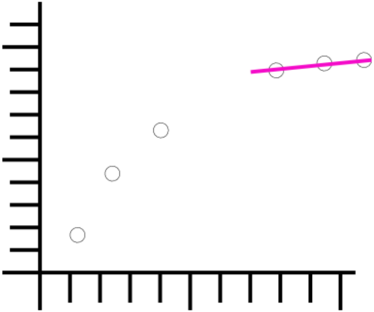
In this case the trick comes in trying to join the two fit equations smoothly.

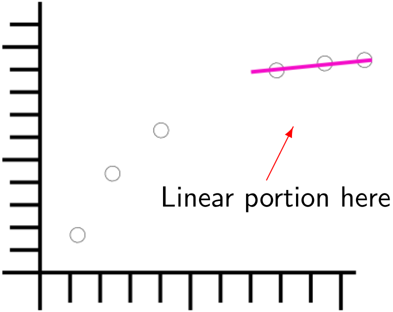
Here's an example where two *lines* have to join.

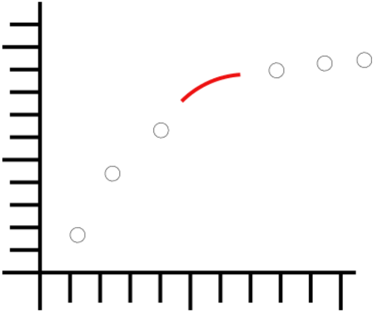


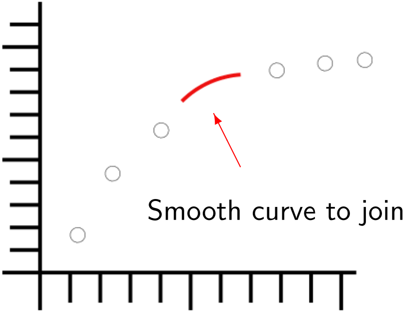


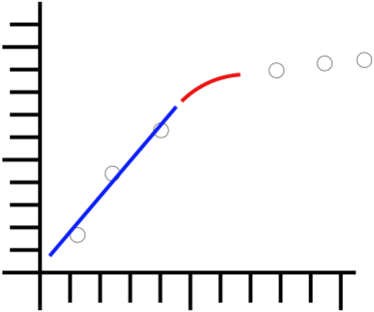

Linear portion here

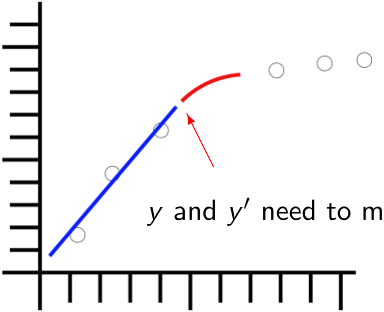



Linear portion here

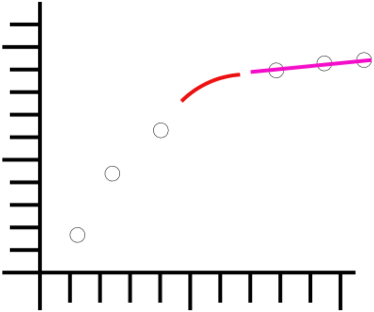


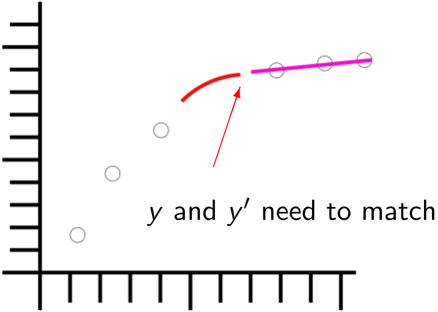

Smooth curve to join

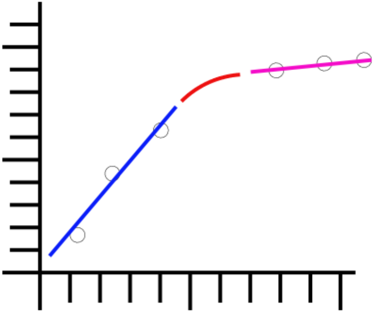




y and y' need to match




y and y′ need to match



In this case,

In this case,

$$f(x) =$$

In this case,

$$f(x) = \begin{cases} F_1(x), & \text{if } x \leq x_1; \\ & \end{cases}$$

In this case,

$$f(x) = \begin{cases} F_1(x), & \text{if } x \leq x_1; \\ F_2(x), & \text{if } x \geq x_2. \end{cases}$$

In this case,

$$f(x) = \begin{cases} F_1(x), & \text{if } x \leq x_1; \\ F_2(x), & \text{if } x \geq x_2. \end{cases}$$

where $x_2 > x_1$.

To make the function $f(x)$ *continuous*, we require that

To make the function $f(x)$ *continuous*, we require that

$$f(x_1) = F_1(x_1)$$

To make the function $f(x)$ *continuous*, we require that

$$f(x_1) = F_1(x_1)$$

and

$$f(x_2) = F_2(x_2)$$

To produce a *smooth* fit, then in addition we require that

To produce a *smooth* fit, then in addition we require that

$$f'(x_1) = {F_1}'(x_1)$$

To produce a *smooth* fit, then in addition we require that

$$f'(x_1) = {F_1}'(x_1)$$

and

$$f'(x_2) = {F_2}'(x_2)$$

For two straight lines, this becomes:

For two straight lines, this becomes:

$$f(x_1) = m_1 x_1 + b_1$$

For two straight lines, this becomes:

$$f(x_1) = m_1 x_1 + b_1$$

$$f(x_2) = m_2 x_2 + b_2$$

For two straight lines, this becomes:

$$f(x_1) = m_1 x_1 + b_1$$

$$f(x_2) = m_2 x_2 + b_2$$

$$f'(x_1) = m_1$$

For two straight lines, this becomes:

$$f(x_1) = m_1 x_1 + b_1$$

$$f(x_2) = m_2 x_2 + b_2$$

$$f'(x_1) = m_1$$

$$f'(x_2) = m_2$$

These 4 equations can be solved exactly by a polynomial in 4 unknowns,

These 4 equations can be solved exactly by a polynomial in 4 unknowns, i.e. an equation of the form

These 4 equations can be solved exactly by a polynomial in 4 unknowns, i.e. an equation of the form

$$f(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$$

These 4 equations can be solved exactly by a polynomial in 4 unknowns, i.e. an equation of the form

$$f(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$$

and therefore

These 4 equations can be solved exactly by a polynomial in 4 unknowns, i.e. an equation of the form

$$f(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$$

and therefore

$$f'(x) = 3\alpha x^2 + 2\beta x + \gamma$$

To solve this, we need to set up a system of equations as follows:

To solve this, we need to set up a system of equations as follows: If we let

To solve this, we need to set up a system of equations as follows: If we let

$$\mathcal{A} = \left(\quad \right)$$

To solve this, we need to set up a system of equations as follows: If we let

$$\mathcal{A} = \begin{pmatrix} {x_1}^3 & {x_1}^2 & x_1 & 1 \\ & & & \\ & & & \end{pmatrix}$$

To solve this, we need to set up a system of equations as follows: If we let

$$\mathcal{A} = \begin{pmatrix} {x_1}^3 & {x_1}^2 & x_1 & 1 \\ {x_2}^3 & {x_2}^2 & x_2 & 1 \\ & & & \\ & & & \end{pmatrix}$$

To solve this, we need to set up a system of equations as follows: If we let

$$\mathcal{A} = \begin{pmatrix} {x_1}^3 & {x_1}^2 & x_1 & 1 \\ {x_2}^3 & {x_2}^2 & x_2 & 1 \\ 3{x_1}^2 & 2x_1 & 1 & 0 \\ & & & \end{pmatrix}$$

To solve this, we need to set up a system of equations as follows: If we let

$$\mathcal{A} = \begin{pmatrix} {x_1}^3 & {x_1}^2 & x_1 & 1 \\ {x_2}^3 & {x_2}^2 & x_2 & 1 \\ 3{x_1}^2 & 2x_1 & 1 & 0 \\ 3{x_2}^2 & 2x_2 & 1 & 0 \end{pmatrix}$$

and

and

$$\mathcal{B} = \left(\qquad \right)$$

and

$$\mathcal{B} = \begin{pmatrix} F_1(x_1) \\ \\ \\ \end{pmatrix}$$

and

$$\mathcal{B} = \begin{pmatrix} F_1(x_1) \\ F_2(x_2) \\ \\ \\ \end{pmatrix}$$

and

$$\mathcal{B} = \begin{pmatrix} F_1(x_1) \\ F_2(x_2) \\ {F_1}'(x_1) \\ \\ \end{pmatrix}$$

and

$$\mathcal{B} = \begin{pmatrix} F_1(x_1) \\ F_2(x_2) \\ {F_1}'(x_1) \\ {F_2}'(x_2) \end{pmatrix}$$

Then it should be clear that

Then it should be clear that

$$\mathcal{A}X = \mathcal{B}$$

Then it should be clear that

$$\mathcal{A}X = \mathcal{B}$$

or

$$X = \mathcal{A}^{-1}\mathcal{B}$$

where

where

$$X = \left(\right)$$

where

$$X = \begin{pmatrix} \alpha \\ \\ \\ \end{pmatrix}$$

where

$$X = \begin{pmatrix} \alpha \\ \beta \\ \\ \\ \end{pmatrix}$$

where

$$X = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \\ \end{pmatrix}$$

where

$$X = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

For two straight lines this becomes

For two straight lines this becomes

$$\mathcal{B} = \left(\qquad \right)$$

For two straight lines this becomes

$$\mathcal{B} = \begin{pmatrix} m_1 x_1 + b_1 \\ \\ \\ \end{pmatrix}$$

For two straight lines this becomes

$$\mathcal{B} = \begin{pmatrix} m_1x_1 + b_1 \\ m_2x_2 + b_2 \\ \\ \\ \end{pmatrix}$$

For two straight lines this becomes

$$\mathcal{B} = \begin{pmatrix} m_1 x_1 + b_1 \\ m_2 x_2 + b_2 \\ m_1 \\ \\ \end{pmatrix}$$

For two straight lines this becomes

$$\mathcal{B} = \begin{pmatrix} m_1 x_1 + b_1 \\ m_2 x_2 + b_2 \\ m_1 \\ m_2 \end{pmatrix}$$

($F_1{}'(x_1)$ can be determined by using x_1 and the fit point immediately to its left, and $F_2{}'(x_2)$ can be determined by using x_2 and the fit point immediately to its right.)

($F_1{}'(x_1)$ can be determined by using x_1 and the fit point immediately to its left, and $F_2{}'(x_2)$ can be determined by using x_2 and the fit point immediately to its right.)
This system can then be solved using the **matrix invert** and **multiply** features of a spreadsheet.

Note: This will produce a *smooth* graph; whether it produces an *accurate* graph remains to be seen, (although smooth transitions are more common than others).